SME MINING REFERENCE HANDBOOK

2ND EDITION

EDITED BY
HEATHER N. DOUGHERTY, P.E. & ANDREW P. SCHISSLER, P.E.

SME

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PUBLISHED BY THE
SOCIETY FOR MINING, METALLURGY & EXPLORATION

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Foreword

As a profession, mining engineering is often viewed as being distinctly unique from other fields of engineering. Paramount among the factors driving this opinion are the multidisciplinary nature of the technical considerations influencing the design and operation of mines and processing facilities, the uncertainty inherent to mineral resources and the geomaterials in which these facilities are constructed, and the economic risk that underlies these investments. Furthermore, all aspects of resource development are governed by specific nonnegotiable values that establish corporate responsibilities toward employee safety and health, environmental stewardship, and community engagement that extend well beyond the law or any regulatory standard. The combination of these realities mandate that professionals in the mining industry have broad-based skill sets, as well as a technical understanding of how to solve complex problems and determine solutions vital to the efficient and safe performance of any activity.

Since the first modern technical mining handbook was published in the United States more than 100 years ago, mining professionals have extensively relied on information obtained from reference guides as an aid in solving these types of problems, as well as other calculations relevant to performing specific tasks and facilitating the basis for making decisions. Over time, changes in mining systems, equipment and processes, economics, and the regulatory environment necessitate the inclusion of new information and revision of established topics. The second edition of the *SME Mining Reference Handbook* is exactly that, and it represents a concise, well-organized reference guide of important information and data for everyday use by engineers and other professionals engaged in mining, exploration, mineral processing, and environmental compliance and reclamation.

As editors of the second edition, Heather N. Dougherty and Andrew P. Schissler have done an exceptional job building on the strengths and success of the previous edition through the introduction of new and updated materials and enhancing the presentation of useful data within tables and figures. Both of these individuals are consummate industry professionals, highly respected for their expertise and breadth of knowledge that extend from engineering and mine operations to economic valuation and project management. Heather and Andy are both registered Professional Engineers, hold advanced academic degrees in mining engineering as well as business administration, and are extremely active in professional service, including SME.

I first met Heather because of her position as a Research Engineer at the Centers for Disease Control and Prevention NIOSH Pittsburgh Mining Research Division and through her exemplary service to numerous SME committees. She has extensive industry experience in underground coal, where she has worked in capacities related to operations and engineering, which has contributed to her success as a researcher at NIOSH. Heather is a recognized expert in mine ventilation, environmental air monitoring, and ground control.

I've had the distinct pleasure of knowing Andy for nearly 15 years, dating back to when he worked as the engineering manager of several potash mines in New Mexico. I am delighted to say that over the years, we have become friends and colleagues. Among his many professional activities, Andy currently serves as an Adjunct Professor at the Colorado School of Mines, where it has been gratifying to watch him interact with students in the courses he teaches in mine plant design, explosives engineering, and mine design. Andy has more than 46 years of global mining experience, during which he has held numerous positions that span a wide range of technical and managerial capacities in nearly every industry sector and commodity.

One of the strengths of this edition of the *Mining Reference Handbook* is the tremendous wealth of talent that has contributed to this publication. These contributors include authors and subject matter experts, as well as technical reviewers. When coupled with the long list of contributors from the first edition, it literally represents a who's who of industry experts; each prominent in one or more specific area(s) of

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technical expertise. The culmination of these efforts is a reference guide that is relevant, well-written, and comprehensive in subject matter. The professional layout, appearance, and detail of the handbook clearly reflect the exceptional skills and contributions of Jane Olivier, SME's Manager of Book Publishing, and her truly talented team.

As with the previous edition, the second edition of the *SME Mining Reference Handbook* is intended to serve as the go-to resource for working professionals throughout the mining industry. I have little doubt that this edition of the handbook will become an industry standard as a reference guide for engineers, scientists, and students today and into the foreseeable future.

Hugh B. Miller
2019 SME President
December 2019

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Preface to the Second Edition

The first edition of the *SME Mining Reference Handbook*, published in 2002, was the first concise reference published in the mining field and quickly became the industry standard. It is placed on every mining engineer's bookshelf, if not open on his or her desk, with worn pages, tabs to find the most used equations, and personal notes. It has been the unequaled single reference and is the first go-to source for countless engineers, whether in the mining, environmental, geological, or processing fields.

That edition is the backbone of this second edition. Raymond L. Lowrie, P.E. and his group of authors, reviewers, and editors worked to ensure that it had the breadth to last. As with everything, time passes, conditions change, and knowledge and technology advance. This second edition updates the text, tables, and figures throughout the book, as well as adds a new chapter on mining methods.

A compact reference, this second edition remains a distillation of key technical information from the mining literature. This handbook is intended as a problem-solving aid for mining engineers, mineral processing engineers, and environmental engineers.

We have worked closely with subject matter experts to again make a concise comprehensive reference for professionals, whether they are in the field, in their vehicles, in their offices, or considering the two examinations for professional licensure: the Fundamentals of Engineering Examination and the Professional Engineers Examination for Mining/Mineral Processing.

The *SME Mining Reference Handbook* will continue to be used by engineers for their daily work, but also for the advancement of their careers. It now becomes the single-source reference for the Professional Licensure of Mining and Mineral Processing Engineers.

As with Lowrie's first effort, we feel as if this is just that, an effort in the progression on advancement in technology and experience in the mining field. We hope that when off-earth mining or other advancements become a reality, the most recent edition of the *SME Mining Reference Handbook* will be packed and ready to accompany you to destinations unknown.

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Preface to the First Edition

Engineers in the mining industry often must solve problems while in the field at prospects, projects, or places far from any personal bookshelf, company office, or public or private library. And it isn't always feasible to bring along the voluminous authoritative books on mining topics so familiar to the profession. This handbook, then, is designed to fill the technical reference gap for the mobile professional who is away from the normal workplace with its comprehensive store of technical information and resources. It is a distillation of key technical information from the mining literature.

To keep this handbook a reasonable and portable size, the volume editor and all chapter editors had to strictly budget the number of pages allocated to particular subjects. We assumed that the reader is already knowledgeable about the topics and may just need a reminder "on the fly." For this reason, many of the ideas, data, graphs, tables, equations, constants, and rules of thumb are presented with little if any explanation. Detailed explanation or elaboration can be found in the original source.

The volume editor and all chapter editors are currently licensed or retired registered professional engineers in one or more states. Although the intended audience for this handbook is primarily mining/mineral engineers who work for mining companies and other mining-oriented firms around the world, we hope that academia, students, and state and federal government agencies will also find it useful. This is a first effort. We anticipate that technological change, along with experience in using the handbook in the field, will allow improvements in future editions.

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About the Editors

Heather N. Dougherty, P.E. has more than 20 years of experience in the mining field. She has held positions in the coal industry ranging from underground miner to Senior Engineer and is currently a Research Engineer with the Pittsburgh Mining Research Division. Her research includes underground mine ventilation, monitoring, ground control, and gas well/mine interaction.

Dougherty holds bachelor's and master's of science degrees and a doctorate in mining engineering, studying at both West Virginia University and Virginia Polytechnic Institute and State University. She also holds an MBA from West Virginia University and is a registered Professional Engineer in West Virginia. Dougherty also spends time volunteering for SME in many capacities, but primarily for the Professional Engineers Exam Committee and Underground Ventilation Committee. She also volunteers her time and expertise with the Pittsburgh Coal Mining Institute of America.

Andrew P. Schissler, P.E. is a mining engineer and manager with 45 years of experience in the industry. He has held positions from miner to executive vice president and interim chief operating officer. He has performed on-site exploration, valuation, mine development, maintenance, operations, and reclamation at mines from southern Argentina to northern Greenland, and from Western Australia to Eastern Europe. Schissler has also consulted to oil and gas companies.

Schissler holds a bachelor's of science and doctorate in mining and earth systems engineering from the Colorado School of Mines and an MBA from Regis University. In academia, Schissler held positions from Assistant Professor (tenure track) at Penn State to Adjunct Professor at the Colorado School of Mines, teaching and researching most subject matter of mining engineering.

Schissler is a founding registered member of SME, licensed in two states as a Professional Engineer, and is a Certified Mine Foreman in three states.

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CHAPTER 1

Conversion Factors, Constants, and Symbols

Raymond L. Lowrie, P.E.

INTERNATIONAL SYSTEM OF UNITS

The International System of Units, abbreviated SI, is the modern metric system. Table 1.1 lists SI conversion factors. Although many engineers prefer SI units, English units remain in widespread use in the United States. Table 1.2 gives SI prefixes and Table 1.3 shows the Greek alphabet.

TABLE 1.1 SI conversion factors (factors in **boldface** are exact)

To Convert From	To	Multiply By	
Acceleration			
acceleration of free fall, standard (g_n)	meter per second squared (m/s^2)	**9.806 65**	**E+00**
foot per second squared (ft/s^2)	meter per second squared (m/s^2)	**3.048**	**E–01**
gal (Gal)	meter per second squared (m/s^2)	**1.0**	**E–02**
inch per second squared ($in./s^2$)	meter per second squared (m/s^2)	**2.54**	**E–02**
Angle			
degree (°)	radian (rad)	1.745 329	E–02
gon (also called grade) (gon)	radian (rad)	1.570 796	E–02
gon (also called grade) (gon)	degree (°)	**9.0**	**E–01**
mil	radian (rad)	9.817 477	E–04
mil	degree (°)	**5.625**	**E–02**
minute (')	radian (rad)	2.908 882	E–04
revolution (r)	radian (rad)	6.283 185	E+00
second (")	radian (rad)	4.848 137	E–06
Area and Second Moment of Area			
acre (based on U.S. survey foot)[1]	square meter (m^2)	4.046 873	E+03
area (a)	square meter (m^2)	**1.0**	**E+02**
barn (b)	square meter (m^2)	**1.0**	**E–28**
circular mil	square meter (m^2)	5.067 075	E–10
circular mil	square millimeter (mm^2)	5.067 075	E–04
foot to the fourth power (ft^4)[2]	meter to the fourth power (m^4)	8.630 975	E–03
hectare (ha)	square meter (m^2)	**1.0**	**E+04**
inch to the fourth power ($in.^4$)[2]	meter to the fourth power (m^4)	4.162 314	E–07
square foot (ft^2)	square meter (m^2)	**9.290 304**	**E–02**
square inch ($in.^2$)	square meter (m^2)	**6.4516**	**E–04**
square inch ($in.^2$)	square centimeter (cm^2)	**6.4516**	**E+00**
square mile (mi^2)	square meter (m^2)	2.589 988	E+06
square mile (mi^2)	square kilometer (km^2)	2.589 988	E+00
square mile (based on U.S. survey foot) (mi^2)[1]	square meter (m^2)	2.589 998	E+06
square mile (based on U.S. survey foot) (mi^2)[1]	square kilometer (km^2)	2.589 998	E+00
square yard (yd^2)	square meter (m^2)	8.361 274	E–01
Capacity (see Volume)			
Density (i.e., Mass Density—see Mass Divided by Volume)			

(continues)

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TABLE 1.1 SI conversion factors (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
Electricity and Magnetism			
abampere	ampere (A)	**1.0**	**E+01**
abcoulomb	coulomb (C)	**1.0**	**E+01**
abfarad	farad (F)	**1.0**	**E+09**
abhenry	henry (H)	**1.0**	**E−09**
abmho	siemens (S)	**1.0**	**E+09**
abohm	ohm (Ω)	**1.0**	**E−09**
abvolt	volt (V)	**1.0**	**E−08**
ampere hour (A·h)	coulomb (C)	**3.6**	**E+03**
biot (Bi)	ampere (A)	**1.0**	**E+01**
EMU of capacitance (abfarad)	farad (F)	**1.0**	**E+09**
EMU of current (abampere)	ampere (A)	**1.0**	**E+01**
EMU of electric potential (abvolt)	volt (V)	**1.0**	**E−08**
EMU of inductance (abhenry)	henry (H)	**1.0**	**E−09**
EMU of resistance (abohm)	ohm (Ω)	**1.0**	**E−09**
ESU of capacitance (statfarad)	farad (F)	1.112 650	E−12
ESU of current (statampere)	ampere (A)	3.335 641	E−10
ESU of electric potential (statvolt)	volt (V)	2.997 925	E+02
ESU of inductance (stathenry)	henry (H)	8.987 552	E+11
ESU of resistance (statohm)	ohm (Ω)	8.987 552	E+11
faraday (based on carbon 12)	coulomb (C)	9.648 531	E+04
franklin (Fr)	coulomb (C)	3.335 641	E−10
gamma (γ)	tesla (T)	**1.0**	**E−09**
gauss (Gs, G)	tesla (T)	**1.0**	**E−04**
gilbert (Gi)	ampere (A)	7.957 747	E−01
maxwell (Mx)	weber (Wb)	**1.0**	**E−08**
mho	siemens (S)	**1.0**	**E+00**
oersted (Oe)	ampere per meter (A/m)	7.957 747	E+01
ohm centimeter (Ω·m)	ohm meter (Ω·m)	**1.0**	**E−02**
ohm circular-mil per foot	ohm meter (Ω·m)	1.662 426	E−09
ohm circular-mil per foot	ohm square millimeter per meter ($\Omega \cdot mm^2/m$)	1.662 426	E−03
statampere	ampere (A)	3.335 641	E−10
statcoulomb	coulomb (C)	3.335 641	E−10
statfarad	farad (F)	1.112 650	E−12
stathenry	henry (H)	8.987 552	E+11
statmho	siemens (S)	1.112 650	E−12
statohm	ohm (Ω)	8.987 552	E+11
statvolt	volt (V)	2.997 925	E+02
unit pole	weber (Wb)	1.256 637	E−07
Energy (includes Work)			
British thermal $unit_{IT}$ (Btu_{IT})[3]	joule (J)	1.055 056	E+03
British thermal $unit_{th}$ (Btu_{th})[3]	joule (J)	1.054 350	E+03
British thermal unit (mean) (Btu)	joule (J)	1.055 87	E+03

(continues)

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TABLE 1.1 SI conversion factors (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
British thermal unit (39°F) (Btu)	joule (J)	1.059 67	E+03
British thermal unit (59°F) (Btu)	joule (J)	1.054 80	E+03
British thermal unit (60°F) (Btu)	joule (J)	1.054 68	E+03
calorie$_{IT}$ (cal$_{IT}$)[3]	joule (J)	**4.1868**	**E+00**
calorie$_{th}$ (cal$_{th}$)[3]	joule (J)	**4.184**	**E+00**
calorie (mean) (cal)	joule (J)	4.190 02	E+00
calorie (15°C) (cal$_{15}$)	joule (J)	4.185 80	E+00
calorie (20°C) (cal$_{20}$)	joule (J)	4.181 90	E+00
calorie$_{th}$, kilogram (nutrition)[4]	joule (J)	**4.1868**	**E+03**
calorie$_{th}$, kilogram (nutrition)[4]	joule (J)	**4.184**	**E+03**
calorie (mean), kilogram (nutrition)[4]	joule (J)	4.190 02	E+03
electronvolt (eV)	joule (J)	1.602 177	E–19
erg (erg)	joule (J)	**1.0**	**E–07**
foot poundal	joule (J)	4.214 011	E–02
foot pound-force (ft·lbf)	joule (J)	1.355 818	E+00
kilocalorie$_{IT}$ (kcal$_{IT}$)	joule (J)	**4.1868**	**E+03**
kilocalorie$_{th}$ (kcal$_{th}$)	joule (J)	**4.184**	**E+03**
kilocalorie (mean) (kcal)	joule (J)	4.190 02	E+03
kilowatt hour (kW·h)	joule (J)	**3.6**	**E+06**
kilowatt hour (kW·h)	megajoule (MJ)	**3.6**	**E+00**
quad (10^{15} Btu$_{IT}$)[3]	joule (J)	1.055 056	E+18
therm (EC)[5]	joule (J)	**1.055 06**	**E+08**
therm (U.S.)[5]	joule (J)	**1.054 804**	**E+08**
ton of TNT (energy equivalent)[6]	joule (J)	**4.184**	**E+09**
watt hour (W·h)	joule (J)	**3.6**	**E+03**
watt second (W·s)	joule (J)	**1.0**	**E+00**
Energy Divided by Area Time			
erg per square centimeter second [erg/(cm^2·s)]	watt per square meter (W/m^2)	**1.0**	**E–03**
watt per square centimeter (W/cm^2)	watt per square meter (W/m^2)	**1.0**	**E+04**
watt per square inch (W/in.^{2})	watt per square meter (W/m^2)	1.550 003	E+03
Flow (see Mass Divided by Time or Volume Divided by Time)			
Force			
dyne (dyn)	newton (N)	**1.0**	**E–05**
kilogram-force (kgf)	newton (N)	**9.806 65**	**E+00**
kilopond (kilogram-force) (kp)	newton (N)	**9.806 65**	**E+00**
kip (1 kip = 1,000 lbf)	newton (N)	4.448 222	E+03
kip (1 kip = 1,000 lbf)	kilonewton (kN)	4.448 222	E+00
ounce (avoirdupois)-force (ozf)	newton (N)	2.780 139	E–01
poundal	newton (N)	1.382 550	E–01
pound-force (lbf)[7]	newton (N)	4.448 222	E+00
pound-force per pound (lbf/lb) (thrust to mass ratio)	newton per kilogram (N/kg)	**9.806 65**	**E+00**
ton-force (2,000 lbf)	newton (N)	8.896 443	E+03
ton-force (2,000 lbf)	kilonewton (kN)	8.896 443	E+00

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TABLE 1.1 **SI conversion factors** (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
Force Divided by Area (see Pressure)			
Force Divided by Length			
pound-force per foot (lbf/ft)	newton per meter (N/m)	1.459 390	E+01
pound-force per inch (lbf/in)	newton per meter (N/m)	1.751 268	E+02
HEAT			
Available Energy			
British thermal unit$_{IT}$ per cubic foot (Btu$_{IT}$/ft^3)	joule per cubic meter (J/m^3)	3.725 895	E+04
British thermal unit$_{th}$ per cubic foot (Btu$_{th}$/ft^3)	joule per cubic meter (J/m^3)	3.723 403	E+04
British thermal unit$_{IT}$ per pound (Btu$_{IT}$/lb)	joule per kilogram (J/kg)	**2.326**	**E+03**
British thermal unit$_{th}$ per cubic foot (Btu$_{th}$/lb)	joule per kilogram (J/kg)	2.324 444	E+03
calorie$_{IT}$ per gram (cal$_{IT}$/g)	joule per kilogram (J/kg)	**4.1868**	**E+03**
calorie$_{th}$ per gram (cal$_{th}$/g)	joule per kilogram (J/kg)	**4.184**	**E+03**
Coefficient of Heat Transfer			
British thermal unit$_{IT}$ per hour square foot degree Fahrenheit [Btu$_{IT}$/(h·ft^2·°F)]	watt per square meter kelvin [W/(m^2·K)]	5.678 263	E+00
British thermal unit$_{th}$ per hour square foot degree Fahrenheit [Btu$_{th}$/(h·ft^2·°F)]	watt per square meter kelvin [W/(m^2·K)]	5.674 466	E+00
British thermal unit$_{IT}$ per second square foot degree Fahrenheit [Btu$_{IT}$/(s·ft^2·°F)]	watt per square meter kelvin [W/(m^2·K)]	2.044 175	E+04
British thermal unit$_{th}$ per second square foot degree Fahrenheit [Btu$_{th}$/(s·ft^2·°F)]	watt per square meter kelvin [W/(m^2·K)]	2.042 808	E+04
Density of Heat			
British thermal unit$_{IT}$ per square foot (Btu$_{IT}$/ft^2)	joule per square meter (J/m^2)	1.135 653	E+04
British thermal unit$_{th}$ per square foot (Btu$_{th}$/ft^2)	joule per square meter (J/m^2)	1.134 893	E+04
calorie$_{th}$ per square centimeter (cal$_{th}$/cm^2)	joule per square meter (J/m^2)	**4.184**	**E+04**
langley (cal$_{th}$/cm^2)	joule per square meter (J/m^2)	**4.184**	**E+04**
Density of Heat Flow Rate			
British thermal unit$_{IT}$ per square foot hour [Btu$_{IT}$/(ft^2·h)]	watt per square meter (W/m^2)	3.154 591	E+00
British thermal unit$_{th}$ per square foot hour [Btu$_{th}$/(ft^2·h)]	watt per square meter (W/m^2)	3.152 481	E+00
British thermal unit$_{th}$ per square foot minute [Btu$_{th}$/(ft^2·min)]	watt per square meter (W/m^2)	1.891 489	E+02
British thermal unit$_{IT}$ per square foot second [Btu$_{IT}$/(ft^2·s)]	watt per square meter (W/m^2)	1.135 653	E+04
British thermal unit$_{th}$ per square foot second [Btu$_{th}$/(ft^2·s)]	watt per square meter (W/m^2)	1.134 893	E+04
British thermal unit$_{th}$ per square inch second [Btu$_{th}$/(in.^{2}·s)]	watt per square meter (W/m^2)	1.634 246	E+06
calorie$_{th}$ per square centimeter minute [cal$_{th}$/(cm^2·min)]	watt per square meter (W/m^2)	6.973 333	E+02
calorie$_{th}$ per square centimeter second [cal$_{th}$/(cm^2·s)]	watt per square meter (W/m^2)	**4.184**	**E+04**
Fuel Consumption			
gallon (U.S.) per horsepower hour [gal/(hp·h)]	cubic meter per joule (m^3/J)	1.410 089	E−09
gallon (U.S.) per horsepower hour [gal/(hp·h)]	liter per joule (L/J)	1.410 089	E−06
mile per gallon (U.S.) (mpg) (mi/gal)	meter per cubic meter (m/m^3)	4.251 437	E+05

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TABLE 1.1 SI conversion factors (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
mile per gallon (U.S.) (mpg) (mi/gal)	kilometer per liter (km/L)	4.251 437	E−01
mile per gallon (U.S.) (mpg) (mi/gal)	liter per 100 kilometer (L/100 km)	divide 235.215 by number of miles per gallon	
pound per horsepower hour [lb/(hp·h)]	kilogram per joule (kg/J)	1.689 659	E−07
Heat Capacity and Entropy			
British thermal unit$_{IT}$ per degree Fahrenheit (Btu$_{IT}$/°F)	joule per kelvin (J/K)	1.899 101	E+03
British thermal unit$_{th}$ per degree Fahrenheit (Btu$_{th}$/°F)	joule per kelvin (J/K)	1.897 830	E+03
British thermal unit$_{IT}$ per degree Rankine (Btu$_{IT}$/°R)	joule per kelvin (J/K)	1.899 101	E+03
British thermal unit$_{th}$ per degree Rankine (Btu$_{th}$/°R)	joule per kelvin (J/K)	1.897 830	E+03
Heat Flow Rate			
British thermal unit$_{IT}$ per hour (Btu$_{IT}$/h)	watt (W)	2.930 711	E−01
British thermal unit$_{th}$ per hour (Btu$_{th}$/h)	watt (W)	2.928 751	E−01
British thermal unit$_{th}$ per minute (Btu$_{th}$/min)	watt (W)	1.757 250	E+01
British thermal unit$_{IT}$ per second (Btu$_{IT}$/s)	watt (W)	1.055 056	E+03
British thermal unit$_{th}$ per second (Btu$_{th}$/s)	watt (W)	1.054 350	E+03
calorie$_{th}$ per minute (cal$_{th}$/min)	watt (W)	6.973 333	E−02
calorie$_{th}$ per second (cal$_{th}$/s)	watt (W)	**4.184**	**E+00**
kilocalorie$_{th}$ per minute (kcal$_{th}$/min)	watt (W)	6.973 333	E+01
kilocalorie$_{th}$ per second (kcal$_{th}$/s)	watt (W)	**4.184**	**E+03**
ton of refrigeration (12,000 Btu$_{IT}$/h)	watt (W)	3.516 853	E+03
Specific Heat Capacity and Specific Entropy			
British thermal unit$_{IT}$ per pound degree Fahrenheit [Btu$_{IT}$/(lb·°F)]	joule per kilogram kelvin [J/(kg·K)]	**4.1868**	**E+03**
British thermal unit$_{th}$ per pound degree Fahrenheit [Btu$_{th}$/(lb·°F)]	joule per kilogram kelvin [J/(kg·K)]	**4.184**	**E+03**
British thermal unit$_{IT}$ per pound degree Rankine [Btu$_{IT}$/(lb·°R)]	joule per kilogram kelvin [J/(kg·K)]	**4.1868**	**E+03**
British thermal unit$_{th}$ per pound degree Rankine [Btu$_{th}$/(lb·°R)]	joule per kilogram kelvin [J/(kg·K)]	**4.184**	**E+03**
calorie$_{IT}$ per gram degree Celsius [cal$_{IT}$/(g·°C)]	joule per kilogram kelvin [J/(kg·K)]	**4.1868**	**E+03**
calorie$_{th}$ per gram degree Celsius [cal$_{th}$/(g·°C)]	joule per kilogram kelvin [J/(kg·K)]	**4.184**	**E+03**
calorie$_{IT}$ per gram kelvin [cal$_{IT}$/(g·K)]	joule per kilogram kelvin [J/(kg·K)]	**4.1868**	**E+03**
calorie$_{th}$ per gram kelvin [cal$_{th}$/(g·K)]	joule per kilogram kelvin [J/(kg·K)]	**4.184**	**E+03**
Thermal Conductivity			
British thermal unit$_{IT}$ foot per hour square foot degree Fahrenheit [Btu$_{IT}$·ft/(h·ft^2·°F)]	watt per meter kelvin [W/(m·K)]	1.730 735	E+00
British thermal unit$_{th}$ foot per hour square foot degree Fahrenheit [Btu$_{th}$·ft/(h·ft^2·°F)]	watt per meter kelvin [W/(m·K)]	1.729 577	E+00
British thermal unit$_{IT}$ inch per hour square foot degree Fahrenheit [Btu$_{IT}$·in./(h·ft^2·°F)]	watt per meter kelvin [W/(m·K)]	1.442 279	E−01
British thermal unit$_{th}$ inch per hour square foot degree Fahrenheit [Btu$_{th}$·in./(h·ft^2·°F)]	watt per meter kelvin [W/(m·K)]	1.441 314	E−01

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TABLE 1.1 **SI conversion factors** (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
British thermal unit$_{IT}$ inch per second square foot degree Fahrenheit [Btu$_{IT}$·in./(s·ft^2·°F)]	watt per meter kelvin [W/(m·K)]	5.192 204	E+02
British thermal unit$_{th}$ inch per second square foot degree Fahrenheit [Btu$_{th}$·in./(s·ft^2·°F)]	watt per meter kelvin [W/(m·K)]	5.188 732	E+02
calorie$_{th}$ per centimeter second degree Celsius [cal$_{th}$/(cm·s·°C)]	watt per meter kelvin [W/(m·K)]	**4.184**	**E+02**
Thermal Diffusivity			
square foot per hour (ft^2/h)	square meter per second (m^2/s)	**2.580 64**	**E–05**
Thermal Insulance			
clo	square meter kelvin per watt (m^2·K/W)	1.55	E–01
degree Fahrenheit hour square foot per British thermal unit$_{IT}$ [°F·h·ft^2/Btu$_{IT}$)]	square meter kelvin per watt (m^2·K/W)	1.761 102	E–01
degree Fahrenheit hour square foot per British thermal unit$_{th}$ [°F·h·ft^2/Btu$_{th}$)]	square meter kelvin per watt (m^2·K/W)	1.762 280	E–01
Thermal Resistance			
degree Fahrenheit hour per British thermal unit$_{IT}$ (°F·h/Btu$_{IT}$)	kelvin per watt (K/W)	1.895 634	E+00
degree Fahrenheit hour per British thermal unit$_{th}$ (°F·h/Btu$_{th}$)	kelvin per watt (K/W)	1.896 903	E+00
degree Fahrenheit second per British thermal unit$_{IT}$ (°F·s/Btu$_{IT}$)	kelvin per watt (K/W)	5.265 651	E–04
degree Fahrenheit second per British thermal unit$_{th}$ (°F·s/Btu$_{th}$)	kelvin per watt (K/W)	5.269 175	E–04
Thermal Resistivity			
degree Fahrenheit hour square foot per British thermal unit$_{IT}$ inch [°F·h·ft^2/(Btu$_{IT}$·in)]	meter kelvin per watt (m·K/W)	6.933 472	E+00
degree Fahrenheit hour square foot per British thermal unit$_{th}$ inch [°F·h·ft^2/(Btu$_{th}$·in)]	meter kelvin per watt (m·K/W)	6.938 112	E+04
Length			
ångström (Å)	meter (m)	**1.0**	**E–10**
ångström (Å)	nanometer (nm)	**1.0**	**E–01**
astronomical unit (AU)	meter (m)	1.495 979	E+11
chain (based on U.S. survey foot) (ch)[1]	meter (m)	2.011 684	E+01
fathom (based on U.S. survey foot)[1]	meter (m)	1.828 804	E+00
fermi	meter (m)	**1.0**	**E–15**
fermi	femtometer (fm)	**1.0**	**E+00**
foot (ft)	meter (m)	**3.048**	**E–01**
foot (U.S. survey) (ft)[1]	meter (m)	3.048 006	E–01
inch (in.)	meter (m)	**2.54**	**E–02**
inch (in.)	centimeter (cm)	**2.54**	**E+00**
kayser (K)	reciprocal meter (m^{-1})	**1**	**E+02**
light year (l.y.)[8]	meter (m)	9.460 73	E+15
microinch	meter (m)	**2.54**	**E–08**
microinch	micrometer (μm)	**2.54**	**E–02**
micron (μ)	meter (m)	**1.0**	**E–06**
micron (μ)	micrometer (μm)	**1.0**	**E+00**

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TABLE 1.1 SI conversion factors (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
mil (0.001 in.)	meter (m)	**2.54**	**E–05**
mil (0.001 in.)	millimeter (mm)	**2.54**	**E–02**
mile (mi)	meter (m)	**1.609 344**	**E+03**
mile (mi)	kilometer (km)	**1.609 344**	**E+00**
mile (based on U.S. survey foot) (mi)[1]	meter (m)	1.609 347	E+03
mile (based on U.S. survey foot) (mi)[1]	kilometer (km)	1.609 347	E+00
mile, nautical[9]	meter (m)	**1.852**	**E+03**
parsec (pc)	meter (m)	3.085 678	E+16
pica (computer) (1/6 in.)	meter (m)	4.233 333	E–03
pica (computer) (1/6 in.)	millimeter (mm)	4.233 333	E+00
pica (printer's)	meter (m)	4.217 518	E–03
pica (printer's)	millimeter (mm)	4.217 518	E+00
point (computer) (1/72 in.)	meter (m)	3.527 778	E–04
point (computer) (1/72 in.)	millimeter (mm)	3.527 778	E–01
point (printer's)	meter (m)	3.514 598	E–04
point (printer's)	millimeter (mm)	3.514 598	E–01
rod (based on U.S. survey foot) (rd)[1]	meter (m)	5.029 210	E+00
yard (yd)	meter (m)	**9.144**	**E–01**
Light			
candela per square inch (cd/in.^{2})	candela per square meter (cd/m^2)	1.550 003	E+03
footcandle	lux (lx)	1.076 391	E+01
footlambert	candela per square meter (cd/m^2)	3.426 259	E+00
lambert[10]	candela per square meter (cd/m^2)	3.183 099	E+03
lumen per square foot (lm/ft^2)	lux (lx)	1.076 391	E+01
phot (ph)	lux (lx)	**1.0**	**E+04**
stilb (sb)	candela per square meter (cd/m^2)	**1.0**	**E+04**
Mass and Moment of Inertia			
carat, metric	kilogram (kg)	**2.0**	**E–04**
carat, metric	gram (g)	**2.0**	**E–01**
grain (gr)	kilogram (kg)	**6.479 891**	**E–05**
grain (gr)	milligram (mg)	**6.479 891**	**E+01**
hundredweight (long, 112 lb)	kilogram (kg)	5.080 235	E+01
hundredweight (short, 100 lb)	kilogram (kg)	4.535 924	E+01
kilogram-force second squared per meter (kgf·s^2/m)	kilogram (kg)	**9.806 65**	**E+00**
ounce (avoirdupois) (oz)	kilogram (kg)	2.834 952	E–02
ounce (avoirdupois) (oz)	gram (g)	2.834 952	E+01
ounce (troy or apothecary) (oz)	kilogram (kg)	3.110 348	E–02
ounce (troy or apothecary) (oz)	gram (g)	3.110 348	E+01
pennyweight (dwt)	kilogram (kg)	1.555 174	E–03
pennyweight (dwt)	gram (g)	1.555 174	E+00
pound (avoirdupois) (lb)[11]	kilogram (kg)	4.535 924	E–01
pound (troy or apothecary) (lb)	kilogram (kg)	3.732 417	E–01

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TABLE 1.1 **SI conversion factors** (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
pound foot squared (lb·ft^2)	kilogram meter squared (kg·m^2)	4.214 011	E−02
pound inch squared (lb·in.^{2})	kilogram meter squared (kg·m^2)	2.926 397	E−04
slug (slug)	kilogram (kg)	1.459 390	E+01
ton, assay (AT)	kilogram (kg)	2.916 667	E−02
ton, assay (AT)	gram (g)	2.916 667	E+01
ton, long (2,240 lb)	kilogram (kg)	1.016 047	E+03
ton, metric (t)	kilogram (kg)	**1.0**	**E+03**
tonne (called "metric ton" in U.S.) (t)	kilogram (kg)	**1.0**	**E+03**
ton, short (2,000 lb)	kilogram (kg)	9.071 847	E+02
Mass Density (see Mass Divided by Volume)			
Mass Divided by Area			
ounce (avoirdupois) per square foot (oz/ft^2)	kilogram per square meter (kg/m^2)	3.051 517	E−01
ounce (avoirdupois) per square inch (oz/in.^{2})	kilogram per square meter (kg/m^2)	4.394 185	E+01
ounce (avoirdupois) per square yard (oz/yd^2)	kilogram per square meter (kg/m^2)	3.390 575	E−02
pound per square foot (lb/ft^2)	kilogram per square meter (kg/m^2)	4.882 428	E+00
pound per square inch (*not* pound force) (lb/in.^{2})	kilogram per square meter (kg/m^2)	7.030 696	E+02
Mass Divided by Capacity (see Mass Divided by Volume)			
Mass Divided by Length			
denier	kilogram per meter (kg/m)	1.111 111	E−07
denier	gram per meter (g/m)	1.111 111	E−04
pound per foot (lb/ft)	kilogram per meter (kg/m)	1.488 164	E+00
pound per inch (lb/in.)	kilogram per meter (kg/m)	1.785 797	E+01
pound per yard (lb/yd)	kilogram per meter (kg/m)	4.960 546	E−01
tex	kilogram per meter (kg/m)	**1.0**	**E−06**
Mass Divided by Time (includes Flow)			
pound per hour (lb/h)	kilogram per second (kg/s)	1.259 979	E−04
pound per minute (lb/min)	kilogram per second (kg/s)	7.559 873	E−03
pound per second (lb/s)	kilogram per second (kg/s)	4.535 924	E−01
ton, short, per hour	kilogram per second (kg/s)	2.519 958	E−01
Mass Divided by Volume (includes Mass Density and Mass Concentration)			
grain per gallon (U.S.) (gr/gal)	kilogram per cubic meter (kg/m^3)	1.711 806	E−02
grain per gallon (U.S.) (gr/gal)	milligram per liter (mg/L)	1.711 806	E+01
gram per cubic centimeter (g/cm^3)	kilogram per cubic meter (kg/m^3)	**1.0**	**E+03**
ounce (avoirdupois) per cubic inch (oz/in.^{3})	kilogram per cubic meter (kg/m^3)	1.729 994	E+03
ounce (avoirdupois) per gallon [Canadian and U.K. (Imperial)] (oz/gal)	kilogram per cubic meter (kg/m^3)	6.236 023	E+00
ounce (avoirdupois) per gallon [Canadian and U.K. (Imperial)] (oz/gal)	gram per liter (g/L)	6.236 023	E+00
ounce (avoirdupois) per gallon (U.S.) (oz/gal)	kilogram per cubic meter (kg/m^3)	7.489 152	E+00
ounce (avoirdupois) per gallon (U.S.) (oz/gal)	gram per liter (g/L)	7.489 152	E+00
pound per cubic foot (lb/ft^3)	kilogram per cubic meter (kg/m^3)	1.601 846	E+01
pound per cubic inch (lb/in.^{3})	kilogram per cubic meter (kg/m^3)	2.767 990	E+04
pound per cubic yard (lb/yd^3)	kilogram per cubic meter (kg/m^3)	5.932 764	E−01

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TABLE 1.1 SI conversion factors (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
pound per gallon [Canadian and U.K. (Imperial)] (lb/gal)	kilogram per cubic meter (kg/m^3)	9.977 637	E+01
pound per gallon [Canadian and U.K. (Imperial)] (lb/gal)	kilogram per liter (kg/L)	9.977 637	E−02
pound per gallon (U.S.) (lb/gal)	kilogram per cubic meter (kg/m^3)	1.198 264	E+02
pound per gallon (U.S.) (lb/gal)	kilogram per liter (kg/L)	1.198 264	E−01
slug per cubic foot ($slug/ft^3$)	kilogram per cubic meter (kg/m^3)	5.153 788	E+02
ton, long, per cubic yard	kilogram per cubic meter (kg/m^3)	1.328 939	E+03
ton, short, per cubic yard	kilogram per cubic meter (kg/m^3)	1.186 553	E+03
Moment of Force or Torque			
dyne centimeter (dyn·cm)	newton meter (N·m)	**1.0**	**E−07**
kilogram-force meter (kgf·m)	newton meter (N·m)	**9.806 65**	**E+00**
ounce (avoirdupois)-force inch (ozf·in.)	newton meter (N·m)	7.061 552	E−03
ounce (avoirdupois)-force inch (ozf·in.)	millinewton meter (mN·m)	7.061 552	E+00
pound-force foot (lbf·ft)	newton meter (N·m)	1.355 818	E+00
pound-force inch (lbf·in.)	newton meter (N·m)	1.129 848	E−01
Moment of Force or Torque, Divided by Length			
pound-force foot per inch (lbf·ft/in.)	newton meter per meter (N·m/m)	5.337 866	E+01
pound-force inch per inch (lbf·in./in.)	newton meter per meter (N·m/m)	4.448 222	E+00
Permeability			
darcy[12]	meter squared (m^2)	9.869 233	E−13
perm (0°C)	kilogram per pascal second square meter [$kg/(Pa·s·m^2)$]	5.721 35	E−11
perm (23°C)	kilogram per pascal second square meter [$kg/(Pa·s·m^2)$]	5.745 25	E−11
perm inch (0°C)	kilogram per pascal second meter [kg/(Pa·s·m)]	1.453 22	E−12
perm inch (23°C)	kilogram per pascal second meter [kg/(Pa·s·m)]	1.459 29	E−12
Power			
erg per second (erg/s)	watt (W)	**1.0**	**E−07**
foot pound-force per hour (ft·lbf/h)	watt (W)	3.766 161	E−04
foot pound-force per minute (ft·lbf/min)	watt (W)	2.259 697	E−02
foot pound-force per second (ft·lbf/s)	watt (W)	1.355 818	E+00
horsepower (550 ft·lbf/s)	watt (W)	7.456 999	E+02
horsepower (boiler)	watt (W)	9.809 50	E+03
horsepower (electric)	watt (W)	**7.46**	**E+02**
horsepower (metric)	watt (W)	7.354 988	E+02
horsepower (U.K.)	watt (W)	7.4570	E+02
horsepower (water)	watt (W)	7.460 43	E+02
Pressure or Stress (Force Divided by Area)			
atmosphere, standard (atm)	pascal (Pa)	**1.013 25**	**E+05**
atmosphere, standard (atm)	kilopascal (kPa)	**1.013 25**	**E+02**
atmosphere, technical (at)[13]	pascal (Pa)	**9.806 65**	**E+04**
atmosphere, technical (at)[13]	kilopascal (kPa)	**9.806 65**	**E+01**

(continues)

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TABLE 1.1 **SI conversion factors** (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
bar (bar)	pascal (Pa)	**1.0**	**E+05**
bar (bar)	kilopascal (kPa)	**1.0**	**E+02**
centimeter of mercury (0°C)[14]	pascal (Pa)	1.333 22	E+03
centimeter of mercury (0°C)[14]	kilopascal (kPa)	1.333 22	E+00
centimeter of mercury, conventional (cm Hg)[14]	pascal (Pa)	1.333 224	E+03
centimeter of mercury, conventional (cm Hg)[14]	kilopascal (kPa)	1.333 224	E+00
centimeter of water (4°C)[14]	pascal (Pa)	9.806 38	E+01
centimeter of water, conventional (cm H_2O)[14]	pascal (Pa)	**9.806 65**	**E+01**
dyne per square centimeter (dyn/cm^2)	pascal (Pa)	**1.0**	**E−01**
foot of mercury, conventional (ft Hg)[14]	pascal (Pa)	4.063 666	E+04
foot of mercury, conventional (ft Hg)[14]	kilopascal (kPa)	4.063 666	E+01
foot of water (39.2°F)[14]	pascal (Pa)	2.988 98	E+03
foot of water (39.2°F)[14]	kilopascal (kPa)	2.988 98	E+00
foot of water, conventional (ft H_2O)[14]	pascal (Pa)	2.989 067	E+03
foot of water, conventional (ft H_2O)[14]	kilopascal (kPa)	2.989 067	E+00
gram-force per square centimeter (gf/cm^2)	pascal (Pa)	**9.806 65**	**E+01**
inch of mercury (32°F)[14]	pascal (Pa)	3.386 38	E+03
inch of mercury (32°F)[14]	kilopascal (kPa)	3.386 38	E+00
inch of mercury (60°F)[14]	pascal (Pa)	3.376 85	E+03
inch of mercury (60°F)[14]	kilopascal (kPa)	3.376 85	E+00
inch of mercury, conventional (in. Hg)[14]	pascal (Pa)	3.386 389	E+03
inch of mercury, conventional (in. Hg)[14]	kilopascal (kPa)	3.386 389	E+00
inch of water (39.2°F)[14]	pascal (Pa)	2.490 82	E+02
inch of water (60°F)[14]	pascal (Pa)	2.4884	E+02
inch of water, conventional (in. H_2O)[14]	pascal (Pa)	2.490 889	E+02
kilogram-force per square centimeter (kgf/cm^2)	pascal (Pa)	**9.806 65**	**E+04**
kilogram-force per square centimeter (kgf/cm^2)	kilopascal (kPa)	**9.806 65**	**E+01**
kilogram-force per square meter (kgf/m^2)	pascal (Pa)	**9.806 65**	**E+00**
kilogram-force per square millimeter (kgf/mm^2)	pascal (Pa)	**9.806 65**	**E+06**
kilogram-force per square millimeter (kgf/mm^2)	megapascal (MPa)	**9.806 65**	**E+00**
kip per square inch (ksi) (kip/$in.^2$)	pascal (Pa)	6.894 757	E+06
kip per square inch (ksi) (kip/$in.^2$)	kilopascal (kPa)	6.894 757	E+03
millibar (mbar)	pascal (Pa)	**1.0**	**E+02**
millibar (mbar)	kilopascal (kPa)	**1.0**	**E−01**
millimeter of mercury, conventional (mm Hg)[14]	pascal (Pa)	1.333 224	E+02
millimeter of water, conventional (mm H_2O)[14]	pascal (Pa)	**9.806 65**	**E+00**
poundal per square foot	pascal (Pa)	1.488 164	E+00
pound-force per square foot (lbf/ft^2)	pascal (Pa)	4.788 026	E+01
pound-force per square inch (psi) (lbf/$in.^2$)	pascal (Pa)	6.894 757	E+03
pound-force per square inch (psi) (lbf/$in.^2$)	kilopascal (kPa)	6.894 757	E+00
psi (pound-force per square inch) (lbf/$in.^2$)	pascal (Pa)	6.894 757	E+03
psi (pound-force per square inch) (lbf/$in.^2$)	kilopascal (kPa)	6.894 757	E+00
torr (Torr)	pascal (Pa)	1.333 224	E+02

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TABLE 1.1 **SI conversion factors** (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
Radiology			
curie (Ci)	becquerel (Bq)	**3.7**	**E+10**
rad (absorbed dose) (rad)	gray (Gy)	**1.0**	**E−02**
rem (rem)	sievert (Sv)	**1.0**	**E−02**
roentgen (R)	coulomb per kilogram (C/kg)	**2.58**	**E−04**
Speed (see Velocity)			
Stress (see Pressure)			
Temperature			
degree Celsius (°C)	kelvin (K)	$T/\mathrm{K} = t/°\mathrm{C} + \mathbf{273.15}$	
degree centigrade[15]	degree Celsius (°C)	$t/°\mathrm{C} \approx t/\text{deg. cent.}$	
degree Fahrenheit (°F)	degree Celsius (°C)	$t/°\mathrm{C} = (t/°\mathrm{F} - \mathbf{32})/\mathbf{1.8}$	
degree Fahrenheit (°F)	kelvin (K)	$T/\mathrm{K} = (t/°\mathrm{F} + \mathbf{459.67})/\mathbf{1.8}$	
degree Rankine (°R)	kelvin (K)	$T/\mathrm{K} = (T/°\mathrm{R})/\mathbf{1.8}$	
kelvin (K)	degree Celsius (°C)	$t/°\mathrm{C} = T/\mathrm{K} - \mathbf{273.15}$	
Temperature Interval			
degree Celsius (°C)	kelvin (K)	**1.0**	**E+00**
degree centigrade[15]	degree Celsius (°C)	1.0	E+00
degree Fahrenheit (°F)	degree Celsius (°C)	5.555 556	E−01
degree Fahrenheit (°F)	kelvin (K)	5.555 556	E−01
degree Rankine (°R)	kelvin (K)	5.555 556	E−01
Time			
day (d)	second (s)	**8.64**	**E+04**
day (sidereal)	second (s)	8.616 409	E+04
hour (h)	second (s)	**3.6**	**E+03**
hour (sidereal)	second (s)	3.590 170	E+03
minute (min)	second (s)	**6.0**	**E+01**
minute (sidereal)	second (s)	5.983 617	E+01
second (sidereal)	second (s)	9.972 696	E−01
shake	second (s)	**1.0**	**E−08**
shake	nanosecond (ns)	**1.0**	**E+01**
year (365 days)	second (s)	**3.1536**	**E+07**
year (sidereal)	second (s)	3.155 815	E+07
year (tropical)	second (s)	3.155 693	E+07
Torque (see Moment of Force)			
Velocity (includes Speed)			
foot per hour (ft/h)	meter per second (m/s)	8.466 667	E−05
foot per minute (ft/min)	meter per second (m/s)	**5.08**	**E−03**
foot per second (ft/s)	meter per second (m/s)	**3.048**	**E−01**
inch per second (in./s)	meter per second (m/s)	**2.54**	**E−02**
kilometer per hour (km/h)	meter per second (m/s)	2.777 778	E−01
knot (nautical mile per hour)	meter per second (m/s)	5.144 444	E−01
mile per hour (mi/h)	meter per second (m/s)	**4.4704**	**E−01**
mile per hour (mi/h)	kilometer per hour (km/h)	**1.609 344**	**E+00**
mile per minute (mi/min)	meter per second (m/s)	**2.682 24**	**E+01**

(continues)

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TABLE 1.1 SI conversion factors (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
mile per second (mi/s)	meter per second (m/s)	**1.609 344**	**E+03**
revolution per minute (rpm) (r/min)	radian per second (rad/s)	1.047 198	E−01
rpm (revolution per minute) (r/min)	radian per second (rad/s)	1.047 198	E−01
Viscosity, Dynamic			
centipoise (cP)	pascal second (Pa·s)	**1.0**	**E−03**
poise (P)	pascal second (Pa·s)	**1.0**	**E−01**
poundal second per square foot	pascal second (Pa·s)	1.488 164	E+00
pound-force second per square foot (lbf·s/ft^2)	pascal second (Pa·s)	4.788 026	E+01
pound-force second per square inch (lbf·s/in.^{2})	pascal second (Pa·s)	6.894 757	E+03
pound per foot hour [lb/(ft·h)]	pascal second (Pa·s)	4.133 789	E−04
pound per foot second [lb/(ft·s)]	pascal second (Pa·s)	1.488 164	E+00
rhe	reciprocal pascal second [(Pa·s)$^{-1}$]	**1.0**	**E+01**
slug per foot second [slug/(ft·s)]	pascal second (Pa·s)	4.788 026	E+01
Viscosity, Kinematic			
centistokes (cSt)	meter squared per second (m^2/s)	**1.0**	**E−06**
square foot per second (ft^2/s)	meter squared per second (m^2/s)	**9.290 304**	**E−02**
stokes (St)	meter squared per second (m^2/s)	**1.0**	**E−04**
Volume (includes Capacity)			
acre-foot (based on U.S. survey foot)[1]	cubic meter (m^3)	1.233 489	E+03
barrel [for petroleum, 42 gallons (U.S.)] (bbl)	cubic meter (m^3)	1.589 873	E−01
barrel [for petroleum, 42 gallons (U.S.)] (bbl)	liter (L)	1.589 873	E+02
bushel (U.S.) (bu)	cubic meter (m^3)	3.523 907	E−02
bushel (U.S.) (bu)	liter (L)	3.523 907	E+01
cord (128 ft^3)	cubic meter (m^3)	3.624 556	E+00
cubic foot (ft^3)	cubic meter (m^3)	2.831 685	E−02
cubic inch (in.^{3})[16]	cubic meter (m^3)	1.638 706	E−05
cubic mile (mi^3)	cubic meter (m^3)	4.168 182	E+09
cubic yard (yd^3)	cubic meter (m^3)	7.645 549	E−01
cup (U.S.)	cubic meter (m^3)	2.365 882	E−04
cup (U.S.)	liter (L)	2.365 882	E−01
cup (U.S.)	milliliter (mL)	2.365 882	E+02
fluid ounce (U.S.) (fl oz)	cubic meter (m^3)	2.957 353	E−05
fluid ounce (U.S.) (fl oz)	milliliter (mL)	2.957 353	E+01
gallon [Canadian and U.K. (Imperial)] (gal)	cubic meter (m^3)	**4.546 09**	**E−03**
gallon [Canadian and U.K. (Imperial)] (gal)	liter (L)	**4.546 09**	**E+00**
gallon (U.S.) (gal)	cubic meter (m^3)	3.785 412	E−03
gallon (U.S.) (gal)	liter (L)	3.785 412	E+00
gill [Canadian and U.K. (Imperial)] (gi)	cubic meter (m^3)	1.420 653	E−04
gill [Canadian and U.K. (Imperial)] (gi)	liter (L)	1.420 653	E−01
gill (U.S.) (gi)	cubic meter (m^3)	1.182 941	E−04
gill (U.S.) (gi)	liter (L)	1.182 941	E−01
liter (L)[17]	cubic meter (m^3)	**1.0**	**E−03**
ounce [Canadian and U.K. fluid (Imperial)] (fl oz)	cubic meter (m^3)	2.841 306	E−05
ounce [Canadian and U.K. fluid (Imperial)] (fl oz)	milliliter (mL)	2.841 306	E+01

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TABLE 1.1 **SI conversion factors** (factors in **boldface** are exact) (continued)

To Convert From	To	Multiply By	
ounce (U.S. fluid) (fl oz)	cubic meter (m^3)	2.957 353	E−05
ounce (U.S. fluid) (fl oz)	milliliter (mL)	2.957 353	E+01
peck (U.S.) (pk)	cubic meter (m^3)	8.809 768	E−03
peck (U.S.) (pk)	liter (L)	8.809 768	E+00
pint (U.S. dry) (dry pt)	cubic meter (m^3)	5.506 105	E−04
pint (U.S. dry) (dry pt)	liter (L)	5.506 105	E−01
pint (U.S. liquid) (liq pt)	cubic meter (m^3)	4.731 765	E−04
pint (U.S. liquid) (liq pt)	liter (L)	4.731 765	E−01
quart (U.S. dry) (dry qt)	cubic meter (m^3)	1.101 221	E−03
quart (U.S. dry) (dry qt)	liter (L)	1.101 221	E+00
quart (U.S. liquid) (liq qt)	cubic meter (m^3)	9.463 529	E−04
quart (U.S. liquid) (liq qt)	liter (L)	9.463 529	E−01
stere (st)	cubic meter (m^3)	**1.0**	**E+00**
tablespoon	cubic meter (m^3)	1.478 676	E−05
tablespoon	milliliter (mL)	1.478 676	E+01
teaspoon	cubic meter (m^3)	4.928 922	E−06
teaspoon	milliliter (mL)	4.928 922	E+00
ton, register	cubic meter (m^3)	2.831 685	E+00
Volume Divided by Time (includes Flow)			
cubic foot per minute (ft^3/min)	cubic meter per second (m^3/s)	4.719 474	E−04
cubic foot per minute (ft^3/min)	liter per second (L/s)	4.719 474	E−01
cubic foot per second (ft^3/s)	cubic meter per second (m^3/s)	2.831 685	E−02
cubic inch per minute ($in.^3$/min)	cubic meter per second (m^3/s)	2.731 177	E−07
cubic yard per minute (yd^3/min)	cubic meter per second (m^3/s)	1.274 258	E−02
gallon (U.S.) per day (gal/d)	cubic meter per second (m^3/s)	4.381 264	E−08
gallon (U.S.) per day (gal/d)	liter per second (L/s)	4.381 264	E−05
gallon (U.S.) per minute (gpm) (gal/min)	cubic meter per second (m^3/s)	6.309 020	E−05
gallon (U.S.) per minute (gpm) (gal/min)	liter per second (L/s)	6 309 020	E−02
Work (see Energy)			

1. The U. S. Metric Law of 1866 gave the relationship 1 m = 39.37 in. (in. is the unit symbol for the inch). From 1893 until 1959, the yard was defined as being exactly equal to (3,600/3,937) m, and thus the foot was defined as being exactly equal to (1,200/3,937) m. In 1959, the definition of the yard was changed to bring the U.S. yard and the yard used in other countries into agreement. Since then, the yard has been defined as exactly equal to 0.9144 m, and thus the foot has been defined as exactly equal to 0.3048 m. At the same time, it was decided that any data expressed in feet derived from geodetic surveys within the United States would continue to bear the relationship as defined in 1893; that is, 1 ft = (1,200/3,937) m (ft is the unit symbol for the foot). The name of this foot is *U.S. survey foot*; the name of the new foot defined in 1959 is "international foot." The two are related to each other through the expression 1 international foot = 0.999 998 U.S. survey foot exactly.
2. This is a unit for the quantity second moment of area, which is sometimes called the *moment of section* or *area moment of inertia* of a plane section about a specified axis.
3. The Fifth International Conference on the Properties of Steam (London, July 1956) defined the International Table calorie as 4.1868 J. Therefore, the exact conversion factor for the International Table Btu is 1.055 055 852 62 kJ. The notation for International Table used in this listing is subscript IT. Similarly, the notation for thermochemical is subscript th. Further, the thermochemical Btu, Btu_{th}, is based on the thermochemical calorie, cal_{th}, where cal_{th} = 4.184 J exactly.
4. The kilogram calorie or large calorie is an obsolete term used for the kilocalorie, which is the calorie used to express the energy content of foods. However, in practice, the prefix *kilo* is usually omitted.
5. The therm (EC) is legally defined in the Council Directive of December 20, 1979, Council of the European Communities (now the European Union [EU]). The therm (U.S.) is legally defined in the *Federal Register* of July 27, 1968. Although the therm (EC), which is based on the International Table Btu, is frequently used by engineers in the United States, the therm (U.S.) is the legal unit used by the U.S. natural gas industry.
6. Defined (not measured) value.

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7. If the local value of the acceleration of free fall is taken as g_n = 9.806 65 m/s^2 (the standard value), the exact conversion factor is 4.448 221 615 260 5 E+00.
8. This conversion factor is based on 1 d = 86,400 s; and 1 Julian century = 36,525 d (USNO 1994).
9. The value of this unit, 1 nautical mile = 1852 m, was adopted by the First International Extraordinary Hydrographic Conference, Monaco, 1929, under the name *international nautical mile*.
10. The exact conversion factor is $10^4/\pi$.
11. The exact conversion factor is 4.535 923 7 E–01.
12. The darcy is a unit for expressing the permeability of porous solids, not area.
13. One technical atmosphere equals one kilogram-force per square centimeter (1 at = kgf/cm^2).
14. Conversion factors for mercury manometer pressure units are calculated using the standard value for the acceleration of gravity and the density of mercury at the stated temperature. Additional digits are not justified because the definitions of the units do not take into account the compressibility of mercury or the change in density caused by the revised practical temperature scale, ITS-90. Similar comments also apply to water manometer pressure units.
15. The centigrade temperature scale is obsolete; the degree centigrade is only approximately equal to the degree Celsius.
16. The exact conversion factor is 1.638 706 4 E–05.
17. In 1964, the General Conference on Weights and Measures reestablished the name *liter* as a special name for the cubic decimeter. Between 1901 and 1964, the liter was slightly larger (1.000 028 dm^3); when using high-accuracy volume data from that time, keep this fact in mind.

Source: Thompson and Taylor 2008

TABLE 1.2 SI prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^1	deka	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Source: Thompson and Taylor 2008

TABLE 1.3 Greek alphabet

Name	Letters	Name	Letters
alpha	A α	nu	N ν
beta	B β	xi	Ξ ξ
gamma	Γ γ	omicron	O ο
delta	Δ δ	pi	Π π
epsilon	E ε	rho	P ρ
zeta	Z ζ	sigma	Σ σ
eta	H η	tau	T τ
theta	Θ θ	upsilon	Y υ
iota	I ι	phi	Φ φ
kappa	K κ	chi	X χ
lambda	Λ λ	psi	Ψ ψ
mu	M μ	omega	Ω ω

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FUNDAMENTAL PHYSICAL CONSTANTS

Table 1.4 is a list of the fundamental physical constants.

TABLE 1.4 Fundamental physical constants based on the 2018 adjustment

Quantity	Symbol	Value and Unit	Relative Standard of Uncertainty, u_r
Speed of light in a vacuum	c, co	299 792 458 ms^{-1}	Exact
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ NA^{-2} = 12.566 370 614… $\times 10^{-7}$ NA^{-2}	Exact
Electric constant, $1/\mu_0 c^2$	ϵ_0	8.854 187 817… $\times 10^{-12}$ Fm^{-1}	Exact
Newtonian constant of gravitation	G	6.674 30(15) $\times 10^{-11}$ $m^3kg^{-1}s^{-2}$	4.7×10^{-5}
Planck constant	h	6.626 070 15 $\times 10^{-34}$ J Hz^{-1}	1.2×10^{-8}
$h/2\pi$	ħ	1.054 571 800(13) $\times 10^{-34}$ J s	1.2×10^{-8}
Elementary charge	e	1.602 176 634 $\times 10^{-19}$ C	Exact
Magnetic flux quantum $2\pi h/(2e)$	Φ_0	2.067 833 848… $\times 10^{-15}$ Wb	6.1×10^{-9}
Conductance quantum $2e^2/h$	G_0	7.748 091 7310(18) $\times 10^{-5}$ S	2.3×10^{-10}
Electron mass	m_e	9.109 383 7015(28) $\times 10^{-31}$ kg	1.2×10^{-8}
Proton mass	m_p	1.672 621 923 69(51) $\times 10^{-27}$ kg	1.2×10^{-8}
Proton–electron mass ratio	m_p/m_e, μ, or β	1836.152 673 43(11)	9.5×10^{-11}
Fine structure constant $e^2/4\pi\epsilon_o\hbar c$	α	7.297 352 5664(17) $\times 10^{-3}$	2.3×10^{-10}
Inverse fine-structure constant	α^{-1}	137.035 999 139(31)	2.3×10^{-10}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	10 973 731.568 160(21) m^{-1}	5.9×10^{-12}
Avogadro constant	N_A, L	6.022 140 76 $\times 10^{23}$ mol^{-1}	Exact
Faraday constant $N_A e$	F	96 485.332 12 C mol^{-1}	6.2×10^{-9}
Molar gas constant $N_A k$	R	8.314 462 618…J mol^{-1} K^{-1}	5.7×10^{-7}
Boltzmann constant R/N_A	k	1.380 649 $\times 10^{-23}$ J K^{-1}	Exact
Stefan–Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	5.670 374 419… $\times 10^{-8}$ W m^{-2} K^{-4}	2.3×10^{-6}
Electronvolt: (e/C)J	eV	1.602 176 6208(98) $\times 10^{-19}$ J	6.1×10^{-9}
(Unified) Atomic mass unit 1u = m_u = $(1/12)m(^{12}C)$ = 10^{-3} kg mol^{-1}/N_A	u	1.660 539 040(20) $\times 10^{-27}$ kg	1.2×10^{-8}

Source: Mohr et al. 2014; with permission of the American Physical Society

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SELECTED CONSTANTS, MEASURES, AND TIME

Table 1.5 shows selected constants, measures, and units of time.

TABLE 1.5 Selected constants, measures, and time

Item	Quantity	Sources
Acceleration of gravity, standard	32.174 0 ft/s^2 = 9.806 65 m/s^2	Thompson and Taylor 2008
Density of dry air (sea level, 70°F)	0.075 0 lb/ft^3 = 1.201 kg/m^3	Tuck 2011
Density of water	62.4 lb/ft^3 = 8.345 lb/gal = 1 g/cm^3 = 1 000 kg/m^3	—
e (natural logarithm base)	2.718 281 828 459 045	Oliver et al. 2010
pi (π)	3.141 592 653 589 793	Oliver et al. 2010
Speed of sound in dry air at 20°C	343.4 m/s	Sytchev et al. 1987
Speed of sound in water at 10°C	1,447.8 m/s	Rumble 2018
Speed of sound in sea water at 10°C (salinity = 3.5%)	1,490.4 m/s	Rumble 2018
Time to nearest second (coordinated universal)	—	NIST and USNO, n.d.

SELECTED UNIT EQUIVALENCIES AND APPROXIMATIONS

Table 1.6 gives selected unit equivalencies and approximations.

TABLE 1.6 Selected unit equivalencies and approximations (factors in **boldface** are exact)

1 acre = **43,560** ft^2 (U.S. survey) = **160** rod^2 = 4,046.873 m^2 = 0.404 687 3 hectare

1 acre-ft = **43,560** ft^3 (U.S. survey) = 325,851 gal (U.S.) = 1,233.489 m^3

1 assay ton = 29.166 667 g = 0.029 166 667 kg

1 atmosphere (standard) = **1.013 25** bar = **1,013.25** millibar = 760 mm Hg = 33.90 ft H_2O = 29.92 in. Hg = 14.696 lb/in.^{2} = 10,332.3 kgf/m^2 = **101,325** pascal (Pa)

1 atmosphere (technical) = **1** kgf/cm^2 = **98,066.5** Pa

1 barrel (U.S. oil) = **42** gal (U.S.) = 0.158 987 3 m^3

1 board foot = 1 ft × 1 ft × 1 in.

1 British thermal unit per pound (Btu/lb) = **2.326** kJ/kg = **0.555 688 3** kcal/kg

1 bushel [U.S.] (bu) = **4** peck (pk) = **32** qt (U.S. dry) = 0.035 239 07 m^3

1 bushel [heaped] (cone ≥ 6 in) ≈ 1.25 struck bushel

1 carat (metric) = **200** mg

1 chain (Gunter's) = **4** rod = **66** ft (U.S. survey) = **100** link = **0.012 5** mile (U.S. statute) = 20.116 84 m

1 cord = 4 ft × 4 ft × 8 ft = 128 ft^3 = 3.625 m^3

1 cubic foot (ft^3) = **1,728** in.^{3} = 0.037 037 04 yd^3 = 7.480 52 gal (U.S.) = 0.028 316 85 m^3

1 cubic foot per second (cusec or ft^3/s) = 448.831 169 gal/min = 2.222 222 yd^3/min = 1.983 474 acre-ft/day = 0.028 316 85 m^3/s

1 cubic inch (in.^{3}) = **16.387 064** cm^3

1 cubic yard (yd^3) = **27** ft^3 = **46,656** in.^{3} = 0.764 554 9 m^3

1 cup (U.S.) = **2** gill (U.S.) = **8** fl oz (U.S.) = **16** tablespoon = 236.588 2 mL

1 day = **24** h = **1,440** min = **86,400** s

1 degree (°) = **60** min (arc) = **3,600** s (arc) = π**/180** rad = 0.017 453 29 radian

1 fathom = **6** ft (U.S. survey) = 1.828 804 m

(continues)

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TABLE 1.6 **Selected unit equivalencies and approximations** (factors in **boldface** are exact) (continued)

1 fluid ounce [U.S.] (fl oz) = **8** fluid dram (fl dr) = 1.804 69 $in.^3$ = 29.573 53 mL

1 foot [International] (ft) = **12** inch (in.) = **0.304 8** m

1 foot [U.S. survey] (ft) = **1,200/3,937** m = 0.304 800 6 m

1 foot pound-force (ft·lbf) = 1.355 818 newton meter (N·m)

1 foot of rock (sp. gr. 2.7) = 1.170 53 $lbf/in.^2$ = 0.082 3 kgf/cm^2

1 foot of water (ft_{H2O}) = 0.433 528 $lb/in.^2$ = 2,989.067 Pa

1 flask of mercury ≈ 34.5 kg ≈ 76 lb (av) [historical variation]

1 furlong = **10** chain = **660** ft (U.S. survey) = 201.168 m

1 g_n (standard acceleration of free fall) = 32.174 05 ft/s^2 = **9.806 65** m/s^2

1 gallon (Imperial) = **8** pints (Imperial) = 4.546 09 L

1 gallon [U.S.] (gal) = **4** qt (U.S. liquid) = **231** $in.^3$ = 0.133 681 ft^3 = 3.785 412 L

1 gill (U.S.) = $\mathbf{2^{-1}}$ cup = **4** fl oz (U.S.) = 118.294 1 mL

1 grain (av) = **1** grain (tr) = **1** grain (ap) = $\mathbf{7{,}000^{-1}}$ lb (av) = $\mathbf{5{,}760^{-1}}$ lb (tr) = **64.798 91** mg

1 hand = 4 in. = 0.101 6 m

1 hertz (Hz) = **1** cycle per second (c/s) = $\mathbf{10^{-6}}$ MHz = $\mathbf{10^{-12}}$ fresnel

1 hogshead (U.S.) = 63 gal (U.S.) = 1.5 barrel (U.S. oil)

1 horsepower (electric) = **746** watt (W)

1 horsepower (hp) = **550** ft·lbf/s = **33,000** ft·lbf/min = 745.699 9 watt (W)

1 inch (in.) = $\mathbf{12^{-1}}$ ft = **2.54** cm

1 karat (1 part in 24 of gold) = 41.666 667 mg/g

1 kilowatt hour (kW·h) = **3,600,000** joule (J) = 3,412.141 16 Btu_{IT}

l league = **3** nautical mile = **5,556** m

1 link (surveyor's) = **0.66** ft (U.S. survey) = 0.201 168 m

1 mile [international] (mi) = **5,280** ft (international) = **1,609.344** m

1 mile [nautical] (nmi) = 1.150 78 mile (international) = 6,076.115 ft (international) = **1,852** m

1 mile [U.S. statute based on U.S. survey foot] (mi) = **5,280** ft (U.S. survey) = **880** fathom = **320** rod = **8** furlong = 1,609.347 m

1 mile per hour (mi/h) = **88** ft/min = 1.466 667 ft/s = 1.609 344 km/h = 0.447 04 m/s

1 mile per minute (mi/min) = **5,280** ft/min = **60** mi/h = **88** ft/s = 26.822 4 m/s

1 miner's inch ≈ 1.5 ft^3 water per minute (historical variation with mining district and state)

1 ounce [apothecary] (ap oz) = **8** dram = **24** scruple = **480** grain = 31.103 48 gram (g)

1 ounce [avoirdupois] (av oz) = **16** dram (dr av) = **437.5** grain (gr) = 0.911 458 tr oz = 28.349 52 gram (g)

1 ounce [troy] (tr oz) = **20** pennyweight (dwt) = **480** grain (gr) = 1.097 143 av oz = 31.103 48 gram (g)

1 ounce (troy) per short ton = 34.285 718 gram/metric ton or tonne

1 part per million (ppm) = **0.0001** percent = **1** g/m^3

1 peck [U.S.] (pk) = **8** qt (U.S. dry) = $\mathbf{4^{-1}}$ bushel = 537.605 $in.^3$ = 8.809 768 liter (L)

1 pennyweight (dwt) = **24** grain = $\mathbf{20^{-1}}$ tr oz = 1.555 174 gram (g)

1 perch of masonry = 16.5 ft × 1.5 ft × 1 ft = 24.75 ft^3 ≈ 25 ft^3

1 pint (Imperial) = 0.568 261 25 L

1 pint [U.S. liquid] (liq pt) = **4** gill (U.S.) = $\mathbf{2^{-1}}$ qt (U.S. liquid) = **2** cup (U.S.) = **16** fl oz (U.S.) = 28.875 0 $in.^3$ = 0.473 176 5 liter (L)

1 pound [avoirdupois] (lb av) = **16** oz av = **256** dram av = **7,000** grain = 1.215 278 lb tr = **453.592 37** g

1 pound [troy] (lb tr) = **12** oz tr = **240** pennyweight (dwt) = **5,760** grain (gr) = 0.822 857 lb av = 373.241 7 g

1 pound (av) per cubic foot (lb/ft^3) = 16.018 463 kg/m^3

1 pound per square inch ($lb/in.^2$) = 2.306 659 ft_{H2O} = 6,894.757 Pa

(continues)

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TABLE 1.6 **Selected unit equivalencies and approximations** (factors in **boldface** are exact) (continued)

1 quart [U.S. liquid] (liq qt) = **2** pt (U.S. liquid) = **32** fl oz (U.S.) = **57.75** $in.^3$ = 0.946 352 9 liter (L)
1 radian = 57.295 78 degree (°) = 57° 17′ 44.8″ = **180/**π (°)
1 revolution (rev) = **1** turn = **360** degree (°) = **21,600** minute (′) = **1 296 000** second (″) = **2**π rad = 6.283 185 307 rad
1 rod (based on U.S. survey foot) = **25** link = **16.5** ft (U.S. survey) = 5.029 210 m
1 slug = 32.174 05 lb = 14.593 90 kg
1 span = 9 in. = 0.228 6 m
1 square foot (ft^2) = **144** $in.^2$ = **0.092 903 04** m^2
1 square inch ($in.^2$) = **6.451 6** cm^2
1 square mile [international] (mi^2) = **27,878,400** ft^2 (international) = 2,589,988 m^2
1 square mile [U.S. statute] (mi^2) = **1** section = **640** acre = **36**$^{-1}$ township = **27,878,400** ft^2 (U.S. survey) = 2,589,998 m^2
1 square yard (yd^2) = **9** ft^2 = **1,296** $in.^2$ = **0.836 127 36** m^2
1 tablespoon = **3** teaspoon = **16**$^{-1}$ cup = 14.786 76 mL
1 ton [long] (t) = **2,240** lb = 1,016.047 kg
1 ton [metric or tonne] (t) = **1,000,000** g = **1,000** kg = 1.102 311 ton (short) = 2,204.623 lb av
1 ton [short] (t) = **2,000** lb = 907.184 7 kg = 0.907 184 7 ton (metric) or tonne
1 ton (short) per cubic yard (t/yd^3) = 1.186 553 tonne/m^3 = 1,186.553 kg/m^3
1 yard (yd) = **36** in. = **3** ft = **0.914 4** m
1 watt (W) = 0.001 341 022 hp (550 ft·lbf/s) = 0.737 562 ft·lbf/s = 0.001 340 483 hp (electric) = **1** joule/s

SAFETY SYMBOLS

Safety symbols are an important part of identifying safety and health hazards in the workplace.

Hazard Assessments

The fire/hazard diamond in Figure 1.1 summarizes common hazard data available on the Safety Data Sheet (SDS) and is frequently shown on chemical labels.

Position A – Health Hazard (Blue)
0 = Normal material
1 = Slightly hazardous
2 = Hazardous
3 = Extreme danger
4 = Deadly

Position B – Flammability Hazard (Red)
0 = Will not burn
1 = Will ignite if preheated
2 = Will ignite if moderately heated
3 = Will ignite at most ambient temperature
4 = Burns readily at ambient conditions

Position C – Reactivity Hazard (Yellow)
0 = Stable and not reactive with water
1 = Unstable if heated
2 = Violent chemical change
3 = Shock short may detonate
4 = May detonate

Position D – Special/Specific Hazard (White)
ACID = Acid
ALKALI = Alkali
Cor = Corrosive
OXY = Oxidizer
W = Use no water
= Radiation hazard

Source: OSHA 2005, as appears in NCEES 2013

Figure 1.1 Hazard diamond

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Globally Harmonized System of Classification and Labeling of Chemicals

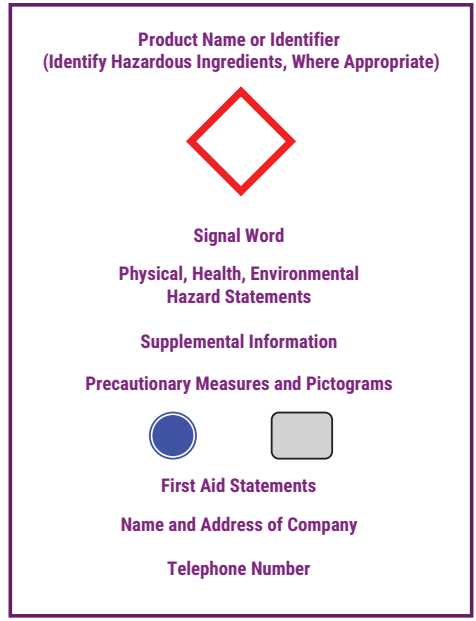
The *Globally Harmonized System of Classification and Labeling of Chemicals*, or GHS, was adopted by the United Nations in 2003. The GHS specifies "what information should be included on labels of hazardous chemicals" (OSHA, n.d.) and on the SDS. GHS is a comprehensive approach to

- Defining health, physical, and environmental hazards of chemicals;
- Creating classification processes that use available data on chemicals for comparison with the defined hazard criteria; and
- Communicating hazard information, as well as protective measures, on labels and SDSs, formerly called Material Safety Data Sheets (MSDSs).

GHS label elements include the following:

- Precautionary statements and pictograms, which are designed to minimize or prevent adverse effects
- Product identifiers (ingredient disclosure) with the names or numbers used for a hazardous product on a label or in the SDS
- Supplier identification with the name, address, and telephone number of the supplier
- Supplemental information, which includes non-harmonized information

Other label elements include symbols, signal words, and hazard statements. Figures 1.2 through 1.5 depict Occupational Safety and Health Administration (OSHA) and GHS labeling information.



Source: OSHA 2005, as appears in NCEES 2013

FIGURE 1.2 GHS label elements

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• OXIDIZERS

• FLAMMABLES
• SELF-REACTIVES
• PYROPHORICS
• SELF-HEATING
• EMITS FLAMMABLE GAS
• ORGANIC PEROXIDES

• EXPLOSIVES
• SELF-REACTIVES
• ORGANIC PEROXIDES

• ACUTE TOXICITY (SEVERE)

• CORROSIVES

• GASES UNDER PRESSURE

• CARCINOGEN
• RESPIRATORY SENSITIZER
• REPRODUCTIVE TOXICITY
• TARGET ORGAN TOXICITY
• MUTAGENICITY
• ASPIRATION TOXICITY

• ENVIRONMENTAL TOXICITY

• IRRITANT
• DERMAL SENSITIZER
• ACUTE TOXICITY (HARMFUL)
• NARCOTIC EFFECTS
• RESPIRATORY TRACT IRRITATION

Source: OSHA 2005, as appears in NCEES 2013

FIGURE 1.3 GHS pictograms and hazard classes

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Source: OSHA 2005, as appears in NCEES 2013

FIGURE 1.4 Transport pictograms

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	CATEGORY 1	CATEGORY 2	CATEGORY 3	CATEGORY 4	CATEGORY 5
LD_{50}	≤ 5 mg/kg	> 5 < 50 mg/kg	≥ 50 < 300 mg/kg	≥ 300 < 2,000 mg/kg	≥ 2,000 < 5,000 mg/kg
PICTOGRAM					NO SYMBOL
SIGNAL WORD	DANGER	DANGER	DANGER	WARNING	WARNING
HAZARD STATEMENT	FATAL IF SWALLOWED	FATAL IF SWALLOWED	TOXIC IF SWALLOWED	HARMFUL IF SWALLOWED	MAY BE HARMFUL IF SWALLOWED

Source: OSHA 2005, as appears in NCEES 2013

FIGURE 1.5 Acute oral toxicity

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CHAPTER

2

Material Properties

Jack W. Burgess, P.E.

Engineering properties of natural earth-related materials are largely compiled based on their important uses to society. Materials considered here are soils, rocks, minerals, and coal; various properties related to each material are also presented. Although the following discussion is not all-inclusive, materials most commonly encountered in mining are covered.

In many cases, several physical properties allow a particular material to be readily identified. For example, color, particle size, crystal system, hardness, and chemical or metal content often assist in identification.

Table 2.1 shows bank and loose densities, angles of repose, and swell factors of common mining-related materials. Table 2.2 presents general swell and void percentages and related load factors.

SOILS

Soils include gravels, sands, silts, clays, organic soils, and permafrost. They are classified on the basis of index properties, such as particle size distribution and plasticity characteristics. From an engineering standpoint, other important physical properties include natural water content, density, permeability, shear strength, and compressibility (Sherman 1973).

Tables 2.3 through 2.8 cover soils. Particle size of a soil is expressed in boulders, cobbles, gravel, sand, silt, and clay; Table 2.3 lists customary sizes. The Unified Soil Classification System, commonly used to classify soils for engineering purposes, is shown in Table 2.4. Table 2.5 shows weight (saturated and dry), friction angle, and cohesion for typical soils and rocks. Strength characteristics of soils are in Table 2.6, and Table 2.7 gives typical soil modulus values. Table 2.8 lists important engineering properties and uses for various soils.

Permafrost, found in northern locations, is frozen ground, no matter what its other soil or rock attributes may be. Associated physical properties can be completely different from unfrozen similar materials and may pose serious problems, such as solifluction or mass creep, in a zone where thawing occurs.

ROCKS

Rock is a naturally occurring solid material consisting of one or more minerals (Figure 2.1). Properties of rocks are shown in Table 2.9. For more information on strength properties, see Table 14.1 in the chapter on ground control and support.

MINERALS

Minerals are solid, naturally occurring chemical elements or compounds that are homogenous, with a definite chemical composition and a very regular arrangement of atoms. More than 3,000 mineral species are known, most of which are characterized by definite chemical composition, crystalline structure, and physical properties. They are classified primarily by chemical composition, crystal class, hardness, and appearance (color, luster, and opacity). Mineral species are generally limited to solid substances; the only liquids are metallic mercury and water. Metalliferous minerals of economic value, which are mined for their metals, are known as *ores*. Mineral properties are shown in Table 2.10, and the properties of major mineral fillers are shown in Table 2.11.

COAL

Coal is a generic designation for many solid organic minerals with different compositions and properties. All are rich in carbon and have a dark color. A genetic relationship exists between peat, brown coal, lignite, bituminous coal, and anthracite. The process of coal formation, or coalification, is a continuous transformation of plant material, with each phase characterized by the degree of coalification (Hower and Parekh 1991). Table 2.12 provides analyses for major seams in the coal-producing states, and Table 2.13 gives classification of coals by rank.

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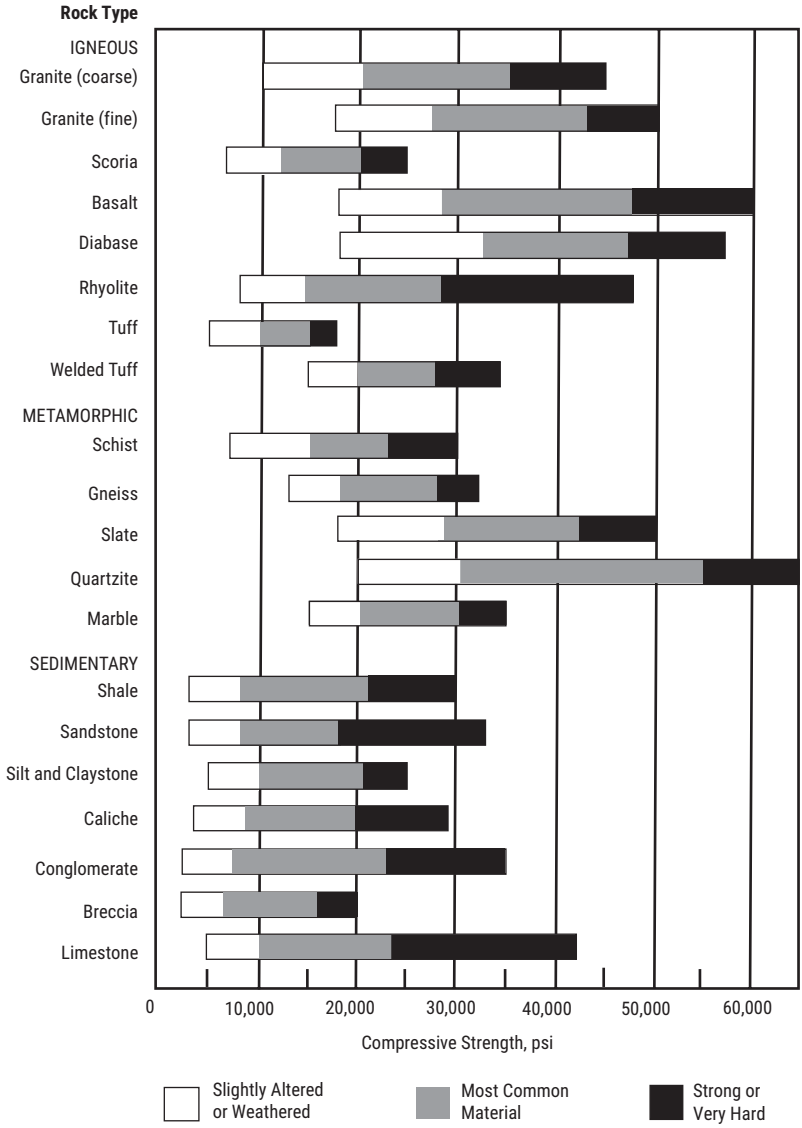

Source: Caterpillar Inc. 1997

FIGURE 2.1 Compressive strength of common rocks

Specific gravity of coal ranges from 1.23 to 1.72 depending on rank, moisture, and ash content; and it tends higher in the range as each increases (Hower and Parekh 1991).

Typical proximate and ultimate analyses are shown in Table 2.14. Table 2.15 shows petrographic and physical properties of coal. Table 2.16 gives sulfur content and forms. For ash content and fusion temperature, see Table 2.17. And for information on strength properties, see Table 14.1 in the chapter covering ground control and support.

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TABLE 2.1 Properties of common mining-related materials

Material	Bank Density, lb/ft^3	Loose Density, lb/ft^3	Angle of Repose, degrees	Swell Factor
Alumina	–	60	22	–
Ammonium nitrate	–	45	–	–
Asbestos ore	–	81	30–44	–
Ashes, dry	–	35–40	40	–
Ashes, wet	–	45–50	50	–
Bauxite, crushed, 3 × 0 in.	–	75–85	30–44	–
Bauxite, ground, dry	–	68	20–31	–
Bauxite, run-of-mine	100–160	75–120	31	0.75
Clay, compact, natural bed	109	82	–	0.75
Clay, dense, tough or wet	111	83	–	0.75
Clay, dry	85	68	–	0.80
Clay, dry excavated	69	–	–	–
Clay, dry in lump, loose	–	60–70	35	–
Clay, fines	–	100–120	35	–
Clay, light (kaolin)	104	80	–	0.77
Clay and gravel, dry	100	71	–	0.71
Clay and gravel, wet	114	81	–	0.71
Chrome ore	–	125–140	30–44	–
Cinders, coal	–	40–45	35	–
Coal, anthracite	81–85	60–63	27	0.74
Coal, anthracite, sized	–	55–60	27	–
Coal, bituminous	80	50–52	45–55	0.62–0.65
Coal, bituminous, mined, run-of-mine	–	45–55	38	–
Coal, bituminous, mined, sized	–	45–55	35	–
Coal, bituminous, mined, slack, ½ in. and under	–	43–50	40	–
Coal, bituminous, strip, not cleaned	–	50–60	–	–
Coal, lignite	–	40–45	38	–
Coke	–	24–31	–	–
Coke, breeze, ¼ in. and under	–	25–34	30–45	–
Coke, loose	–	23–35	30–44	–
Coke, petroleum	–	35–40	–	–
Copper ore	141	100	–	0.71
Earth, dry	104	57–83	35	0.55–0.80
Earth, dry, loam	78	57–68	–	0.73–0.87
Earth, moist	100	75–85	–	0.75–0.85
Earth, rock	93–119	71–91	–	0.76
Earth, sand, gravel	115	98	–	0.85
Earth, wet	125	100–104	–	0.80–0.83
Earth, wet, containing clay	–	100–110	45	–
Feldspar, ½-in. screenings	–	70–85	38	–
Feldspar, 1½–3 in.	–	90–100	34	–
Feldspar, 200 mesh	–	100	30–44	–
Gneiss	168	96	–	0.57
Granite	167	90–111	–	0.54–0.66

(continues)

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TABLE 2.1 Properties of common mining-related materials (continued)

Material	Bank Density, lb/ft^3	Loose Density, lb/ft^3	Angle of Repose, degrees	Swell Factor
Granite and porphyry	170	97	–	0.57
Graphite ore	–	65–75	30–44	–
Gravel, dry	91–120	46–107	–	0.51–0.89
Gravel, dry, screened	–	90–100	40	–
Gravel, run-of-bank	–	90–100	38	–
Gravel, wet	144	131	–	0.91
Gypsum	163–167	100–111	–	0.61–0.66
Gypsum, ½-in. screenings	–	70–80	40	–
Gypsum, 1½–3 in.	–	70–80	30	–
Iron ore	–	100–200	35	–
Iron ore, hematite	241–322	144–145	–	0.60–0.45
Iron ore, pellets	–	116–130	30–44	–
Iron ore, taconite	150–200	107–143	–	0.71–0.72
Kaolin	104	80	–	0.77
Lead ore	–	200–270	30	–
Lime, pebble	–	53–56	30	–
Limestone	163	99	–	0.61
Limestone, blasted	156	89–93	–	0.57–0.60
Limestone, crushed	–	85–90	38	–
Limestone, marble	170	97–101	–	0.57–0.59
Manganese ore	–	125–140	39	–
Mud, dry	80–110	66–91	–	0.82–0.83
Mud, wet	110–130	91–108	–	0.83
Nickel–cobalt sulfate ore	–	80–150	30–44	–
Rock, crushed	–	125–145	20–29	–
Rock, soft, excavated with shovel	–	100–110	30–44	–
Rock, stone, crushed	120–145	89–107	–	0.74
Rock, well-blasted	148	99	–	0.67
Sand, bank, damp	–	105–130	45	–
Sand, bank, dry	–	90–110	35	–
Sand, dry	81–126	70–115	–	0.86–0.91
Sand, moist	126	110	–	0.87
Sand and gravel, dry	123	108	–	0.88
Sand and gravel, wet	144	125	–	0.87
Sandstone	144–153	96–110	–	0.67–0.72
Sandstone, broken	–	85–90	30–44	–
Shale, broken	–	90–100	20–29	–
Shale, crushed	–	85–90	39	–
Shale, riprap	104	78	–	0.75
Slag	136	110	–	0.81
Slate	170–180	131–139	–	0.77
Stone, crushed	120–145	89–107	–	0.74
Sulfur ore	–	87	–	–
Trap rock	185	122–124	–	0.66–0.67
Zinc ore, crushed	–	160	38	–

Adapted from Hartman 1992

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TABLE 2.2 Load factors from swell and void percentages

Swell, %	Voids, %	Load Factor
5	4.8	0.952
10	9.1	0.909
15	13.0	0.870
20	16.7	0.833
25	20.0	0.800
30	23.1	0.769
35	25.9	0.741
40	28.6	0.714
45	31.0	0.690
50	33.3	0.667
55	35.5	0.645
60	37.5	0.625
65	39.4	0.606
70	41.2	0.588
75	42.9	0.571
80	44.4	0.556
85	45.9	0.541
90	47.4	0.526
95	48.7	0.513
100	50.0	0.500

TABLE 2.3 Particle sizes of soils

Material		Sizes, mm
Boulders		More than 200
Cobbles		60–200
Gravel	Coarse	20–60
	Medium	6–20
	Fine	2–6
Sand	Coarse	0.6–2
	Medium	0.2–0.6
	Fine	0.06–0.2
Silt	Coarse	0.02–0.06
	Medium	0.006–0.02
	Fine	0.002–0.006
Clay		Less than 0.002

Source: Wagner 1957

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TABLE 2.4 Unified Soil Classification System

Major Divisions			Group Symbols[1]	Typical Names	Field Identification Procedures (excluding particles larger than 3 in. and basing fractions on estimated weights)	Information Required for Describing Soils	Laboratory Classification Criteria	
1	2		3	4	5	6	7	
Coarse-Grained Soils More than 50% of material is *larger* than No. 200 sieve size.[2] The No. 200 sieve size is about the smallest particle visible to the naked eye.	Gravels More than 50% of coarse fraction retained is larger than No. 4 sieve size. (For visual classification, the ¾-in. size may be used as equivalent to the No. 4 sieve size.)	Clean Gravels (less than 5% fines)[3]	GW	Well-graded gravels, gravel-sand mixtures, little or no fines[4]	Wide range in grain sizes and substantial amounts of all intermediate particle sizes	For undisturbed soils, add information on stratification, degree of compactness, cementation, moisture conditions, and drainage characteristics. Give typical name; indicate approximate percentages of sand and gravel, maximum size; angularity, surface condition, and hardness of the coarse grains; local or geologic name and other pertinent descriptive information; and symbol in parentheses. Example: Silty sand, gravelly; about 20% hard, angular gravel particles, ½-in. maximum size; rounded and subangular sand grains, coarse to fine; about 15% nonplastic fines with low dry strength; well compacted and moist in place; alluvial sand (SM).	Use grain-size curve in identifying the fractions as given under field identification.	$C_u = \frac{D_{60}}{D_{10}}$ Greater than 4 $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			GP	Poorly graded gravels or gravel-sand mixtures, little or no fines[4]	Predominantly one size or a range of sizes with some intermediate sizes missing			Not meeting all gradation requirements for GW
		Gravels with Fines (more than 12% fines)[3]	GM	Silty gravels, gravel-and-silt mixtures[4,5,6]	Nonplastic fines or fines with low plasticity (for identification procedures, see ML)		Use grain-size curve in identifying the fractions as given under field identification. Determine percentages of gravel and sand from grain-size curve. Depending on percentage of fines (fraction smaller than No. 200 sieve size), coarse-grained soils are classified as follows: Less than 5% = GW, GP, SW, SP More than 12% = GM, GC, SM, SC 5% to 12% = *Borderline* cases requiring use of dual symbols	Atterberg limits below "A" line or PI less than 4 — Above "A" line with PI between 4 and 7 are *borderline* cases requiring use of dual symbols.
			GC	Clayey gravels, gravel-sand-clay mixtures[4,5,6]	Plastic fines (for identification procedures, see CL)			Atterberg limits above "A" line with PI greater than 7 — Above "A" line with PI between 4 and 7 are *borderline* cases requiring use of dual symbols.
	Sands More than 50% of coarse fraction retained is smaller than No. 4 sieve size.	Clean Sands (less than 5% fines)[7]	SW	Well-graded sands, gravelly sands, little or no fines	Wide range in grain size and substantial amounts of all intermediate particle sizes			$C_u = \frac{D_{60}}{D_{10}}$ Greater than 4 $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			SP	Poorly graded sands or gravelly sands, little or no fines with some intermediate sizes missing	Predominantly one size or a range of sizes with some intermediate sizes missing			Not meeting all gradation requirements for SW
		Sands with Fines (more than 12% fines)[7]	SM	Silty sands, sand-silt mixtures[5,6,8]	Nonplastic fines or fines with low plasticity (for identification procedures, see ML)			Atterberg limits below "A" line or PI less than 4 — Above "A" line with PI between 4 and 7 are *borderline* cases requiring use of dual symbols.
			SC	Clayey sands, sand-clay mixtures[5,6,8]	Plastic fines (for identification procedures, see CL)			Atterberg limits above "A" line with PI greater than 7 — Above "A" line with PI between 4 and 7 are *borderline* cases requiring use of dual symbols.

(continues)

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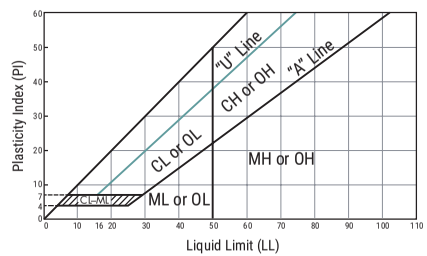
TABLE 2.4 Unified Soil Classification System (continued)

Major Divisions			Group Symbols[1]	Typical Names	Identification Procedures on Fractions Smaller than No. 40 Sieve Size			Information Required for Describing Soils	Laboratory Classification Criteria
1	2		3	4	5			6	7
					Dry Strength (crushing characteristics)	Dilatancy (reaction to shaking)	Toughness (consistency near PL)	For undisturbed soils, add information on structure, stratification, consistency in undisturbed and remolded states, moisture, and drainage conditions. Give typical name; indicate degree and character of plasticity; amount and maximum size of coarse grains; color in wet condition; odor, if any; local or geologic name and other pertinent descriptive information; and symbol in parentheses. Example: Clayey silt, brown; slightly plastic; small percentage of fine sand; numerous vertical root holes; firm and dry in place; loess (ML).	Use grain-size curve in identifying the fractions as given under field identification.
Fine-Grained Soils More than 50% of material is *smaller* than No. 200 sieve size.[2] The No. 200 sieve size is about the smallest particle visible to the naked eye.	Silts and Clays Liquid limit is less than 50.	ML	Inorganic silts and very fine sands, rock flour, silty or clayey fine sands or clayey silts with slight plasticity[9,10,11]	None to slight	Quick to slow	None			
		CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays[9,10,11]	Medium to high	None to very slow	Medium			
		OL	Organic silts[9,10,11,13] and organic silty clays of low plasticity[9,10,11,12]	Slight to medium	Slow	Slight			
	Silts and Clays Liquid limit is greater than 50.	MH	Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts[9,10,11]	Slight to medium	Slow to none	Slight to medium			
		CH	Inorganic clays of high plasticity, fat clays[9,10,11]	High to very high	None	High			
		OH	Organic clays of medium to high plasticity,[9,10,11,13] organic silts[9,10,11,14]	Medium to high	None to very slow	Slight to medium			
Highly Organic Soils		Pt	Peat and other highly organic soils	Readily identified by color, odor, spongy feel, and frequently by fibrous texture					

For classification of fine-grained soils and fine-grained fraction of coarse-grained soils.

Equation of "A" – line
Horizontal at PI = 4 to LL = 25.5,
then PI = 0.73 (LL – 20)

Equation of "U" – line
Vertical at LL = 16 to PI = 7,
then PI = 0.9 (LL – 8)



Notes:

1. Boundary classifications: Soils possessing characteristics of two groups are designated by combinations of group symbols. For example, GW–GC, well-graded gravel-sand mixture with clay binder.
2. All sieve sizes on this chart are U.S. standard.
3. Gravels with 5%–12% fines require dual symbols:
   GW–GM = well-graded gravel with silt
   GW–GC = well-graded gravel with clay
   GP-GM = poorly graded gravel with silt
   GP–GC = poorly graded gravel with clay.
4. If soil contains ≥15% sand, add "with sand" to group name.
5. If fines classify as CL–ML, use dual symbol GC–GM or SC–SM.
6. If fines are organic, add "with organic fines" to group name.
7. Sands with 5%–12% fines require dual symbols:
   SW–SM = well-graded gravel with silt
   SW–SC = well-graded gravel with clay
   SP–SM = poorly graded gravel with silt
   SP–SC = poorly graded gravel with clay.
8. If soil contains ≥15% gravel, add "with gravel" to group name.
9. If soil contains 15% to <30% plus No. 200, add "with sand" or "with gravel," whichever is predominant.
10. If soil contains ≥30% plus No. 200, predominantly sand, add "sand" to group name.
11. If soil contains ≥30% plus No. 200, predominantly gravel, add "gravelly" to group name.
12. PI ≥ 4 and plots on or above "A" line.
13. PI < 4 or plots below "A" line.
14. PI plots below "A" line.

Source: U.S. Army Corps of Engineers 1953 and ASTM D2487-17

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TABLE 2.5 Unit weight, friction angle, and cohesion for soils and rocks

Description		Unit Weight (saturated/dry)		Friction Angle, degrees	Cohesion	
Type	Material	kN/m^3	lb/ft^3		kPa	lb/ft^2
Cohesionless						
Sand	Loose sand, uniform grain size	19/14	118/90	28–34*	–	–
	Dense sand, uniform grain size	21/17	130/109	32–40*	–	–
	Loose sand, mixed grain size	20/16	124/99	34–40*	–	–
	Dense sand, mixed grain size	21/18	135/116	38–46*	–	–
Gravel	Gravel, uniform grain size	22/20	140/130	34–37*	–	–
	Sand and gravel, mixed grain size	19/17	120/110	48–45*	–	–
Blasted/ broken rock	Basalt	22/17	140/110	40–50*	–	–
	Chalk	13/10	80/62	30–40*	–	–
	Granite	20/17	125/110	45–50*	–	–
	Limestone	19/16	120/100	35–40*	–	–
	Sandstone	17/13	110/80	35–45*	–	–
	Shale	20/16	125/100	30–35*	–	–
Cohesive						
Clay	Soft bentonite	13/6	80/30	7–13	10–20	200–400
	Very soft organic clay	14/6	90/40	12–16	10–30	200–600
	Soft, slightly organic clay	16/10	100/60	22–27	20–50	400–1,000
	Soft glacial clay	17/12	110/76	27–32	30–70	600–1,500
	Stiff glacial clay	20/17	130/105	30–32	70–150	1,500–3,000
	Glacial till, mixed grain size	23/20	145/130	32–35	150–250	3,000–5,000
Rock	Hard igneous rocks—granite, basalt, porphyry	25–30	160–190†	35–45	35,000–55,000	720,000–1,150,000
	Metamorphic rocks—quartzite, gneiss, slate	25–28	160–180	30–40	20,000–40,000	400,000–800,000
	Hard sedimentary rocks—limestone, dolomite, sandstone	23–28	150–180	35–45	10,000–30,000	200,000–600,000
	Soft sedimentary rock—sandstone, coal, chalk, shale	17–23	110–150	25–35	1,000–20,000	20,000–400,000

* Higher friction angles in cohesionless materials occur at low confining or normal stresses.

† For intact rock, the density of the material does not significantly vary between saturated and dry states with the exception of some materials such as porous sandstones.

Adapted from Houk and Bray 1977

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TABLE 2.6 Typical strength characteristics of soils

Group Symbol*	Cohesion (compacted), psf c	Cohesion (saturated), psf c_{sat}	Effective Stress Envelope, degrees ϕ
GW	0	0	>38
GP	0	0	>37
GM	—	—	>34
GC	—	—	>31
SW	0	0	38
SP	0	0	37
SM	1,050	420	34
SM–SC	1,050	300	33
SC	1,550	230	31
ML	1,400	190	32
ML–CL	1,350	460	32
CL	1,800	270	28
OL	—	—	—
MH	1,500	420	25
CH	2,150	230	19
OH	—	—	—

* See Tables 2.4 and 2.8 for group symbol definitions.

Source: Lindeburg 1992

TABLE 2.7 Typical soil modulus values

Type of Soil	Depth of Cover, ft	Standard AASHTO* Relative Compaction			
		85%	90%	95%	100%
Fine-grained soils with less than 25% sand content (CL, ML, CL–ML†)	0–5	500	700	1,000	1,500
	5–10	600	1,000	1,400	2,000
	10–15	700	1,200	1,600	2,300
	15–20	800	1,300	1,800	2,600
Coarse-grained soils with fines (SM, SC)	0–5	600	1,000	1,200	1,900
	5–10	900	1,400	1,800	2,700
	10–15	1,000	1,500	2,100	3,200
	15–20	1,100	1,600	2,400	3,700
Coarse-grained soils with little or no fines (SP, SW, GP, GW)	0–5	700	1,000	1,600	2,500
	5–10	1,000	1,500	2,200	3,300
	10–15	1,050	1,600	2,400	3,600
	15–20	1,100	1,700	2,500	3,800

* AASHTO = American Association of State Highway and Transportation Officials.

† See Tables 2.4 and 2.8 for group symbol definitions.

Notes: Values of modulus of soil reaction, E' (psi) based on depth of cover, type of soil, and relative compaction. Soil type symbols are from the Unified Soil Classification System.

Source: Hartley and Ducan 1987

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TABLE 2.8 Soil engineering properties and uses

Typical Names of Soil Groups	Group Symbols	Important Properties				Relative Desirability for Various Users (graded from 1 [highest] to 14 [lowest])									
						Rolled Earth Dams			Canal Sections		Foundations		Roadways		
													Fills		
		Permeability When Compacted	Shearing Strength When Compacted and Saturated	Compressibility When Compacted and Saturated	Workability as a Construction Material	Homogeneous Embankment	Core	Shell	Erosion Resistance	Compacted Earth Lining	Seepage Important	Seepage Not Important	Frost Heave Not Possible	Frost Heave Possible	Surfacing
Well-graded gravels, gravel-sand mixtures, little or no fines	GW	Pervious	Excellent	Negligible	Excellent	–	–	1	1	–	–	1	1	1	1
Poorly graded gravels, gravel-sand mixtures, little or no fines	GP	Very pervious	Good	Negligible	Good	–	–	2	2	–	–	3	3	3	–
Silty gravels, poorly graded gravel-sand-silt mixtures	GM	Semipervious to impervious	Good	Negligible	Good	2	4	–	4	4	1	4	4	9	5
Clayey gravels, poorly graded gravel-sand-clay mixtures	GC	Impervious	Good to fair	Very low	Good	1	1	–	3	1	2	6	5	5	1
Well-graded sands, gravelly sands, little or no fines	SW	Pervious	Excellent	Negligible	Excellent	–	–	3 if gravelly	6	–	–	2	2	2	4
Poorly graded sands, gravelly sands, little or no fines	SP	Pervious	Good	Very low	Fair	–	–	4 if gravelly	7 if gravelly	–	–	5	6	4	–
Silty sands, poorly graded sand-silt mixtures	SM	Semipervious to impervious	Good	Low	Fair	4	5	–	8 if gravelly	5 erosion critical	3	7	8	10	6
Clayey sands, poorly graded sand-clay mixtures	SC	Impervious	Good to fair	Low	Good	3	2	–	5	2	4	8	7	6	2
Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity	ML	Semipervious to impervious	Fair	Medium	Fair	6	6	–	–	6 erosion critical	6	9	10	11	–
Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays	CL	Impervious	Fair	Medium	Good to fair	5	3	–	9	3	5	10	9	7	7
Organic silts and organic silt-clays of low plasticity	OL	Semipervious to impervious	Poor	Medium	Fair	8	8	–	–	7 erosion critical	7	11	11	12	–
Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	MH	Semipervious to impervious	Fair to poor	High	Poor	9	9	–	–	–	8	12	12	13	–
Inorganic clays of high plasticity, fat clays	CH	Impervious	Poor	High	Poor	7	7	–	10	8 volume change	9	13	13	8	–
Organic clays of medium to high plasticity	OH	Impervious	Poor	High	Poor	10	10	–	–	–	10	14	14	14	–
Peat and other highly organic soils	Pt	–	–	–	–	–	–	–	–	–	–	–	–	–	–

Source: Wagner 1957

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TABLE 2.9 Properties of rocks

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Igneous															
Amphibolite	McLeese Lake, British Columbia	2,920	2.65	8				0.24			1.75	10			8
Amphibolite, fine	Oorguam, Mysore State, India	3,070	4.23	3	92	5.79	4		4.58	10	1.04	11			20
Andesite, hypersthene	Palisades Dam, Idaho	2,570	1.29–1.32	9				0.18					5.45	10	1
Anorthosite, Labradorite, C.	Ukrainian Shield, USSR	2,770	2.27	3				0.36	3.41	10	9.28	10			2
Basalt	Painesdale, Michigan	2,850	2.30	3	69	4.63	4		2.68	10	6.15	10			20
	Ahmeek, Michigan	2,940	2.58–3.59	3	79	5.15	4		3.17	10	7.79	10			20
Basalt, dense		2,593	1.13	3				0.20			5.21	10			19
		2,761	1.32	3		5.56	4	0.17			7.65	10			19
		2,752	1.25	3		4.70	4				5.79	10			19
Basalt, Lower Granite	Pullman, Washington	2,727	2.27–3.55	3	57:412:116	5.27	4				5.02	10			7
Basalt, Olivine, dense	Nevada Test Site, Nevada	2,720									2.47	10			15
Basalt, Olivine, sl., vesicular		2,660									2.86	10			15
Basalt, Olivine, Western Cascade	Medford, Oregon	2,730	1.69–2.20	3				0.25					4.21	10	1
Basalt, subaqueous	Eniwetok, PTT	2,860	1.94	3	71			0.18			6.93	10			5
Basalt, vesicular	Bergstrom, Texas	2,550	7.44	8		4.65	4	0.13			3.74	10			19
		2,580	8.34	8		5.04	4	0.19			4.05	10			19
Charnockite (hypersthene granite)	Ukrainian Shield, USSR	2,730	2.47	3				0.22	2.75	10	6.73	10			2
Diabase, altered	Clinton County, New York	2,940	3.21	3	92	5.70	4		3.73	10	9.58	10			20
Diabase, French Creek	St. Peters, Pennsylvania	3,060	3.01	3	58						9.94	10			3
Diabase, Medford	Cambridge, Massachusetts	2,882	1.77	3	44:2.13:60										3
Diabase, Palisades	West Nyack, New York	2,932	2.41	3	59						8.19	10			3
Diorite, augite, altered		2,720	2.15	3	71	4.94	4	0.30	2.56	10	6.64	10			21
Diorite, augite, fresh	Keetley, Utah	2,740	3.33	3	82	5.55	4	0.25	3.37	10	8.41	10			21
Diorite, augite, slightly altered		2,720	2.79	3	83	5.43	4	0.26	3.18	10	8.00	10			21
Diorite, biotite, porph., s. altered		2,660	1.80	3	67	4.75	4	0.22	2.45	10	6.01	10			21
Diorite, biotite, porph., sl. altered		2,690	2.28	3	77	4.97	4	0.27	2.83	10	6.68	10			21
Diorite, gneissic	Mineville, New York	3,030	1.86	3	90	4.27	4		2.78	10	5.53	10			20
Diorite, hornblende	Ishpeming, Michigan	3,010	2.74	3	84	6.00	4	0.29	4.22	10	1.07	11			5
Diorite, Kensington	Washington, District of Columbia	2,820	8.09(7)–2.76[a]		08:07.7										3

(continues)

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Gabbro/diabase	Clinton County, New York	2,930	2.77	3	82	5.36	4		3.36	10	8.48	10			20
	Ukrainian Shield, USSR	3,000	3.09	3				0.33	4.41	10	1.19	11			2
	Karelian SSS, USSR	3,190	3.14	3							1.17	11			2
Gabbro, Salem	Beverly, Massachusetts	3,060	1.33–1.49	3	52:6.47:129						8.76	10			3
Granite	Woodstock, Maryland	2,650	2.51	3	98	4.51	4		2.54	10	5.46	10			20
	Tem Piute District, Nevada	2,630	2.72	3	100	4.42	4		2.25	10	5.13	10			20
	Mount Airy, North Carolina	2,600	2.10	3	90	2.44	4		1.02	10	1.57	10			20
Granite, Barre	Barre, Vermont	2,643	1.94	3	53						6.15	10			15
Granite, biotite, Lithonia	Lithonia, Georgia	2,640	1.93	3	85	2.71	4	−0.19	1.18	10	1.91	10			4
		2,640	2.13	3	85	2.50	4	−0.23	1.09	10	1.64	10			4
		2,660	2.09	3	89	2.62	4	0.02	8.96	9	1.86	10			4
		2,620	2.05	3	85	1.08	4	−0.28	7.10	9	1.04	10			4
Granite, biotite, m.	Karelian SSS, USSR	2,700	2.39	3				0.25	2.41	10	6.93	10			2
Granite, c.		2,627	1.61	3	52:9.5:161	4.08	4				5.24	10			6
Granite, f.	Grand Coulee, Washington	2,571	1.94	3	53:10.5:172	4.64	4				5.48	10			6
Granite, f.-m; unaweep	Grand Junction, Colorado	2,670	1.74	3	59	3.17	4	−0.19	1.68	10	2.72	10			5
		2,710	1.59	3	53	3.75	4	0.00	1.91	10	3.82	10			5
Granite, par. to foliation; unaweep		2,730	1.61	3	44	3.93	4	0.12	1.90	10	4.23	10			5
Granite, Pikes Peak	Colorado Springs, Colorado	2,675	1.57	3	58						7.06	10			3
Granite, pink	Bergstrom, Texas	2,710				6.47	4	0.29	6.84	10	8.57	10			19
Granite, Pre-Cambrian	Loveland, Colorado	2,630	7.21	8				0.14					2.69	10	1
Granite, unaweep		2,660	1.74	3	37	3.17	4	−0.13	1.55	10	2.72	10			5
Granite, weathered		2,650				5.83	10	0.29	4.72	10	5.75	10			19
		2,620				5.33	10	0.30	4.65	10	5.36	10			19
Granodiorite	Bergstrom, Texas	2,689	4.07	8		5.72	4	0.70			5.84	10			19
		2,703	1.39	3		5.89	4	0.22			7.99	10			19
		2,699	8.51	8		5.80	4	0.17			6.87	10			19
		2,702	1.29	3		5.72	4	0.19			7.30	10			19
		2,700	1.15	3		5.93	4	0.19			7.10	10			20
Magnetite, ore	Mineville, New York	4,230	1.41	3	72	2.72	4		1.86	10	3.14	10			20
Monzonite, porphyritic, Colville	Grand Coulee, Washington	2,575	1.49	3				0.18					4.14	10	1
		2,575	1.71	3				0.15					4.21	10	1

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Pegmatite	Star Lake, New York	2,590	2.14	3	87	4.88	4		2.28	10	6.16	10			20
Pyroxenite	Clinton County, New York	3,450	1.70	3	70	1.98	4		1.03	10	1.31	10			20
Pyroxenite, fresh	Star Lake, New York	3,430	1.82	3	60	6.03	4		5.03	10	1.24	11			20
Pyroxenite, heavily altered		2,530	5.86	8	28	2.96	4		7.58	9	2.20	10			20
Quartz, diorite	Mountain Home, Idaho		8.74	8				0.05					2.14	10	1
Quartz, Monzonite	Bergstrom, Texas	2,669	1.48	3				0.70			6.74	10			19
		2,680	1.55	3				0.22			7.24	10			19
		2,670	1.30	3				0.17			6.68	10			19
		2,673	1.29	3				0.19			7.65	10			19
		2,667	1.39	3				0.19			7.72	10			19
Rapakivi (granite)	Ukrainian Shield, USSR	2,640	2.72	3				0.20	2.43	10	5.81	10			2
Shonkinite (dark syenite)	Clinton County, New York	3,350	1.85	3	78	3.23	4		1.94	10	3.54	10			20
Syenite	Kirkland Lake, Ontario	2,820	3.03	3		5.12	4		2.83	10	7.38	10			20
Syenite, porphyritic		2,700	4.34	3		5.12	4		3.03	10	7.10	10			20
Metamorphic															
Argillite, Cambridge	Dorchester, Massachusetts	2,810	1.36	3							8.41	10			3
	Cambridge, Massachusetts	2,642	6.61	8											3
		2,510	3.15	8	15:0.45:10										3
		2,759	1.55	3	26:1.2						4.83	10			3
		2,715	1.55	3							3.86	10			3
Gneiss	Bergstrom, Texas	2,710		3		4.58	4	0.15	2.59	10	5.38	10			19
		2,810		3		6.28	4	0.29	6.74	10	8.32	10			19
Gneiss, augite	Hackettstown, New Jersey	3,360	2.19	3	74	5.55	4	0.27	4.07	10	1.03	11			21
Gneiss, biotite		2,910	1.61	3	74	4.79	4	0.24	2.71	10	6.72	10			21
Gneiss, Diorite, Idaho Springs	Montezuma Quad., Colorado	2,865	8.41	8				0.06			6.41	10			18
Gneiss, Dworshak	Orofino, Idaho	2,804	1.62	3	48:						5.36	10			3
Gneiss, granite	Mineville, New York	2,750	2.12	3	99	3.63	4		1.96	10	3.85	10			20
Gneiss, granite, pegmatitic	Star Lake, New York	3,040	1.53	3	75	4.66	4		2.88	10	6.67	10			20
Gneiss, pegmatitic		2,650	1.96	3	81	4.11	4		2.12	10	4.46	10			20
Gneiss, quartz diorite	Bethesda, Maryland	2,775	9.60	8	64:5.17:139						7.24	10			3
Gneiss, schistose; Wissahickon	Washington, District of Columbia	2,980	7.01	8	46:2.90:79										3

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Greenstone	Mount Weather, Virginia	3,020	2.69	3	81	5.85	4		4.21	10	1.05	11			20
		2,960	3.05	3	80	5.21	4		3.86	10	8.07	10			20
Greenstone, amygdaloidal	Catoctin, Pennsylvania	3,040	2.01	3	64	3.99	4	−0.21	3.07	10	4.90	10			4
Hematite, ore	Soudan, Minnesota	5,070	6.07	3	74	6.28	4		7.79	10	2.00	11			20
Hematite, ore; par. bedding	Bessemer, Alabama	3,780	1.19	3	51	4.30	4		2.69	10	6.69	10			20
		3,670	1.39	3	50	4.30	4		2.70	10	6.73	10			20
Hornfels	Tem Piute District, Nevada	3,190	5.33	3		5.49	4		4.09	10	9.58	10			20
Marble, Cherokee	Tate, Georgia	2,707	6.69	8	36						5.59	10			3
Marble, dolomitic, f.	Karelian SSR, USSR	2,820	2.74	3				0.26	3.00	10	8.94	10			2
Marble, Oro Grande	Oro Grande, California	2,720	1.65	3	56	5.40	4	0.30	3.03	10	7.86	10			4
		2,680	5.52	8	42	4.90	4	0.16	2.80	10	6.52	10			4
Marble, Paleozoic	Ural Mountains, USSR	2,710	1.49	3							7.67	10			2
Marble, par. bedding		2,870	2.23	3	27				2.83	10	6.74	10			20
Marble, perp. bedding	Cockeysville, Maryland	2,870	2.12	3	56	4.18	4		2.61	10	4.93	10			20
Marble, Taconic	Rutland, Vermont	2,707	6.21	8	31						4.79	10			3
Metarhyolite	Soudan, Minnesota	2,840	1.25	3	47	5.06	4		3.16	10	7.86	10			20
Phyllite, graphitic	El Dorado County, California	2,350	6.69	7									9.65	9	1
Phyllite, green	Ishpeming, Michigan	3,240	1.26	3	40	4.85	4		3.28	10	7.65	10			20
Phyllite, quartzose		2,180	9.38	7									7.58	9	1
Phyllite, sericite	El Dorado County, California	2,340	9.79	7									1.79	10	1
Quartzite, altered		2,590				2.50	4				1.21	10			13
Quartzite, Biwabik	Babbitt, Minnesota	2,750	6.29	3		5.55	4	0.10	3.86	10	8.48	10			4
Quartzite, ferruginous	Kursk, USSR	3,510	3.43	3							1.71	11			2
Quartzite, hematic	Ishpeming, Michigan	4,070	2.93	3	71	5.21	4	0.20	4.06	10	9.79	10			5
Quartzite, phyllite lenses	Raven, Yugoslavia	2,590				8.22	2				1.27	9			13
Quartzite, Wissahickon	Washington, District of Columbia	2,804	4.71	8	38:2.78:63										3
Schist, biotite, Idaho Springs	Montezuma Quad., Colorado	2,720	2.09	8									2.48	10	1
Schist, chlorite	Bethesda, Maryland	2,813	2.53	8	37:1.89:51						3.10	10			3
Schist, sericite	Superior, Arizona	2,700	1.62	3	82	4.72	4		2.62	10	6.00	10			20
Skarn, garnet-pyroxene	Star Lake, New York	3,280	1.30	3	61	5.12	4		3.48	10	8.62	10			20
Slate, par. bedding, calcareous	Bangor, Pennsylvania	2,740	1.83	3	56						8.88	10			20
Tactite, epidote	Ophir, Utah	2,870	2.66	3	65	4.60	4	0.11	2.77	10	6.14	10			21

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Sedimentary															
Borax, ore: Ricardo	Boron, California	2,140	4.41	8	22								4.21	9	5
Chert, chalcedonic; Boone	Picker, Oklahoma	2,560	3.60	3	96			0.09			5.34	10			4
Chert, chalcedonic; Fort Payne	Smithville, Tennessee	2,630	2.10	3	74	3.35	4	0.00	1.65	10	3.54	10			5
		2,670	2.02	3	67	4.48	4	0.14	2.37	10	5.62	10			5
Conglomerate	Kirkland Lake, Ontario	2,670	1.65	3		5.40	4		3.24	10	7.79	10			20
Conglomerate; Roxbury	Boston, Massachusetts	2,679	8.28	8	41:6.37:102										3
Dolomite	Jefferson City, Tennessee	2,760	3.59	3	69:	5.30	4		3.17	10	7.79	10			20
	Mascot, Tennessee	2,840	3.22	3	74	5.46	4		3.52	10	8.48	10			20
	Wood County, West Virginia	2,833						0.22	2.95	10	7.23	10			17
		3,004						0.22	3.05	10	7.50	10			17
		2,783						0.26	3.75	10	9.50	10			17
		2,832						0.19	3.64	10	8.65	10			17
Dolomite, Bonne Terre	Bonne Terre, Missouri	2,673	1.52	3	49:						6.63	10			3
Dolomite, jointed Jurassic	Gojak, Yugoslavia	2,800		8		2.51	4				1.27	10			13
Dolomite, Lockport	Rochester, New York	2,765	2.12	3	50:3.24:86						4.48	10			3
	Niagara Falls, New York	2,579	9.10	8	44:						5.10	10			3
Dolomite, Maple Mill	Omaha, Nebraska		3.47	8				0.36			4.79	10			18
		2,827	4.32	8				0.05			2.74	10			18
		2,818	1.13	3				0.12			6.13	10			18
		2,528	4.45	8				0.51			4.39	10			18
		2,507	7.08	8				0.09			2.14	10			18
		2,531	6.09	8				0.40			8.62	10			18
Graywacke, m., Chico	Monticello Dam, California	2,440	4.88	8				0.03					1.24	10	1
		2,490	5.07	8				0.02					9.65	9	1
Gypsum	Buffalo, New York	2,262	1.25	8	18										3
Jaspilite, ferrug., silli. sandstone	Ishpeming, Michigan	3,390	3.42	3	85	5.55	4		4.83	10	1.03	11			20

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Limestone	Bedford, Indiana	2,206	5.10	8	33:0.43:20	3.91	4				2.85	10			7
Limestone, Black River	Trenton, West Virginia	2,688		3				0.16	2.45	10	5.70	10			17
Limestone, chalky, Smokey Hill	Pickstown, South Dakota	1,410	8.27	7	10	1.34	4	0.30	1.59	9	2.90	9			4
		1,710	1.65	8	13	1.74	4	−0.13	2.55	9	4.48	9			4
Limestone, Chickamauga	Smithville, Tennessee	2,740	1.73	3	53	4.39	4	0.14	2.33	10	5.30	10			5
		2,730	1.73	3	52	3.08	4	0.22	1.17	10	2.72	10			5
Limestone, detrital	Moscow Syncline, USSR	2,160	5.20	8							2.90	10			2
Limestone, dolomitic; Bonne Terre	Bonne Terre, Missouri	2,660	1.75	3	51	5.09	4	0.22	2.85	10	6.96	10			4
		2,780	1.98	3	59	5.88	4	0.29	3.76	10	9.72	10			4
		2,710	1.96	3	49			0.05			1.99	10			4
		2,690	1.96	3	33	5.36	4	0.22	3.13	10	7.65	10			4
		2,670	1.46	3	48	3.78	4	−0.07	2.10	10	3.87	10			4
Limestone, dolomitic; Mesozoic	Turkmenia SSR, USSR	2,700	2.10	3							7.62	10			2
Limestone, dolomitic; well-cemented	Pondera County, Montana	2,710	1.68	3				0.31					7.65	10	16
Limestone, fossiliferous	Bedford, Indiana	2,370	7.52	8	27	3.78	4		1.42	10	3.34	10			20
Limestone, fossiliferous, pa. bed	Bedford, Indiana	2,370	6.85	8	27				1.56	10	3.91	10			20
Limestone, fossiliferous; St. Louis	St. Genevieve, Missouri	2,670	1.64	3	48	5.00	4	0.24	2.68	10	6.67	10			5
Limestone, jointed; Jurassic	Gojak, Yugoslavia	2,700				1.92	4				9.16	9			13
Limestone, limonitic	Bessemer, Alabama	2,920	1.72	3	61	4.75	4		2.82	10	4.45	10			20
Limestone, marly	Rifle, Colorado	2,250	1.10	3	56	2.38	4		6.90	9	1.25	10			20
Limestone, marly, par. bedded	Rifle, Colorado	2,180				3.11	4		6.76	9	2.14	10			20
Limestone, Martinsburg	Martinsburg, West Virginia	2,680	1.59	3	61	5.00	4	0.21	2.73	10	6.59	10			21
Limestone, Ozark Tavern	Carthage, Missouri	2,659	9.79	8	49						5.59	10			3
Limestone, porous; redwall	Lee's Ferry, Arizona	2,440	1.33	3				0.18					1.65	10	1
Limestone, reef	Eniwetok, PTT	2,300	3.42	8				0.16					3.79	10	1
Limestone, Silurian	Omaha, Nebraska	2,352	9.60	8				0.19			3.07	10			18
Limestone, Solnhofen	Bavaria, FRG	2,621	2.45	3	54:1.75:72	5.78	4				6.38	10			7
Limestone; Wyandotte	Omaha, Nebraska	2,546	1.15	8				0.24			2.11	9			18
		2,605	49.0	7				0.64			16.1	10			18

(continues)

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Marlstone, mahogany	Rifle, Colorado	2,220	8.14	8	49	3.20	4	0.17	1.02	10	2.41	10			4
Marlstone, Maxville	East Fultonham, Ohio	2,190	5.59	8	23	3.38	4	0.13	1.10	10	2.50	10			5
Marlstone, par. bed.; mahogany	Rifle, Colorado	2,360	1.72	3	61	4.18	4	0.33	1.53	10	4.10	10			4
Oil Shale, Parachute Creek	Rio Blanco, Colorado	2,044	8.28	8				0.33					6.24	9	12
		2,220	1.10	3				0.37					1.12	10	12
		2,190	1.81	3				0.30					1.08	10	12
		2,124	9.35	8				0.24					7.03	9	12
Quartzite	Bergstrom, Texas	2,610	6.45	8							2.76	7			19
		2,570	1.26	3							3.56	10			19
		2,610	1.75	3							5.91	10			19
		2,640	2.23	3							6.36	10			19
		2,570	1.64	3							5.44	10			19
Quartzite, Baraboo	Baraboo, Wisconsin	2,627	3.21	3	59						8.84	10			3
Salt	Bergstrom, Texas	2,167	1.81	8		3.76	4				6.14	9			19
		2,168	1.89	8		3.37	4	0.06			3.45	9			19
		2,167	2.85	8		4.08	4				3.45	10			19
		2,298	2.20	8		4.07	4	0.189			2.05	10			19
		2,317	3.07	8				0.03			3.28	10			19
Salt, diamond crystal	Jefferson Island, Louisiana	2,163	2.14	8	23						4.90	9			3
Sandstone	Huntington, Utah	2,200	1.07	3		2.44	4	−0.10	7.03	9	1.31	10			21
		2,170	7.93	8		2.56	4	0.04	7.03	9	1.45	10			21
		2,140	9.79	8		2.19	4	0.04	4.83	9	1.01	10			21
		2,350	2.23	3		2.96	4	−0.11	1.17	10	2.07	10			21
		2,330	1.91	3		2.87	4	−0.07	1.02	10	1.86	10			21
Sandstone, Berea	Amherst, Ohio	2,182	7.38	8	42:0.47:29	2.64	4				1.93	10			7
Sandstone, c.	Amherst, Ohio	2,170	4.21	8	20	1.20	4		4.00	9	7.10	9			20
Sandstone, c., par. bedded	Amherst, Ohio	2,170	3.55	8	20				4.65	9	1.09	10			20
Sandstone, calcareous, nonesuch	White Pine, Montana	2,600	1.58	3	62	4.63	4	0.16	2.39	10	5.53	10			4
Sandstone, Cambridge	Cambridge, Massachusetts	2,582	3.85	8	37:1.84:51						1.03	10			3
Sandstone, carboniferous	Donets Basin, USSR	2,650	2.56	3				0.14	2.43	10	5.55	10			21

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TABLE 2.9 Properties of rocks (continued)

Rock Type: Geologic Unit	Location	γ (N/m³)	C_o (N/m²)[a]	Ref.	H[b]	V_p (Km/s)	Ref.	u	G (N/m²)[a]	Ref.	E_r (N/m²)[a]	Ref.	Es (N/m²)[a]	Ref.	Ref.
Sandstone, cem.; obl. bed; Navajo	Huntington, Utah	2,370	3.38	8	54	3.38	4	0.05	1.41	10	2.71	10			4
Sandstone, cemented, Navajo	Huntington, Utah	2,880	1.24	3	50	2.77	4	−0.07	9.45	9	1.75	10			4
Sandstone, Crab Orchard	Crossville, Tennessee	2,531	2.14	3	47						3.92	10			7
Sandstone, f.; Morrison/Bushy Basin	Long Park, Colorado	2,540				2.62	4	−0.04	9.10	9	1.76	10			5
Sandstone, f. Tensleep	Casper, Wyoming	2,325	7.25	8				0.06					1.31	10	1
Sandstone, ferruginous	Bessemer, Alabama	2,930	2.35	3	65	4.05	4		2.42	10	4.96	10			20
	Monongalia County, West Virginia	2,600	1.32	3	53	3.42	4	0.22	1.51	10	3.83	10			21
Sandstone, Graywacke, Kanawha	Dehue, West Virginia	2,600	1.41	3	55	2.93	4	−0.17	1.34	10	2.23	10			4
Sandstone, Homewood	Franklin, Pennsylvania	2,200	8.69	8	43	1.92	4	−0.11	4.69	9	8.27	9			5
Sandstone, Navajo	Page, Arizona	2,015	4.35	8	30:0.04:6	2.52	4						1.53	10	7
Sandstone, shaly; St. Peter	Omaha, Nebraska	2,344	3.73	8				0.05			7.19	9			18
		2,450	3.46	8				0.06			1.25	10			18
Sandstone, silty; Seminole	Tulsa, Oklahoma	2,500	7.45	8	31	2.87	4		1.08	10	2.19	9			5
Sandstone, Thorold	Niagara Falls, Ontario	2,460						−0.12			2.13	10			11
		2,510						−0.18			3.31	9			11
Sandstone, uncem., par. bed; Navajo	Huntington, Utah	2,130	3.31	8	36	2.10	4	−0.04	4.96	9	9.58	9			4
Sandstone, uncem.; obl. bed; Navajo	Huntington, Utah	2,130	5.59	8	32	2.29	4	−0.05	5.86	9	1.12	10			4
Shale, Bertie	Buffalo, New York	2,712	1.97	3	42:1.92:59						5.03	10			3
Shale, Brunswick	Highland Park, New Jersey	2,631	8.29	8	38:0.70:31						1.38	10			3
Shale, calcareous; Sheffield	Omaha, Nebraska	2,602	5.98	7				0.14			3.09	10			18
Shale, calcareous; Wyandotte	Omaha, Nebraska	2,177	1.19	8				0.32			1.97	9			18
Shale, carbonaceous; Chattanooga	Smithville, Tennessee	2,300	1.12	3	50	2.38	4	0.00	6.55	9	1.39	10			5
		2,300	1.10	3	48	2.38	4	−0.02	7.10	9	1.34	10			5
Shale, Maquoketa	Omaha, Nebraska	2,618	4.25	8				0.01			7.32	9			18
Shale, Rochester	Rochester, New York	2,738	1.22	3	45:0.73:39						3.79	10			3
Shale, siderite, banded; Kanawha	Dehue, West Virginia	2,760	1.12	3	38	2.16	4	−0.43	1.17	10	1.33	10			4
Shale, sl. weathered; Cherokee	Omaha, Nebraska	2,496	8.34	7				0.15			1.67	9			
Siltstone, Hackensack	Hackensack, New Jersey	2,595	1.23	3	47:154:58	3.99	4				2.63	10			7
Siltstone, par. bedding; Maxville	East Fultonham, Ohio	2,660	3.65	8	20			0.13					4.81	10	5
		2,680	3.45	8	19			0.26					8.68	10	5
Siltstone, poorly cemented, Bandera	Omaha, Nebraska	2,304	3.54	7				0.35			1.25	3			18

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TABLE 2.9 Properties of rocks (Legend)

Engineering Property	Symbol	Nature: Field (F), Laboratory (L), Derived (D)	Method of Measurement	Units of Measure	Use in Engineered Construction	Limitations
Unit weight	g	L	Volumetric displacement in water; weight per unit volume, usually weighed as oven dried but may be specified on several other bases	kg/m^3	Weight, per volume, of entire rock; primary term in many computations; useful for computing in situ stress	Expected statistical variances in the more porous rocks
Compressive strength	C_o	L	Uniaxial or triaxial conditions, in universal test machine; strain gauges used for moduli determinations	P_a	Index classification test; load-bearing capacity; other properties according to Mohr failure concept; slope stability; mine pillar stress; subsidence; excavation, blasting, drilling, and mole-boring performance	Avoid unrepresentative anisotropic fabric elements of discontinuities; select representative sample; consider data scatter; length-to-diameter (L/D) ratio is quite important (standard is 2:1); peak strength is obtained
Hardness values are in order:			Small laboratory test holding devices for impact; height or rebound of small diamond-tipped device	Dimensionless	Indicator of relative hardness; useful in tunnel boring rate estimates	
Schmidt rebound	H_s	L		Dimensionless	As above	Softer rock breaks on impact; must use type L device of minimal energy
Tabor	H_a	L		Dimensionless	As above	
Total	H_t	L		Dimensionless		
Seismic velocity		F	Mechanical or explosive wave arrival sensed by geophone, timer, and recorder; measured on ground surface or in borehole configurations	km/s	As above, indicator of overall maturity of rock caused by averaging effect on wave travel paths, depending on geophone/energy source array	Degree of saturation important; test does not introduce nonlinear, time-dependent strain variation in derived properties; most valid in homogeneous and isotropic rock
Compression	V_p					
Shear	V_s					
Poisson's ratio	u	D	Calculated from sonic/seismic velocity tests or by use of electrical resistance strain gauges on compression tests	Dimensionless; in ranges from 0 to 0.5	Computational input for calculation of stress-distribution patterns and of predicted strain in elastic media; required for finite element modeling	Difficult to extrapolate laboratory measurement to field conditions; often estimated without testing; best approximation is from triaxial compression test at confinement equivalent to in situ conditions
Modulus of rigidity	G	D	As above	P_a	Indicator of seismic design stiffness	Strain related
Tangent modulus of elasticity (Young's modulus, or modulus of deformation)	E_r	L	Triaxial compression in universal test machine; electrical resistance strain gauges	P_a	The fundamental stress–strain relationship; input for static displacement computations and for dynamic, seismic analyses	Requires accommodation of any anisotropy of rock fabric and model in situ conditions
Secant modulus of elasticity	E_s	L	As above	P_a	Alternate expression of the fundamental stress–strain relationship	As above

(continues)

TABLE 2.9 Properties of rocks (References)

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Source: Carmichael 1982

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TABLE 2.10 Properties of minerals

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Albite feldspar	$NaSi_3AlO_8$		Triclinic	Colorless or white; sometimes yellow, pink, green, or black	Vitreous to pearly	White	Transparent to subtranslucent	Brittle	Perfect	Uneven to conchoidal	6.3	2.61	In igneous rocks	Soda feldspar, cryptoclase
Alunite	$KAl_3(SO_4)_2(OH)_6$	K_2O 11.37 Al_2O_3 36.92	Hexagonal; ditrigonal pyramidal	White, grayish, yellow, or reddish brown	Vitreous	White	Transparent to subtranslucent	Brittle	Distinct	Flat-conchoidal uneven	3.7	2.67	Secondary mineral in acid volcanic rocks that have been altered	Alum stone
Amblygonite	$LiAl(PO_4)F$	Li_2O 10.10 AL_2O_3 34.46 P_2O_5 48.00 F 12.85	Triclinic pinacoidal	White, yellowish, beige, salmon pink, greenish, bluish, gray	Vitreous to greasy	White	Subtransparent to translucent	Brittle	Perfect	Uneven to subconchoidal	6.0	3.05	In granite pegmatite veins	Hebronite
Andalusite	$Al_2O(SiO_4)$	Al_2O_3 60.16–62.70	Orthorhombic	Pink, white, or rose red	Vitreous	White	Transparent to opaque	Brittle	Distinct	Uneven to subconchoidal	7.5	3.18	Contact mineral in clay, slate, and argillaceous schists	Chiastolite, viridine
Anglesite	$PbSO_4$	PbO 73.6	Orthorhombic dipyramidal	Colorless, white, often tinged gray	Adamantine to resinous	Colorless	Transparent to opaque	Very brittle	Distinct but interrupted	Conchoidal	2.8	6.35	Secondary mineral in the oxidation zone of lead veins. Alteration product of galena	

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Anhydrite	$CaSO_4$	CaO 41.19	Orthorhombic dipyramidal	Colorless to violet. Also white, mauve, rose, brownish	Pearly to vitreous	White or grayish white	Translucent to opaque	Brittle	Very perfect	Uneven, sometimes splintery	3.3	2.95	As evaporite usually associated with gypsum	Cube spar, tripe stone
Anorthite feldspar	$CaAl_2Si_2O_8$		Triclinic	Colorless or white; sometimes yellow, pink, green, or black	Vitreous to pearly	White	Transparent to translucent	Brittle	Perfect	Conchoidal to uneven	6.3	2.75	In basic igneous rocks	Lime feldspar, calciclase
Antlerite	$Cu_3(SO_4)(OH)_4$	Cu 54.0	Orthorhombic dipyramidal	Green to blackish green	Vitreous	Pale green			Perfect		3.0	3.90	Copper deposits, an alteration of brochantite	Stelznerite, arnimite
Apatite	$Ca_5(PO_4)_3(F, Cl, OH)$	CaO 55.38 P_2O_5 42.06 F 1.25 Cl 2.33	Hexagonal; hexagonal dipyramidal	Usually sea green	Vitreous, to subresinous	White	Transparent to opaque	Brittle	Imperfect	Conchoidal and uneven	5.0	3.20	Most common in metamorphic crystalline rocks, often associated with beds of iron ore	Asparagus stone, collophane
Aragonite	$CaCO_3$	CaO 56.03	Orthorhombic dipyramidal	Colorless to white; also gray, yellowish, blue, green, rose red	Vitreous, resinous on fracture	Uncolored	Transparent to translucent	Brittle	Distinct	Subconchoidal	3.7	2.94	Hot springs deposit, precipitate from saline solution with gypsum in cavities in lavas	Flowers of iron, oserskit
Argentite	Ag_2S	Ag 87.06	Isometric hexoctahedral	Blackish lead gray	Metallic	Blackish lead gray	Opaque	Sectile	Traces	Conchoidal	2.5	7.30	Usually with galena and other sulfide ores	Silver glance, argyrite

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Arsenic	As	100	Hexagonal scalenohedral	Tin white tarnishing to dark gray	Nearly metallic on fresh surface	Same as color	Opaque	Brittle	Perfect	Granular	3.5	5.70	Metallic veins with silver, cobalt, nickel ores	
Arsenopyrite	FeAsS	Fe 34.30 As 46.01	Monoclinic prismatic	Silver white to steel gray	Metallic	Dark grayish black	Opaque	Brittle	Distinct	Uneven	6.0	6.10	Usually veins with other sulfides	Mispickel, arsenical pyrites
Atacamite	$Cu_2(OH)_3Cl$	Cu 14.88	Orthorhombic dipyramidal	Green to blackish green	Adamantine to vitreous	Apple green	Transparent to translucent	Brittle	Highly perfect	Conchoidal	3.3	3.77	Secondary mineral derived from malachite and cuprite	Remolinite, halochalzit
Autunite	$Ca(UO_2)_2(PO_4)_2{\cdot}10–12H_2O$	CaO 5.69 UO_3 58.00 P_2O_5 14.39	Tetragonal ditetragonal-dipyramidal	Lemon yellow to sulfur yellow, sometimes greenish	Vitreous, pearly	Yellowish	Transparent to translucent	Brittle	Eminent		2.3	3.10	Secondary mineral usually associated with uraninite and other uranium minerals	Lime uranite
Azurite	$Cu_3(OH)_2(CO_3)_2$	Cu 55.3	Monoclinic prismatic	Azure blue, very dark blue	Vitreous, almost adamantine	Blue, lighter than color	Transparent to subtranslucent	Brittle	Perfect, but interrupted	Conchoidal	3.8	3.83	With malachite as secondary mineral in the oxidized zone of copper deposits	Chessylite, blue spar
Barite	$BaSO_4$	BaO 65.70	Orthorhombic dipyramidal	Colorless, white, yellow, brown, red	Vitreous, to resinous	White	Transparent to opaque	Brittle	Perfect	Uneven	3.0	4.45	Veins or beds, gangue mineral in veins, cement in sandstones	Barytes, heavy spar, desert roses

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Beryl	$Be_3Al_2(Si_6O_{18})$	BeO 10.54–13.76 Al_2O_3 17.10–19.00	Hexagonal; dihexagonal dipyramidal	White, bluish green, greenish yellow, yellow	Vitreous	White	Transparent to subtranslucent	Brittle	Imperfect	Conchoidal to uneven	7.6	2.70	In granitic rocks and pegmatites	Aquamarine, emerald, goshenite
Biotite mica	$K_2(Mg, Fe^{+2})6–4(Fe^{+3}, Al, Ti)0–2(Al_{2–3}O_{20})$; $O_{0–2}(OH, F)_{4–2}$	–	Monoclinic	Colorless, shades of pink, purple	Vitreous to pearly	–	Transparent to opaque	Elastic	Basal, highly perfect	–	2.7	2.9	Important constituent of many igneous rocks, and as an alteration product	Black mica
Bismuth	Bi	100	Hexagonal scalenohedral	Silver white, with reddish hue. Iridescent tarnish	Metallic	Same as color	Opaque	Sectile	Perfect	Uneven	2.5	9.80	Veins in granite, gneiss, with ores of cobalt, nickel, silver, lead	–
Bismuthinite	Bi_2S_3	Bi 81.3	Orthorhombic dipyramidal	Lead gray to tin white	Metallic	Lead gray	Opaque	Sectile	Perfect	–	2.0	6.40	With igneous rocks, magnetite, garnet, pyrite, tin, and tungsten	Bismuthine, wismuthglanz
Borax	$Na_2B_4O_7{\cdot}_{10}H_2O$	Na_2O 16.26 B_2O_3 36.51	Monoclinic prismatic	Colorless, white, also grayish, bluish or greenish	Vitreous to resinous	White	Translucent to opaque	Rather brittle	Perfect	Conchoidal	2.3	1.70	In the waters of saline lakes and in the beds resulting from the evaporation of these lakes	Tincal
Bornite	Cu_5FeS_4	Cu 63.33	Isometric hexoctahedral	Brownish bronze	Metallic	Pale grayish black	Opaque	–	Traces	Uneven	3.0	5.20	Usually primary with other copper minerals	Purple copper ore, peacock ores

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Bournonite	$PbCuSbS_3$	Pb 42.40 Cu 13.01 Sb 24.91	Orthorhombic dipyramidal	Steel gray to iron black	Metallic, often brilliant	Same as color	Opaque	Brittle	Imperfect	Uneven	3.0	5.80	Veins with other sulfide minerals	Cogwheel ore, endellione
Brucite	$Mg(OH)_2$	41.7	Hexagonal scalenohedral	White to pale green, gray or blue	Pearly on cleavages, elsewhere, waxy	White	Transparent to translucent	Fibrous, folia flexible sectile	Basal eminent	Fibrous	2.5	2.40	Secondary origin in serpentine and chlorites	Nemalite, texalite
Calaverite	$AuTe_2$	Au 43.59	Monoclinic prismatic	Brass yellow to silver white	Metallic	Yellowish to greenish gray	–	–	–	–	2.5	9.00	Veins with gold, pyrite, and quartz	None
Calcite	$CaCO_3$	CaO 56.03	Hexagonal; hexagonal scalenohedral	Colorless or white when pure	Vitreous; sometimes pearly or iridescent	White to grayish	Transparent to opaque	Brittle	Highly perfect	Conchoidal	3.0	2.71	Minor secondary constituent of igneous rocks; widespread constituent of sedimentary rocks; generally deposited by lime-bearing fluids	Iceland spar, limestone
Carnallite	$KMgCl_3{\cdot}6H_2O$	K 14.07 Mg 8.75	Orthorhombic dipyramidal	Colorless to milk white; often reddish	Greasy, dull to shining	–	Transparent to translucent	Brittle	Not distinct	Conchoidal	2.5	1.60	A deposit from low-temperature solutions	None
Carnotite	$K_2(UO_2)_2(VO_4)_2{\cdot}3H_2O$	K_2O 10.44 U 52.7 V 28.5	Orthorhombic or monoclinic	Bright yellow, lemon yellow, greenish yellow	Earthy	–	–	–	Basal	–	–	–	Occurs as a yellow crystalline powder or in loosely cohering masses mixed with quartzose material	Kalio-carnotite, potassio-carnotite

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Cassiterite	SnO_2	Sn 78.6	Tetragonal ditetragonal dipyramidal	Yellowish or reddish brown to black	Adamantine to metallic	White, grayish, brownish	Transparent to opaque	Brittle	Imperfect	Uneven	6.5	6.90	Veins of quartz near granite or pegmatite, often in gravel sands	Tin stone, stannolite
Celestite	$SrSO_4$	SrO 56.42	Orthorhombic dipyramidal	Colorless to pale blue	Vitreous, pearly on cleavages	White	Transparent to subtranslucent	Brittle	Perfect	Uneven	3.3	3.96	Usually in limestone or sandstone with gypsum, rock salt, etc.	Celestine, coelestine
Cerussite	$PbCO_3$	Pb 76.5	Orthorhombic dipyramidal	Colorless to white and gray or smoky	Adamantine to vitreous	Colorless to white	Transparent to subtranslucent	Very brittle	Distinct	Conchoidal	3.3	6.52	Oxidized zones of lead veins	White lead, lead spar
Chalcanthite	$CuSO_4 \cdot 5H_2O$	CuO 31.87	Triclinic pinacoidal	Sky blue	Vitreous	Colorless	Subtransparent to translucent	Brittle	Imperfect	Conchoidal	2.5	2.21	Formed by the oxidation of chalcopyrite and other copper sulfides	Blue vitriol, blue stone, cyanose
Chalcocite	Cu_2S	Cu 79.86	Orthorhombic dipyramidal	Blackish lead gray	Metallic	Blackish lead gray	Opaque	Sectile	Indistinct	Conchoidal	2.5	5.70	Secondary, usually with pyrite, chalcopyrite, etc.	Copper glance, vitreous copper
Chalcopyrite	$CuFeS_2$	Cu 34.64 Fe 30.42	Tetragonal scalenohedral	Brass yellow, often tarnished iridescent	Metallic	Greenish black	Opaque	Brittle	Distinct	Uneven	3.5	4.20	Primary, veins or disseminated often with pyrite, quartz	Copper pyrites, cupropyrite
Chlorite	$(Mg, Al, Fe)_{12}[(Si, Al)_8O_{20}](OH)_{16}$	–	Monoclinic	Green, white, yellow, pink, red, brown	Vitreous to pearly	White, pale green	Transparent to subtranslucent	Flexible	Perfect	–	2.3	2.72	In chlorite schist and other crystalline schists	–

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Chromite	$FeCr_2O_4$	Cr 46.4	Isometric hexoctahedral	Black	Metallic	Brown	Translucent to opaque	Brittle	–	Uneven	5.5	4.50	Occurs in veins in peridotites or serpentines derived from them	Eisenchrom, chromoferrite
Chrysoberyl	$BeAl_2O_4$	BeO 19.71 Al_2O_3 80.29	Orthorhombic dipyramidal	Green, greenish white, yellowish green, yellow	Vitreous	Colorless	Transparent to translucent	Brittle	Quite distinct	Uneven to conchoidal	8.5	3.67	In granite rocks and pegmatites and in sands and gravels	Alexandrite, cat's-eye, cymophane
Chrysocolla	$CuSiO_3{\cdot}nH_2O$	CuO 32.4–42.2	Amorphous	Green to greenish blue	Vitreous	–	Translucent to opaque	Sectile or brittle		Conchoidal	2.4	2.12	Secondary mineral found in the upper portions of copper veins	Kieselmalachit, chalcostaktite
Cinnabar	HgS	Hg 86.2	Hexagonal; trigonal trapezohedral	Cochineal red, often brownish red	Adamantine to metallic when dark colored to dull	Scarlet	Transparent to opaque	Sectile	Perfect	Uneven	2.5	8.10	Veins in sediments often with pyrite and marcasite	Cinnabarite, zinnober, hepatic-cinnabar
Cobaltite	CoAsS	Co 35.53 As 45.15	Isometric tetragonal	Silver white, to red. Also steel gray, with violet tinge	Metallic	Grayish, black	Opaque	Brittle	Cubic perfect	Uneven	5.5	6.20	Contact deposits, in gneiss, schists and diopside, with cobalt and nickel, copper and silver	Cobaltine, sehta, bright white cobalt
Copper	Cu	100	Isometric hexoctahedral	Fresh copper red. Tarnishes to brown, red, black, green	Metallic	Copper red metallic and shining	Opaque	Malleable and ductile	None	Hackly	2.7	8.80	Secondary, associated with copper minerals frequently near igneous rocks	–
Cordierite	Al_3 (mg, Fe^{+2}) (Si_5AlO_{18})	–	Orthorhombic dipyramidal	Grayish blue, lilac blue, dark blue	Vitreous	White	Transparent to translucent	Brittle	Distinct	Subconchoidal	7.3	2.63	In contact metamorphic zones	Iolite, dichroite, water sapphire

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Corundum	Al_2O_3	Al 52.91	Hexagonal scalenohedral	Blue to colorless, yellow to golden, pink to deep red	Adamantine to vitreous, sometimes pearly	Uncolored	Transparent	Brittle	Interrupted	Uneven	9.0	4.00	Usually in limestone, dolomite, or gneiss, with minerals of chlorite group	Ruby, sapphire, emery, oriental amethyst
Covellite	CuS	Cu 66.48	Hexagonal; dihexagonal dipyramidal	Indigo blue. Often highly iridescent	Crystals—submetallic inclining to resinous	Lead gray to black, shining	Opaque	Flexible	Perfect	Uneven	2.0	4.60	Secondary with other copper sulfides	Blue copper, indigo copper, covelline
Cristobalite	SiO_2	—	Tetragonal trapezohedral	White, colorless	Vitreous	White	Transparent to opaque	Brittle to tough	None	Conchoidal	7.0	2.27	In acidic volcanic rocks and in meteorites	Lussatite
Crocoite	$Pb(CrO)_4$	PbO 69.06 CrO 30.94	Monoclinic prismatic	Hyacinth red, deep orange red, orange, yellow	Adamantine to vitreous	Orange yellow	Translucent	Sectile	Rather distinct	Small conchoidal to uneven	2.7	6.00	Secondary mineral from hot solutions	Callochrome, crocoise, red lead ore
Cryolite	Na_3AlF_6	Na 32.86 Al 12.85	Monoclinic prismatic	Colorless to white; also brownish; reddish or brick red	Vitreous to greasy	White	Transparent to translucent	Brittle	—	Uneven	2.5	2.97	In granite veins	Eisstein, ice-stone
Cuprite	Cu_2O	Cu 88.82	Isometric gyriodal	Cochineal red; sometimes almost black	Adamantine or submetallic to earthy	Several shades of brownish red	Translucent	Brittle	Interrupted	Conchoidal	3.5	6.00	Secondary, often with malachite, azurite, limonite	Ruby copper, ruberite, red copper ore
Diamond	C	100	Isometric hextetrahedral	Pale yellow to deep yellow, pale to deep brown, white to blue white	Adamantine to greasy	—	Transparent	Brittle	Perfect	Conchoidal	10.0	3.50	Alluvial deposits of sand and clay. Volcanic pipes	Bort, carbonado

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Diaspore	$HA10_2$	Al_2O_3 84.98	Orthorhombic dipyramidal	White, grayish white, colorless	Brilliant; pearly on cleavage	–	Transparent to subtranslucent	Very brittle	Eminent	Conchoidal	6.7	3.40	Alteration product of corundum. Also in limestones	–
Dolomite	$CaMg(CO_3)_2$	CaO 30.41 MgO 21.86	Hexagonal rhombohedral	Colorless or white, sometimes gray or greenish	Vitreous to pearly	White	Transparent to translucent	Brittle	Perfect	Subconchoidal	3.7	2.85	Vein mineral or altered limestone	Pearl spar, rhomb spar, bitter spar
Enargite	Cu_3AsS_4	Cu 48.42 As 19.02	Orthorhombic pyramidal	Grayish black to iron black	Metallic, tarnishing dull	Grayish black	Opaque	Brittle	Distinct	Uneven	3.0	4.40	Primary, usually with other copper minerals	Garbyite, clarite, guayacanite
Epidote	$Ca_2Fe^{+3}Al_2O \cdot OH(Si_2O_7)(SiO_4)$	CaO 22, 18–24.15 Fe_2O_3 11.07–23.42 Al_2O_3 13.10–24.36	Monoclinic prismatic	Green, yellow, gray	Vitreous	White or grayish white	Transparent to opaque	Brittle	Perfect	Uneven	6.5	3.42	Formed by the metamorphism of impure calcareous sedimentary rocks	Pistacite
Epsomite	$MgSO_4 \cdot 7H_2O$	MgO 16.36	Orthorhombic disphenoidal	Colorless white, pink, or greenish	Vitreous	White	Transparent to translucent	–	Very perfect	Conchoidal	2.3	1.75	In mineral waters and on cave and mine walls	Epsom salt, bitter salt, gletschersalz
Fluorite	CaF_2	Ca 51.33	Isometric hexoctahedral	Yellow, green, greenish blue, violet blue; also white, gray, yellow	Vitreous; glimmering to dull in massive varieties	–	Transparent to subtranslucent	Brittle	Perfect	Flat-conchoidal or splintery	4.0	3.13	In veins and sedimentary rocks	Fluorspar, fluor, chlorophane
Forsterite	Mg_2SiO_4	SiO_2 41.72 FeO 1.11 MgO 57.83	Orthorhombic	Green, lemon yellow to greenish yellow, yellow amber	Vitreous	White or gray	Transparent to translucent	Brittle	Rather distinct	Conchoidal	6.7	3.32	In igneous rocks from low silica melts and metamorphic rocks formed from impure dolomites	–

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Franklinite	$ZnFe_2O_4$	Zn 5.4–18.7	Isometric hexoctahedral	Black to brownish black	Metallic to semimetallic	Reddish brown	Opaque	Brittle	Pseudo-cleavage (parting) octahedral	Conchoidal to uneven	6.0	5.14	Crystallized from igneous melts	Zinkoferrite, isophane, francklinite
Galena	PbS	Pb 86.60	Isometric hexoctahedral	Lead gray	Metallic	Lead gray	Opaque	Brittle	Cubic	Even	2.5	7.50	Veins, often with pyrite, sphalerite, chalcopyrite, intrusive replacement	Gelenite, lead glance, plumbago
Garnierite	(Ni, Mg) $SiO_3 \cdot nH_2O$	NiO 15.56	Amorphous monoclinic	Apple green to white	Earthy and dull	–	Opaque	Soft and friable	–	–	–	2.52	A variation of serpentine	Noumeite, genthite, népouite
Gibbsite	$Al(OH)_3$	–	Monoclinic prismatic	White; grayish, greenish, or reddish white	Pearly on cleavage, vitreous other surfaces	–	Translucent	Tough	Eminent	–	3.0	2.30	Usually with bauxite	–
Glauberite	$Na_2Ca(SO_4)_2$	Na_2O 22.29 CaO 20.16	Monoclinic prismatic	Gray or yellowish	Vitreous, pearly on cleavage	White	–	Brittle	Perfect	Conchoidal	2.7	2.77	In salt deposits	Brongniartine
Goethite	$HFeO_2$	Fe 62.9	Orthorhombic dipyramidal	Crystals—blackish brown. Massive—yellowish or reddish brown. Earthy—brownish yellow, ocher yellow	Crystals—imperfect adamantine metallic, sometimes dull; fibrous variety often silky	Brownish yellow, orange yellow, ocher yellow	Opaque	Brittle	Very perfect	Uneven	5.3	4.28	Found with limonite as alteration product of a sulfide, usually pyrite	Bog iron ore, yellow ocher
Gold	Au	100	Isometric hexoctahedral	Gold yellow when pure. Silver white to orange red when impure	Metallic	Same as color	Opaque	Malleable and ductile	None	Hackly	2.7	19.3	In significant amounts in hydrothermal veins and related rocks, in consolidated placer deposits and unconsolidated placer deposits	Moss gold, wire or sponge gold

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Graphite	C	100	Hexagonal; dihexagonal dipyramidal	Black to steel gray	Metallic, sometimes dull, earthy	Black	Opaque	Flexible	Perfect	–	1.5	2.1	Veins in granite, gneiss, quartzite, and limestone	Plumbago, black lead, graphitite
Greenockite	CdS	Cd 77.81	Hexagonal; dihexagonal pyramidal	Yellow, orange	Adamantine to resinous	Orange yellow	Transparent	Brittle	Distinct	Conchoidal	3.5	5.0	Usually coating on sphalerite	Cadmium-blended, cadmium ocher
Gypsum	$CaSo_4 \cdot 5H_2O$	CaO 32.57	Monoclinic prismatic	Colorless; also white, gray, yellowish or brownish	Subvitreous	White	Transparent to opaque	Flexible to brittle	Eminent	Conchoidal	1.7	2.32	Forms extensive sedimentary beds	Satin spar, alabaster, selenite
Halite	NaCl	Na 39.34	Isometric hexoctahedral	Colorless; also white, red, yellow, blue, purple	Vitreous	Colorless	Transparent to translucent	Rather brittle	Cubic, perfect	Conchoidal	2.5	2.3	An evaporite	Rock salt, muriate of soda
Hematite	Fe_2O_3	Fe 69.94	Hexagonal scalenohedral	Steel gray (crystals) dull red to bright red for earthy material	Metallic to submetallic to dull	Cherry red or reddish brown	Opaque	Brittle	Parts due to lamellar structure	Uneven	6.0	5.1	Often in granites, syenites, andesites. Altered limonite	Martite, red ocher, specularite
Ilmenite	$FeTiO_3$	Fe 36.8 Ti 31.6	Hexagonal rhombohedral	Iron black	Metallic to submetallic	Black	Opaque	Brittle		Conchoidal	5.5	4.7	Veins near igneous rocks	Titanic iron ore, menaccanite
Jamesonite	$Pb_4FeSb_6S_{14}$	Pb 40.16 Fe 2.71 Sb 35.39	Monoclinic prismatic	Gray black, tarnishes iridescent	Metallic	Gray black	Opaque	Brittle	Perfect	Uneven	2.5	5.8	Veins with galena, sphalerite, quartz	Brittle feather ore
Kaolinite	$Al_4(Si_4O_{10})(OH)_8$	–	Triclinic	White with reddish, brownish, or bluish tints	Dull earthy	White	Transparent to translucent	Flexible	Perfect	–	2.3	2.6	A result of decomposition of aluminous minerals	Kaolin, China clay, smelite

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Kernite	$Na_2B_9O_7{\cdot}4H_2O$	Na_2O 22.66 B_2O_3 51.02	Monoclinic prismatic	Colorless; white	Vitreous	White	Transparent		Perfect	–	3.0	1.95	In salt marshes as an evaporite	Rasorite
Kyanite	$Al_2O(SiO_4)$	Al_2O_3 60.43–62.74	Triclinic	Blue to white	Vitreous to pearly	White	Translucent to transparent		Very perfect	–	6.2	3.6	In gneiss and mica schist	Diathene, cyanite
Lazulite	(Mg, Fe) $Al_2(PO_4)_2(OH)_2$	Mg:Fe = 1:0 MgO 13.34 Al_2O_3 33.73 P_2O_5 46.97	Monoclinic prismatic	Azure blue, bluish white, or bluish green	Vitreous	White	Subtranslucent to opaque	Brittle	Prismatic indistinct	Uneven	5.5	3.1	In quartz or pegmatite veins	blue spar, blue feldspar
Lazurite	(Na, $Ca)_8(Al_6Si_6O_{24})$ $(SO_4, S, Cl)_2$	–	Isometric	Deep azure blue, greenish blue	Vitreous	White	Translucent	Brittle	Dodecahedral	Uneven	5.3	2.4	Contact metamorphism limestone	–
Lepidolite mica	$K_2(Li, Al)_{5-6}(Si_{6-7}Al_{2-1}O_{20})(OH, F)_4$	–	Monoclinic	Colorless, shades of pink, purple	Vitreous to pearly	–	Translucent	Elastic	Basal, highly eminent	–	3.2	3.0	In granite pegmatites	Lithia mica
Leucite	$K(AlSi_2O_6)$	–	Tetragonal (pseudocubic)	White or gray	Vitreous to dull	White	Translucent to opaque	Brittle	Very imperfect	Conchoidal	5.7	2.50	In recent lavas	
Limonite	Largely $HFeO_2{\cdot}nH_2O$ also $Fe_2O_3{\cdot}nH_2O$ and other hydrous iron oxides	–	Amorphous or cryptocrystalline	Shades of brown, commonly dark brown to brownish black. When earthy, dull brown, yellow, ocher	Vitreous to dull	Yellowish brown to reddish	Opaque	Brittle	None	Uneven	5.3	3.8	Secondary iron mineral	Brown ocher, bog iron ore
Magnesite	$MgCo_3$	MgO 47.81	Hexagonal; hexagonal scalenohedral	Colorless, white, grayish white, yellowish to brown	Vitreous	Nearly white	Transparent to opaque	Brittle	Perfect	Flat-conchoidal	4.0	3.06	Alteration product of magnesium-rich rocks by carbonic fluids	Baudisserite, magnesianite

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Magnetite	$FeFe_2O_4$	Fe 72.4	Isometric hexoctahedral	Black to brownish black	Metallic to semimetallic	Black	Opaque	Brittle	Not distinct	Subconchoidal to uneven	6.0	5.17	Common constituent of crystalline rocks	Lode stone, siderite
Malachite	$Cu_2(OH)_2(CO_3)$	Cu 57.4	Monoclinic	Bright green, blackish green	Adamantine to vitreous	Pale green	Translucent to opaque	Brittle	Perfect	Subconchoidal, uneven	3.7	3.96	Oxidation zone of copper deposits. Alteration product of other copper minerals	Mountain green
Manganite	MnO(OH)	Mn 62.4	Monoclinic prismatic	Dark steel gray to iron black	Submetallic	Reddish brown to black	Opaque	Brittle	Perfect	Uneven	4.0	4.3	With other manganese oxides, barite, calcite	Sphenomanganite, newkirkite
Marcasite	FeS_2	Fe 46.55	Orthorhombic dipyramidal	Pale bronze yellow	Metallic	Grayish or brownish black	Opaque	Brittle	Poor	Uneven	6.5	4.9	Formed near surface, with galena, sphalerite, calcite, dolomite	White iron pyrites, cockscomb
Microcline feldspar	$K(AlSi_3O_8)$	–	Triclinic	Colorless or white; sometimes pink, yellow, red, or green	Vitreous	White	Transparent to translucent	Brittle	Perfect	Uneven	6.3	2.55	In igneous rocks	Amazon stone, moonstone
Millerite	NiS	Ni 64.67	Hexagonal scalenohedral	Pale brass yellow	Metallic	Greenish black	Opaque	Elastic	Perfect	Uneven	3.5	5.4	Capillary crystals among other sulfides	Harkise, capillose
Molybdenite	MoS_2	Mo 59.94	Hexagonal; dihexagonal dipyramidal	Lead gray	Metallic	Greenish on porcelain; bluish gray on paper	Opaque	Flexible, sectile	Perfect	–	1.5	4.7	Veins often with quartz, and copper sulfides	Moly, molybdaena

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Muscovite mica	$K_2Al_4(Si_6Al_2O_{20})(OH,F)_4$	–	Monoclinic prismatic	Colorless; light shades of green, red, or brown	Vitreous to silky or pearly	White	Transparent to translucent	Elastic	Basal eminent	–	2.3	2.88	Original constituent of granite permatites and other potash and alumina-rich rocks	White mica, adamsite, didymite, isinglass
Niccolite	NiAs	Ni 43.92	Hexagonal; dihexagonal dipyramidal	Pale copper red	Metallic	Pale brownish black	Opaque	Brittle	None	Uneven	5.0	7.5	With sulfides and silver-arsenic minerals	Copper nickel, nickeline
Niter	KNO_3	K_2O 46.5	Orthorhombic dipyramidal	Colorless to white, gray	Vitreous	Colorless to white	–	Brittle	Perfect		2.0	2.1	Occurs on the surface of the earth	Saltpeter, nitrokalite
Opal	$SiO_2 \cdot nH_2O$	–	Submicro-crystalline aggregate	Milky white or bluish white; also yellow to brown, orange, green, and blue	Vitreous, often somewhat resinous	White	Transparent to nearly opaque	Brittle	None	Conchoidal	6.0	2.1	In seams and fissures of igneous rocks; deposited at low temperature by silica-bearing waters	Girasol, hydrophane, tabasheer, geyserite
Orpiment	As_2S_3	As 60.91	Monoclinic prismatic	Lemon yellow; golden yellow; brownish yellow	Pearly on cleavage surfaces, elsewhere resinous	Pale lemon yellow	Translucent	Sectile, flexible	Perfect	–	2.0	3.5	Vein with realgar	Yellow arsenic, arsenblende
Orthoclase feldspar	$K(AlSi_3O_8)$	–	Monoclinic	Colorless or white; sometimes pink, yellow, red, or green	Vitreous	White	Transparent to translucent	Brittle	Perfect	Conchoidal to uneven	6.0	2.57	In igneous rocks	Common feldspar, moonstone

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Pentlandite	$(Fe, Ni)_9S_8$	Fe 32.55 Ni 34.22	Isometric hexoctahedral	Light bronze yellow	Metallic	Light bronze brown	Opaque	Brittle	Octahedral	Uneven	4.0	5.0	Intergrown with pyrrhotite	Micropyrite, folgerite
Phenacite	$Be_2(SiO_4)$	BeO 45.55	Hexagonal rhombohedral	Colorless, white, yellow, rose red, brown	Vitreous		Transparent to subtranslucent	Brittle	Distinct	Conchoidal	7.8	2.98	Granite pegmatites	Phenakite
Phlogopite mica	$K_2(Mg,Fe_{+2})(Si_6Al_2O_{20})(OH,F)_4$	–	Monoclinic prismatic	Colorless, yellowish, brown, green, reddish brown, dark brown	Pearly to vitreous	White to gray	Transparent to opaque	Elastic	Basal highly eminent	–	2.8	2.8	In crystalline limestone or dolomite and also in serpentine	Amber mica, flogopite
Platinum	Pt	100	Isometric hexoctahedral	Whitish steel gray to dark	Metallic	Whitish steel gray	Opaque	Malleable and ductile	None	Hackly	4.3	19.0	Placer deposits, with gold, chromite	
Polyhalite	$K_2Ca_2Mg(SO_4)_4{\cdot}2H_2O$	K_2O 15.62 CaO 18.60 MgO 6.69	Triclinic pinacoidal	White or gray; often salmon pink to brick red	Vitreous to resinous	–	–	–	Distinct	–	2.8	2.78	–	Mamanite, ischelite
Proustite	Ag_3AsS_3	Ag 65.42 As 15.14	Hexagonal ditrigonal pyramidal	Scarlet vermilion	Adamantine	Vermilion	Translucent	Brittle	Distinct	Uneven	2.5	5.6	Usually with pyrargerite	Light ruby silver
Psilomelane	$BaMn_2Mn_8{}^4O_{16}(OH)_4$	Ba 14.35 Mn 51.75	Amorphous	Black	Submetallic	Brownish black	Opaque	–	None	–	6.0	4.0	Usually with pyrolusite	Black hematite, psilomelanite
Pyrargyrite	Ag_3SbS_3	Ag 59.76 Sb 22.48	Hexagonal ditrigonal pyramidal	Deep red	Adamantine	Purplish red	Opaque	Brittle	Distinct	Uneven	2.5	5.8	Veins with silver, galena, sphalerite	Dark ruby silver

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Pyrite	FeS_2	Fe 46.55	Isometric diploidal	Pale brass yellow	Metallic, splendent to glistening	Greenish or brownish black	Opaque	Brittle	Indistinct	Uneven	6.0	5.0	Primary, veins or disseminated, usually crystalline	Fool's gold, iron pyrites, mundic
Pyrolusite	MnO_2	Mn 63.19	Tetragonal ditetragonal-dipyramidal	Light steel gray	Metallic	Black, bluish black	Opaque	Soft, soils fingers	None	–	2.0	4.8	Usually secondary, often in clays	Polianite, varvicite
Pyrophyllite	$Al_4(Si_8O_{20})(OH)_2$	–	Monoclinic	White, yellow, pale blue, grayish or brownish green	Pearly	White	Subtransparent to opaque	Flexible	Eminent	–	1.5	2.8	In schistose rocks	Pyrauxite
Pyrrhotite	$Fe_{1-x}S$ (x between 0 and 2)	Fe 63.53	Hexagonal; dihexagonal dipyramidal	Bronze yellow to pinchbeck brown	Metallic	Dark grayish black	Opaque	Brittle	Distinct	Uneven	4.0	4.6	In basic igneous, with sulfides and magnetite	Magnetic pyrites, pyrrhotine
Quartz	SiO_2	Si 46.71	Hexagonal; trigonal trapezohedral	Varies widely	Vitreous; sometimes greasy	White	Transparent to translucent	Brittle	None	Conchoidal	7.0	2.65	In igneous rocks, sediments, and metamorphics	Rock crystal, chalcedony, agate, flint, chert, jasper
Realgar	AsS	As 70.0	Monoclinic	Aurora red to orange yellow	Resinous to greasy	Orange red to aurora red	Transparent to translucent	Sectile	Fair	Conchoidal	2.0	3.6	Vein with orpiment, stibnite lead, silver, and gold	Red orpiment, red arsenic, ruby sulfur
Rhodochrosite	$MnCO_3$	MnO 61.71	Hexagonal; hexagonal scalenohedral	Pink, rose, red; fawn colored; brown	Vitreous, inclining to pearly	White	Translucent to subtranslucent	Brittle	Perfect	Uneven	4.0	3.52	Gangue mineral of primary origin in sediments and metesediments	Manganese spar, dialogite

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TABLE 2.10 Properties of minerals (continued)

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Rutile	TiO_2	Ti 60	Tetragonal; ditetrago-nal-dipyra-midal	Reddish brown, passing into red	Metallic to adamantine	Pale brown to yellowish	Transparent to opaque	Brittle	Distinct	Uneven	6.5	4.2	Frequently secondary in micas or igneous rocks. Black sands	Edisonite, titanite
Scheelite	$CaWO_4$	WO_3 80.53	Tetragonal dipyramidal	Yellowish white, pale yellow, or brownish	Vitreous	White	Transparent to translucent	Brittle	Distinct	Uneven	4.8	6.0	Pegmatite veins or in veins associated with granite or gneiss	Tungstein, scheelspath
Serpentine	$Mg_3(Si_2O_5)(OH)_2$	MgO 43.0 Si_2O 44.1 H_2O 12.9	Monoclinic prismatic	Green, green blue, white, gray, yellow	Waxy, greasy, or silky	White	Translucent to opaque	–	Distinct	Conchoidal or splintery	4.0	2.58	Secondary mineral formed by alteration of nonalu-minous silicates containing magnesia	Verd antique, bowenite, ophite
Siderite	$FeCO_3$	Fe 48.2	Hexagonal; hexagonal scalenohedral	Yellowish brown to reddish brown	Vitreous, inclining to pearly or silky	White	Translucent to subtranslucent	Brittle	Perfect	Uneven or subconchoidal	3.7	3.85	Sometimes in sedimentary deposits and in veins	Spathic iron, clay ironstone
Silver	Ag	100	Isometric hexoctahedral	Silver white, gray to black due to tarnish	Metallic	Silver white	Opaque	Malleable and ductile	None	Hackly	2.8	10.5	Usually secondary in upper part of silver-bearing veins	White gold
Smithsonite	$ZnCO_3$	Zn 52.3	Hexagonal; hexagonal scalenohedral	Grayish white to dark gray, greenish or brownish white	Vitreous	White	Subtransparent to translucent	Brittle	Perfect	Uneven to imperfectly conchoidal	5.5	4.38	Both in veins and beds with galena and sphalerite in calcareous rocks	Dry bone ore, turkey-fat ore, zinc spar

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Sodalite	$Na_8(Al_6Si_6O_{24})Cl_2$	–	Isometric	Pale pink, gray, yellow, blue, green	Vitreous	White	Transparent to translucent	Brittle	Dodecahedral distinct	Conchoidal to uneven	5.8	2.17	Igneous rocks of the nepheline-syenite groups	–
Soda niter	$NaNO_3$	Na_2O 36.5	Hexagonal scalenohedral	Colorless; also white	Vitreous	White	Transparent	Rather sectile	Perfect	Conchoidal	1.8	2.27	In deserts as an evaporite	Chile saltpeter, nitratine
Sphalerite	ZnS	Zn 67.10	Isometric hextetrahedral	Commonly brown, black, yellow; also red, green to white to nearly colorless	Resinous to adamantine	Brownish to light yellow and white	Translucent	Brittle	Perfect	Conchoidal	3.5	4.0	Often in limestone with other sulfides	Zinc blende, black jack, ruby zinc
Spinel	$MgAl_2O_4$	MgO 27.49–13.65	Isometric hexoctahedral	Variable; red to blue, green, brown to nearly colorless	Vitreous, splendent to nearly dull	White	Transparent to nearly opaque	Brittle	Imperfect	Conchoidal	8.0	3.8	In sands and gravels, accessory mineral in basic rocks	Balas ruby, picotite, rubicelle
Stannite	Cu_2FeSnS_4	Cu 29.58 Fe 12.99 Sn 27.61	Tetragonal scalenohedral	Steel gray to iron black	Metallic	Blackish	Opaque	Brittle	Indistinct	Uneven	3.5	4.4	Veins with cassiterite, chalcopyrite, pyrite	Bell metal ore, zinnkies
Staurolite	$(Fe^{+2}, Mg)_2(Al, Fe^{+3})_9O_6(SiO_4)_4(O,OH)_2$	–	Monoclinic prismatic	Dark brown, reddish brown, yellow brown	Vitreous to resinous	Gray	Translucent to nearly opaque	Brittle	Distinct but interrupted	Subconchoidal	7.3	3.70	In crystalline schists or phyllites as a result of regional metamorphosis	Cross stones, fairy stone crosses
Stephanite	Ag_5SbS_4	Ag 68.33 Sb 15.42	Orthorhombic pyramidal	Iron black	Metallic	Iron black	Opaque	Brittle	Imperfect	Uneven	2.5	6.2	Veins with silver, galena, sphalerite	Brittle silver, melanglance

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Stibnite	Sb_2S_3	Sb 71.69	Orthorhombic dipyramidal	Lead gray to steel gray	Metallic, splendent on cleavage	Lead gray	Opaque	Sectile	Perfect	Subconchoidal	2.0	4.6	Veins with quartz often in granite	Antimonite, antimony glance
Strontianite	$SrCO_3$	SrO 70.19	Orthorhombic dipyramidal	Colorless to gray, yellowish or greenish	Vitreous, resinous on fracture	White	Transparent to translucent	Brittle	Nearly perfect	Uneven	3.8	3.70	In veins in limestones and marls	Strontian
Sulfur	S	100	Orthorhombic dipyramidal	Yellow, brown, green, red, gray	Resinous to greasy	White	Translucent	Brittle	Imperfect	Uneven	2.0	2.07	Volcanic activity, usually with gypsum, limestone	–
Sylvanite	$(Au, Ag)Te_2$	Au 24.19 Ag 13.22	Monoclinic prismatic	Steel gray to silver white	Metallic brilliant	Same as color	Opaque	Brittle	Perfect	Uneven	2.0	8.1	Veins with gold, pyrite, and quartz	Aurotellurite
Sylvite	KCl	K 52.44	Isometric hexoctahedral	Colorless or white; also grayish, bluish, yellowish red, or red	Vitreous	White	Transparent to translucent	Brittle	Cubic perfect	Uneven	2.0	1.98	An evaporite	Muriate of potash, hoevelite
Talc	$Mg_6(Si_8O_{20})(OH)_4$	MgO 29.13–31.76 SiO_2 60.06–62.67	Monoclinic	Colorless, white, green, brown	Pearly to greasy	White	Subtransparent to translucent	Sectile	Perfect	–	1.3	2.75	Secondary mineral formed by alteration of nonaluminous magnesium silicates	Steatite, soapstone
Tennantite	$(Cu, Fe)_{12}As_4S_{13}$ $Cu_{13}As_4S_{13}$	Cu 51.57 As 20.26	Isometric hextetrahedral	Flint gray to iron black to dull black	Metallic, often splendent	Black to brown	Opaque	Brittle	–	–	3.5	4.4	Veins with other copper minerals	Arsenicalfahlerz
Tetrahedrite	$(Cu, Fe)_{12}Sb_4S_{13}$ $Cu_{12}Sb_4S_{13}$	Cu 45.77 Sb 29.22	Isometric hextetrahedral	Flint gray to iron black to dull black	Metallic, often splendent	Black to brown	Opaque	Brittle	None	Uneven	3.5	4.7	Veins with copper, silver, pyrite, galena, sphalerite, quartz	Gray copper, fahlore, panabase

(continues)

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TABLE 2.10 Properties of minerals (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Topaz	$Al_2(SiO_4)(OH, F)_4$	Al_2O_3 55.67–56.76 F 13.23–20.7	Orthorhombic	Colorless, white, yellow, light shades of gray, green, red, or blue	Vitreous	White	Transparent to subtranslucent	Brittle	Highly perfect	Subconchoidal to uneven	8.0	3.5	In veins and cavities in igneous rocks	Brazilian ruby, chrysolithos
Tourmaline	$NaMg_3Al_6B_3Si_6O_{27}(OH, F)_4$	–	Hexagonal; ditrigonal pyramidal	Black to brown	Vitreous to resinous	White	Transparent to opaque	Brittle	Difficult	Subconchoidal to uneven	7.8	3.10	In granites or gneisses or in pegmatite veins	Brazilian emerald, peridot, rubellite, siberite, schorl
Turquoise	$CuAl_6(PO_4)_4(OH)_8 \cdot H_2O$	CuO 9.78 Al_2O_3 37.60	Triclinic pinacoidal	Sky blue, bluish green, to apple green, greenish gray	Waxy to vitreous	White or greenish	Subtranslucent to opaque	Rather brittle	Two directions in crystals, none in massive material	Small conchoidal	5.5	2.7	Secondary mineral occurring in veins in highly altered rocks	Agaphite, calaite
Uraninite	UO_2 to U_3O_8	UO_2 varies from 70.09 to 23.07 while UO_3 varies 22.69 to 40.60	Isometric hexoctahedral	Steely to velvety black and brownish black, grayish, greenish	Submetallic to pitchlike or greasy and dull	Brownish black, grayish olive green, a little shining	Opaque	Brittle	–	Uneven	5.5	9.4	Granitic pegmatites, or with ores of silver, lead, copper	Pitchblende, ulrichite
Vanadinite	$Pb_5(VO_4)_3Cl$	PbO 78.80 V_2O_5 19.26	Hexagonal; hexagonal dipyramidal	Orange red, ruby red, brownish red	Subresinous	White or Yellowish	Subtranslucent to opaque	–	–	Uneven, flat or conchoidal	2.9	6.88	In altered lead deposits	Vanadate of lead
Wavellite	$Al_3(OH)_3(PO_4)_2 \cdot 5H_2O$	Al_2O_3 37.11 P_2O_5 34.4	Orthorhombic	Greenish white, green, yellow	Vitreous to resinous	White	Translucent	Brittle	Rather perfect	Uneven to subconchoidal	3.6	2.32	Secondary mineral associated with many rock types	Devonite, hydrargillite

(continues)

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TABLE 2.10 **Properties of minerals** (continued)

Name	Chemical Formula	Metal, %	Crystal Structure	Color	Luster	Streak	Degree of Transparency	Tenacity	Cleavage	Fracture	Mohs Hardness	Specific Gravity	Occurrence	Common Names or Synonyms
Willemite	$Zn_2(SiO_4)$	ZnO 73.0	Hexagonal rhombohedral	White or greenish yellow	Vitreous to resinous	White or colorless	Transparent to opaque	Brittle	Easy	Conchoidal to uneven	5.5	4.04	In zinc ore deposits	Villemite, hebertine
Witherite	$BaCO_3$	BaO 77.70	Orthorhombic dipyramidal	Colorless to milky white or grayish	Vitreous, resinous on fracture	White	Subtransparent to translucent	Brittle	Distinct	Uneven	3.4	4.31	In veins with galena, direct crystallization from barium carbonate-rich fluids	Barolite
Wolframite	$(Fe, Mn)WO_4$	WO 74.78–76.58	Monoclinic prismatic	Grayish or brownish black	Submetallic	Reddish brown to black	Opaque	Brittle	Very perfect	Uneven	5.3	7.3	In granite and pegmatite veins	Wolfram, mock-lead
Wulfenite	$PbMoO_4$	PbO 60.79 MoO_3 39.21	Tetragonal pyramidal	Orange yellow to yellowish gray, grayish white	Resinous	White	Subtransparent to subtranslucent	Brittle	Very smooth	Subconchoidal	2.8	6.85	Oxidation zone of lead and zinc deposits	Yellow lead ore, melinose
Zincite	ZnO	Zn 80.34	Hexagonal; dihexagonal pyramidal	Orange yellow to deep red	Subadamantine	Orange yellow	Translucent	Brittle	Perfect	Subconchoidal	4.5	5.5	Usually with franklinite and willemite. Sometimes in calcite	Spartalite
Zircon	$Zr(SiO)_4$	ZrO_2 67.2	Tetragonal	Reddish brown, yellow, gray, green, or colorless	Adamantine	Uncolored	Transparent to subtranslucent or opaque	Brittle	Imperfect	Conchoidal	7.5	4.69	Accessory mineral in igneous rocks	Hyacinth, azorite

Source: Bolles and McCullough 1985

TABLE 2.11 Properties of major mineral fillers

Mineral	Theoretical Chemical Composition	Specific Gravity	Bulk Density, kg/m³*	Hardness Mohs Scale	Refractive Index	Reaction, pH	Oil Absorption, cc/100 g	Particle Characteristics
Asbestos (chrysotile)	$3MgO{\cdot}2SiO_2{\cdot}2H_2O$	2.5–2.6	160–640	2.5–4.0	1.51–1.55	8.5–10.3	40–90	Fibers fine, easily separable; fibrils hexagonally close packed 100–300 Å
Barite	$BaSO_4$	4.3–4.6	1280–2400	2.5–3.5	1.64	7	6–10	Generally equi-dimensional
Bentonite	$(Mg,Ca)O{\cdot}Al_2O_35SiO_2{\cdot}xH_2O$	2.3–2.8	800–960	1.5+	1.55–1.56	6.2–9.0	20–30	Porous microaggregates, irregular shapes; ultimate plate structure
Diatomite	$SiO_2{\cdot}xH_2O$	2.0–2.35	96–320	4.5–6.0	1.42–1.49	6–8.5	100–300	Unique diatom structure; micro- and ultra-microporosity
Fuller's earth	$(Mg,Ca)O{\cdot}Al_2O_35SiO_2{\cdot}xH_2O$	2.2–2.6	432–608	4	1.50	7.5–8.2	30	Apparently equi-dimensional; electron-microscopically fibrous, lathlike
Gypsum	$CaSO_4{\cdot}2H_2O$	2.3	400–640	1.5–2.0	1.52	6.5–7	17–25	Irregular, roughly equi-dimensional
Kaolin	$Al_2SO_3{\cdot}2SiO_2{\cdot}2H_2O$	2.6	320–640	2.0–2.5	1.56–1.58	4.5–7	25–50	Thin, flat hexagonal plates, 0.05–2 μ size and stacks of same
Limestone	$CaCO_3$	2.7	640–1600	3	1.63–1.66	7.8–8.5	6–30	Variable size particles; ultimate rhombs
Mica (muscovite)	$H_2KAl_3(SiO_4)_3$	2.7–3.0	192–320	2.0–3.0	1.59±	7.4–9.4	25–50	Platelike particles
Nepheline syenite	$K_2O{\cdot}7d\_{2}O{\cdot}4.5Al_2O_3{\cdot}2OSiO_2$	2.61	800–1280	5.5–6.0	1.53	9.9	21–29	Nodular and irregular
Perlite	Like rhyolite	2.5–2.6	64–320	5.0	1.48–1.49	9	50–275	Expanded *glass* bubbles and fragments
Portland cement	Essentially carbon silicates and aluminates	2.9–3.15	1440–1600	5.6	17.2±	11.0–12.6	20	Variable, smooth, rounded, angular, and flake particles
Pumicite	A silicate, like rhyolite	2.2–2.63	640–800	5–6	1.49–1.50	7–9	30–40	Vesicular
Pyrophyllite	$Al_2O_3{\cdot}4SiO_2{\cdot}H_2O$	2.8–2.9	400–480	1–2	1.57–1.59	6–8	40–70	Minute foliated plates or scales and extra-long particles
Rock dusts	Variable	2.6–3.3	800–1600	4–6.5	Variable	Usually above 7	20–40	Variable
Silicas, crystalline and microcrystalline	SiO_2	2.60–2.65	800–1280	6.5–7.0	1.53–1.54	6–7	20–50	Variable sized, angular and equi-dimensional particles, or minute particles to porous masses
Slate	Mixture of mineral silicates	2.7–2.8	640–1280	4–6	–	6.8	20–25	Flat or wedge-shaped, or spherical grains
Talc	$H_2Mg_3(SiO_3)_4$	2.6–3.0	416–960	1–1.5	1.57–1.59	8.1–9.0	20–50	Lamellar, foliated, or microfibrous
Vermiculite	$(Mg,Fe)_3(Si,Al)_4O_{10}(OH)_2{\cdot}4H_2O$	2.2–2.7	96–160; fines-320	1.5	1.56	Pract. neutral	–	Platelets or lamellar structure
Wollastonite	$CaSiO_3$	2.8–3.0	320–640	4.5–5.0	1.63	9.9	25–30	Brilliant white powder with acicular nature

* Metric equivalent: 1 lb/ft³ × 16.01846 = kg/m³.

Source: Trivedi and Hagemeyer 1994

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TABLE 2.12 Typical analyses of U.S. primary coal seams by state

State	Time Period	Basin	County	Seam	Analysis Type	Moisture, %	Ash, %	Sulfur, %	Vol Matter, %	Fixed Carbon	BTU/lb
Alabama	–	Warrior	–	Blue Creek	As received	2.4	9	1.3	31	57.7	13,364
Colorado	1864–2001	Uinta	–	Uinta	As received	12.0	6.9	0.5	–	–	–
Illinois	–	Southeastern	LaSalle, Grundy	Herrin	As received	13–16	7–11	3–5	36–41	35–40	10,500–11,400
Indiana	–	Raccoon Creek Group	–	Sullivan	Moist free	–	15	2.1	–	–	11,500
Kansas	2001	Bandera	Linn	Mulberry	As received	9	14	4	–	–	11,300
Kentucky	–	Hazard	–	Hazard 7	As received	3.6	11.1	1.7	34.5	50.9	12,537
Louisiana	–	Norbiton	–	Wilcox	As received	30.00	16.10	0.64	–	–	7,000
Maryland	–	–	–	Upper Freeport	As received	1.7–5.3	5.3–14.1	0.8–4.2	15.4–21.9	63.6–72.8	12,890–14,480
Missouri	–	–	–	Bevier	As received	13.4	10.6	4.6	–	–	10,691
Montana	–	–	–	Decker	Moist free	–	4	0.4	–	–	9,652
New Mexico	–	–	–	Navajo	As received	13	19.3	0.8	31.3	35.9	9,124
North Dakota	–	–	–	23 Analyses	As received	37.9	6.2	26.7	26.7	29.2	6,783
Ohio	1946–2001	Monongalia	–	Pittsburgh No. 8	Moist free	–	10.47	3.04	–	–	12,948
Oklahoma	115 years	–	Pittsburg	Lower Hartshorne	As received	3.6	6.5	1.5	37.2	52.8	13,490
Pennsylvania	–	–	–	Pittsburgh No. 8	As received	3.2	8.4	1.5	–	–	12,920
Tennessee	–	–	Claiborne	Jellico	As received	2.0–6.8	2.1–12.4	0.6–2.2	30.8–40.1	51.1–59.6	12,490–14,560
Texas	–	–	Sabine	Wilcox	As received	32	15	1	28	24	6,460
Utah	–	Wasatch Plateau	Scofield Area	Scofield	As received	7.2	6.1	0.64	41.4	45.3	12,200
Virginia	–	–	Buchanan	Pocahontas No. 3	As received	1.32	6.92	0.66	18.76	71.31	14,103
Washington	–	Whatcom	–	Bellingham No. 1	As received	7.3	15.7	0.3	35.8	41.3	10,542
West Virginia	–	–	–	Upper Kittanning	Moist free	–	12.49	2.03	31.27	55.15	13,199
Wyoming	–	Powder River	Eastern Portion	Anderson Bed	As received	29.8	5.1	0.36	–	–	7,710

Data from Keystone Coal Industry Manual 1987 and 2004

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TABLE 2.13 Classification of coals by rank*

Class/Group	Fixed Carbon Limits (dry, mineral-matter-free basis), %		Volatile Matter Limits (dry, mineral-matter-free basis), %		Gross Calorific Value Limits (moist† mineral-matter-free basis)				Agglomerating Character
					MJ/kg‡		Btu/lb		
	Equal or Greater Than	Less Than	Greater Than	Equal or Less Than	Equal or Greater Than	Less Than	Equal or Greater Than	Less Than	
Anthracitic									
Meta-anthracite	98	—	—	2	—	—	—	—	Nonagglomerating
Anthracite	92	98	2	8	—	—	—	—	
Semianthracite§	86	92	8	14	—	—	—	—	
Bituminous									
Low volatile bituminous coal	78	86	14	22	—	—	—	—	Commonly agglomerating††
Medium volatile bituminous coal	69	78	22	31	—	—	—	—	
High volatile A bituminous coal	—	69	31	—	32.557		14,000**		
High volatile B bituminous coal	—	—	—	—	30.232	32.557	13,000**	14,000	
High volatile C bituminous coal	—	—	—	—	26.743	30.232	11,500	13,000	
					24.418	26.743	10,500	11,500	Agglomerating
Subbituminous									
Subbituminous A coal	—	—	—	—	24.418	26.743	10,500	11,500	Nonagglomerating
Subbituminous B coal	—	—	—	—	22.090	24.418	9,500	10,500	
Subbituminous C coal	—	—	—	—	19.300	22.090	8,300	9,500	
Lignitic									
Lignite A	—	—	—	—	14.650	19.300	6,300	8,300	
Lignite B	—	—	—	—	—	14.650	—	6,300	

* This classification does not apply to certain coals.

† *Moist* refers to coal that contains its natural inherent moisture but not including visible water on the coal's surface.

‡ Megajoules per kilogram; to convert British thermal units per pound to megajoules per kilogram, multiply by 0.002326.

§ If agglomerating, classify in low volatile group of the bituminous class.

** Coals having 69% or more fixed carbon on the dry, mineral-matter-free basis are classified according to fixed carbon, regardless of gross calorific value.

†† There may be nonagglomerating varieties in these groups of the bituminous class, and there are notable exceptions in the high volatile C bituminous group.

Source: ASTM D388-18a

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TABLE 2.14 Typical proximate and ultimate analyses and heating values of coals in the United States

Coal Type	Ultimate Analysis (dry), wt %						Proximate Analysis (as received), wt %				
	Carbon	Hydrogen	Sulfur	Nitrogen	Ash	Oxygen	Volatile Matter	Fixed Carbon	Moisture	Ash	Heating Value (as received), Btu/lb
Lignite	65.7	4.5	1.0	1.2	9.2	18.4	31.4	25.9	35.5	7.2	7,100
Western subbituminous	62.6	4.0	1.0	1.0	13.6	17.8	36.6	42.8	8.1	12.5	9,400
Illinois No. 6	70.0	4.9	3.8	1.4	9.2	10.7	41.1	39.6	11.2	8.1	11,300
Eastern bituminous (high-sulfur)	70.4	4.6	4.6	1.4	10.5	8.5	37.0	46.4	6.9	9.7	11,700
Eastern bituminous	79.9	5.5	1.3	1.5	5.4	6.4	36.9	53.9	4.0	5.2	14,000

Source: Levy et al. 1981

TABLE 2.15 Petrographic and physical properties for various U.S. coal seams

Coal Seam	Location of Band Within Seam (from top to bottom)	MM*	VM*	FSI*	Btu*	BSG*	T*	HGI*
Kittanning Coal, Preston County, West Virginia	10 in. of coal at top of seam	16.3	34.9	9	13,020	1.42	116	78
	7 in. of dark gray shale†	–	–	–	–	–	–	–
	2¾ in. of coal and shale lenses	–	–	–	–	–	–	–
	5½ in. of dark brown shale	–	–	–	–	–	–	–
	7½ in. friable dull coal	16.1	37.7	9	13,192	1.38	121	85
	1 in. of shale	–	–	–	–	–	–	–
	9 in. of coal	26.5	34.3	5	11,525	1.48	102	73
	3¼ in. of coal with shale lenses	–	–	–	–	–	–	–
	8 in. of coal grading into shale	–	–	–	–	–	–	–
	24 in. of cross-bedded shale	–	–	–	–	–	–	–
	18 in. of blocky coal	6.5	32.2	9	14,655	1.30	107	89
	1 in. of black shale	–	–	–	–	–	–	
	9 in. of blocky coal	11.1	33.7	9	14,016	1.35	104	86
Pond Creek Coal, Stone County, Kentucky	4 in. of dull blocky coal†	–	–	–	–	–	–	–
	8 in. of coal with lenses of clay	–	–	–	–	–	–	–
	12½ in. of coal with nodules and bonds of shale	–	–	–	–	–	–	–
	19½ in. of blocky detrital coal	18.2	33.3	1	12,331	1.43	7	58
	6 in. of blocky coal	15.4	34.6	1	12,936	1.39	32	52
	2½ in. of dull blocky coal	13.1	31.0	1	13,354	1.41	41	52
	5½ in. of finely banded coal	6.9	35.8	1	14,157	1.30	51	58
Jawbone Coal, Dickenson County, Virginia	10½ in. of blocky coal	3.6	27.9	8.5	15,366	1.30	89	93
	11 in. of bright banded coal	11.6	27.3	8	13,957	1.38	84	88
	16 in. of blocky coal	9.6	28.5	8.5	14,359	1.32	93	84
Sewickley Coal, Fayette County, Pennsylvania	Bulk sample of coal	12.7	38.9	7.5	13,485	1.37	84	72
Pittsburgh No. 8 Coal, Athens County, Ohio	Bulk sample of coal 48 in. thick	15.5	48.7	3	12,950	–	–	–
Herrin No. 6 Coal, Jefferson County, Illinois	Bulk sample of coal 6 ft thick	10.5	42.1	–	12,890	–	–	–
Crowburg Coal, Randolph County, Missouri	4 in. of bright coal	–	–	–	–	–	–	–
	1 in. of pyritized coal	–	–	–	–	–	–	–
	1 in. of bright coal	–	–	–	–	–	–	–
	2 in. of pyritized coal	–	–	–	–	–	–	–
	11 in. of banded coal analysis of bulk sample	16.7	48.3	4	11,708	–	–	–
Blue Creek Coal, Jefferson County, Alabama	Bulk sample coal	16.8	28.9	8.5	14,008	–	–	–
Anderson-Canyon (Wyodak) Coal, Campbell County, Wyoming	1 ft of coal; vitrain bands + attritus	–	27.3	8	13,957	1.38	84	88
	21 ft of coal; vitrain bands + bright attritus	9.6	28.5	8.5	14,359	1.32	93	84
	13 ft of banded to nonbanded coal; vitrain, attritus, wood grain	12.7	38.9	7.5	13,485	1.37	84	72
	25 ft of coal; vitrain bands, attritus, fusain	–	–	–	–	–	–	–
	0.1 ft of carbonaceous claystone	–	–	–	–	–	–	–
	6.5 ft of banded coal; vitrain, attritus	–	–	–	–	–	–	–
	7.8 ft of banded to nonbanded coal; vitrain, attritus, wood grain	–	–	–	–	–	–	–
	Bulk sample coal	10.1	48.8	–	11,500	–	–	–

* MM = mineral matter = 1.1 (ash) + 0.1 (sulfur); VM = dry mineral matter-free volatile matter; FSI = free-swelling index; Btu = moisture-free heating value; BSG = bulk specific gravity; T = Gieseler maximum plastic temperature range; HGI = Hardgrove grindability index.

† Bands that are reported but not analyzed have specific gravities above 1.60.

Source: Beasley et al. 1991

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TABLE 2.16 Sulfur content and sulfur forms for various coals

Mine Location	Coal Seam	Percentage, Moisture-Free Basis*			
		Total Sulfur	Pyritic Sulfur	Organic Sulfur	Organic Sulfur as Percentage of Total Sulfur
Washington County, Pennsylvania†	Pittsburgh	1.13	0.35	0.78	69.0
Clearfield County, Pennsylvania	Upper Freeport	3.56	2.82	0.74	20.8
Allegheny County, Pennsylvania	Thick Freeport	0.92	0.46	0.45	48.9
Somerset County, Pennsylvania	B	0.78	0.19	0.57	73.1
Somerset County, Pennsylvania	C prime	2.00	1.43	0.54	27.0
Clearfield County, Pennsylvania	B	1.90	1.12	0.75	39.5
Cambria County, Pennsylvania	Miller	1.25	0.56	0.65	52.0
Franklin County, Illinois	No. 6	2.52	1.50	1.02	40.5
Franklin County, Illinois	No. 6	1.50	0.81	0.69	46.0
Montgomery County, Illinois	No. 6	4.97	2.53	2.40	48.3
Williamson County, Illinois	No. 6	4.01	2.17	1.80	44.9
Union County, Kentucky	No. 9	3.28	1.05	2.23	68.0
Union County, Kentucky	No. 9	3.46	1.65	1.81	52.3
Webster County, Kentucky	No. 12	1.48	0.70	0.78	52.7
Pike County, Kentucky	Freeburn	0.46	0.13	0.33	71.7
Letcher County, Kentucky	Elkhorn	0.68	0.13	0.51	75.0
McDowell County, West Virginia	Pocahontas No. 3	0.55	0.08	0.46	83.6
Boone County, West Virginia	Eagle	2.48	1.47	1.01	40.7
Walker County, Alabama	Pratt	1.62	0.81	0.81	50.0
Jefferson County, Alabama	Pratt	1.72	0.97	0.72	41.9
Jefferson County, Alabama	Mary Lee	1.05	0.33	0.69	65.7
Clay County, Indiana	No. 3	3.92	2.13	1.79	45.7
Cumnock, North Carolina	Deep River	2.32	1.52	0.80	34.5
Cumnock, North Carolina	Deep River	2.08	1.53	0.55	26.4
Allegany County, Maryland	Big Vein	0.86	0.18	0.67	77.9
Meigs County, Ohio	8-A	2.51	1.61	0.86	34.3
Natal, South Africa	No data	1.51	0.47	0.97	64.2
Transvaal, South Africa	No data	1.39	0.59	0.70	50.4
Transvaal, South Africa	No data	0.44	0.06	0.37	84.1
Brazil, South America	No data	2.39	1.78	0.50	20.9
Istria, Italy‡	No data	9.01	1.09	7.90	87.7
Germany, bituminous	No data	1.78	0.92	0.76	42.7
Germany, brown	No data	3.15	0.02	3.06	97.1
Germany	No data	4.77	0.15	4.57	95.8
Czechoslovakia, Bohemia, brown	No data	0.76	0.27	0.46	60.5
Great Britain, Tamworth	No data	4.30	2.11	1.87	43.5
Great Britain, Derbyshire	No data	2.61	1.55	0.87	33.3
Great Britain, Parkgate	No data	3.15	2.71	0.36	11.4
Great Britain, anthracite	No data	1.06	0.75	0.23	21.7

* Sulfate sulfur values are not recorded in this table. Where the sum of pyritic and organic sulfur is not equal to total sulfur, the difference is sulfate sulfur. In other cases, sulfate sulfur is included with the pyritic sulfur. Organic sulfur by difference.

† Average of two mines.

‡ Total iron in ash calculated to pyrite.

Source: Hower and Parekh 1991

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TABLE 2.17 Ash content and fusion temperature of various coals

Rank	Low Volatile Bituminous		High Volatile Bituminous			Subbituminous	Lignite
Seam	Pocahontas No. 3	No. 9	Pittsburgh	No. 6			
Location	West Virginia	Ohio	West Virginia	Illinois	Utah	Wyoming	Texas
Ash, dry basis, %	12.3	14.10	10.87	17.36	6.6	6.6	12.8
Sulfur, dry basis, %	0.7	3.30	3.53	4.17	0.5	1.0	1.1
Analysis of ash, percent by weight							
SiO_2	60.0	47.27	37.64	47.52	48.0	24.0	41.8
Al_2O_3	30.0	22.96	20.11	17.87	11.5	20.0	13.6
TiO_2	1.6	1.00	0.81	0.78	0.6	0.7	1.5
Fe_2O_3	4.0	22.81	29.28	20.13	7.0	11.0	6.6
CaO	0.6	1.30	4.25	5.75	25.0	26.0	17.6
MgO	0.6	0.85	1.25	1.02	4.0	4.0	2.5
Na_2O	0.5	0.28	0.80	0.36	1.2	0.2	0.6
K_2O	1.5	1.97	1.60	1.77	0.2	0.5	0.1
Total	98.8	98.44	95.74	95.20	97.5	86.4	84.3
Ash fusibility							
Initial deformation							
Temperature, °F							
Reducing	2,900+	2,030	2,030	2,000	2,060	1,990	1,975
Oxidizing	2,900+	2,420	2,265	2,300	2,120	2,190	2,070
Softening temperature, °F							
Reducing		2,450	2,175	2,160		2,180	2,130
Oxidizing		2,605	2,385	2,430		2,220	2,190
Hemispherical temperature, °F							
Reducing		2,480	2,225	2,180	2,140	2,250	2,150
Oxidizing		2,620	2,450	2,450	2,220	2,240	2,210
Fluid temperature, °F							
Reducing		2,620	2,370	2,320	2,250	2,290	2,240
Oxidizing		2,670	2,540	2,610	2,460	2,300	2,290

Adapted from Anon. 1972

ACKNOWLEDGMENTS

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CHAPTER

3

Mathematics, Statistics, and Probability

R. Karl Zipf Jr., P.E.

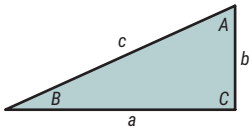
The contents of this chapter are common knowledge and available from a wide variety of published sources. For example, additional information can be found in Abramowitz and Stegun (1972) and NCEES (2013) or in the *Handbook of Mathematical, Scientific, and Engineering Formulas, Tables, Functions, Graphs, Transforms* (Staff of Research and Education Association 1988).

GEOMETRY

The areas and volumes of common shapes are given in this section.

Right Triangle

$A + B = C = 90°$



The Pythagorean theorem is

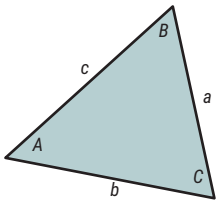
$$a^2 + b^2 = c^2$$

$$\text{area} = \frac{1}{2}ab$$

General Triangle

$A + B + C = 180°$

$$\text{area} = \frac{1}{2}ab \sin C$$

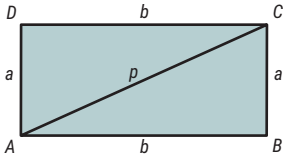


Rectangle

$A = B = C = D = 90°$

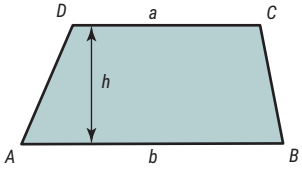
$$\text{area} = ab$$

$$\text{diagonal} = p = \sqrt{a^2 + b^2}$$



Trapezoid

$$\text{area} = \frac{1}{2}(a + b)h$$



Parallelogram

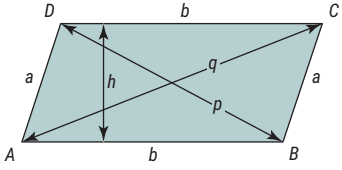
$A = C, B = D, A + B = 180°$

$$\text{area} = b \cdot h = b \cdot a \sin A$$

$$h = a \sin A$$

$$p = \sqrt{a^2 + b^2 + 2ab \cos A}$$

$$q = \sqrt{a^2 + b^2 - 2ab \cos A}$$



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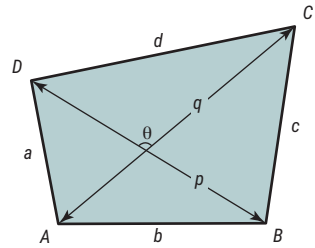
General Quadrilateral

$$\text{area} = \frac{1}{2}pq\sin\theta$$

$$\text{area} = \sqrt{(s-a)(s-b)(s-c)(s-d) - a\cdot b\cdot c\cdot d\,\cos^2\left(\frac{A+C}{2}\right)}$$

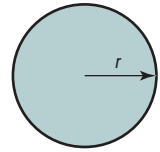
where $s = \frac{1}{2}(a+b+c+d)$



Circle

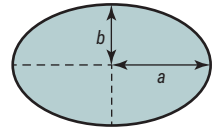
$$\text{area} = \pi r^2$$

$$\text{perimeter} = 2\pi r$$



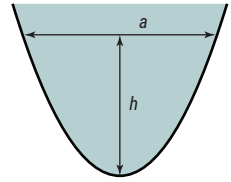
Ellipse

$$\text{area} = \pi a\cdot b$$



Parabola

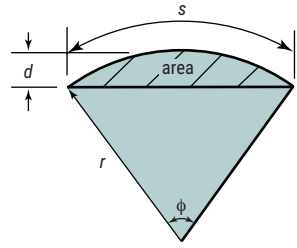
$$\text{area} = \frac{2}{3}a\cdot h$$



Circular Segment

$$\text{area} = \frac{r^2(\phi - \sin\phi)}{2}$$

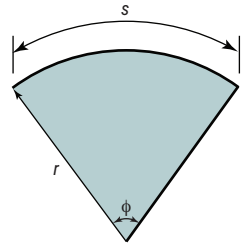
$$\phi = \frac{s}{r} = 2\left\{\arccos\left(\frac{r-d}{r}\right)\right\}$$



Circular Sector

$$\text{area} = \frac{\phi r^2}{2} = \frac{s\cdot r}{2}$$

$$\phi = \frac{s}{r}$$



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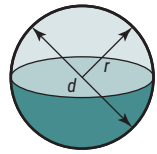
Sphere

volume = $\frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$

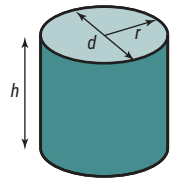
area = $4\pi r^2 = \pi d^2$



Right Circular Cylinder

volume = $\pi r^2 h = \frac{\pi}{4} d^2 h$

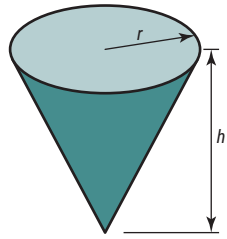
area = $2\pi r(h + r)$



Right Circular Cone

volume = $\frac{1}{3}\pi r^2 h$

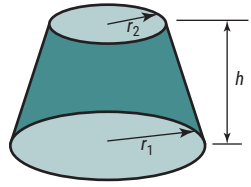
area = $\pi r\left(r + \sqrt{r^2 + h^2}\right)$



Frustum of Right Circular Cone

volume = $\frac{\pi}{3} h\left(r_1^2 + r_2^2 + r_1 \cdot r_2\right)$

area = $\pi(r_1 + r_2)\sqrt{(r_1 - r_2)^2 + h^2}$

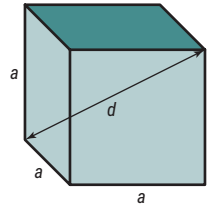


Cube

volume = a^3

diagonal = $d = a\sqrt{3}$

area = $6a^2$

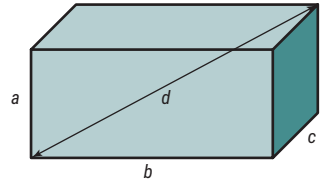


Rectangular Prism

volume = $a \cdot b \cdot c$

diagonal = $d = \sqrt{a^2 + b^2 + c^2}$

area = $2(a \cdot b + b \cdot c + c \cdot a)$



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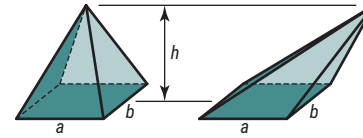
Pyramid

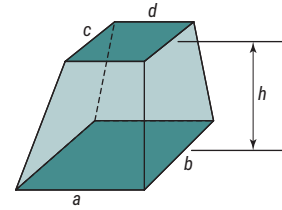
$$\text{volume} = \frac{1}{3}a \cdot b \cdot h$$

$$\text{area} = a \cdot b + (a + b)h$$



Frustum of Pyramid

$$\text{volume} = \frac{1}{3}h(a \cdot b + c \cdot d + \sqrt{a \cdot b \cdot c \cdot d})$$



PLANAR AREAS BY APPROXIMATION

Divide the planar area into n strips by equidistant parallel chords of length $y_0, y_1, y_2, \ldots, y_n$ (where y_0 and y_n may be zero), and let h denote the common distance between chords.

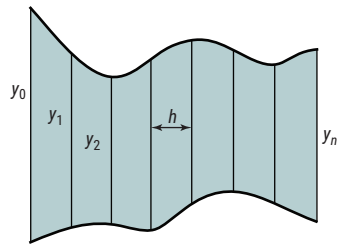
Trapezoidal Rule

$$\text{area} = h\left(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n\right)$$

Simpson's Rule

Where n is even,

$$\text{area} = \frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-2} + 2y_{n-1} + y_n)$$



ALGEBRA

Following are the basic algebraic laws.

Basic Laws

Commutative	$a + b = b + a$ and $a \cdot b = b \cdot a$
Associative	$a + (b + c) = (a + b) + c$ and $a(b \cdot c) = (a \cdot b)c$
Distributive	$c(a + b) = c \cdot a + c \cdot b$

Special Products and Factors

$(x + y)^2 = x^2 + 2xy + y^2$ and $(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

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Factorial n

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$$

Proportion

If $\frac{a}{b} = \frac{c}{d}$, then

$$\frac{a+b}{b} = \frac{c+d}{d}$$
$$\frac{a-b}{b} = \frac{c-d}{d}$$
$$\frac{a-b}{a+b} = \frac{c-d}{c+d}$$

Straight Line in Two Dimensions

In two dimensions, the general form for the equation of a straight line is

$$Ax + By + C = 0$$

The standard form for the equation of a straight line is

$$y = mx + b$$

which is also known as the slope intercept form.

The point slope form for the equation of a straight line is

$$y - y_1 = m(x - x_1)$$

Given two points, the slope of the line joining the two points is

$$m = (y_2 - y_1)/(x_2 - x_1)$$

The angle between lines with slopes m_1 and m_2 is

$$\alpha = \arctan\left[\frac{(m_2 - m_1)}{(1 + m_2 \cdot m_1)}\right]$$

Two lines are perpendicular if

$$m_1 = -1/m_2$$

In two dimensions, the distance between two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In three dimensions, the distance between two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Quadratic Equation

An algebraic equation of the form

$$ax^2 + bx + c = 0$$

has roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$, the two roots are real.

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If $b^2 - 4ac = 0$, the two roots are equal.

If $b^2 - 4ac < 0$, the two roots are complex conjugates.

Equation of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

where r is the radius of the circle with center at (h,k).

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Equation of a Plane in Three Dimensions

$$Ax + By + Cz + D = 0$$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Logarithms

The logarithm of x to the base b is defined as

$$\log_b(x) = c$$

where $b^c = x$. Special definitions for $b = e$ or $b = 10$ are

$\ln x$, where base $= e$

$\log x$, where base $= 10$

To change from one base to another,

$$\log_b x = \frac{\log_a x}{\log_a b}, \text{ for example, } \ln x = \frac{\log_{10} x}{\log_{10} e} = 2.302585\ (\log_{10} x)$$

Logarithmic Identities

$$\log_b b^n = n$$

$$\log x^c = c \log x$$

$$x^c = \text{antilog}\ (c \log x)$$

$$\log xy = \log x + \log y$$

$$\log_b b = 1$$

$$\log 1 = 0$$

$$\log \frac{x}{y} = \log x - \log y$$

Vectors

A and B are vectors, and i, j, and k are orthogonal unit vectors.

Components of a Vector

$$A = a_1 i + a_2 j + a_3 k$$

$$B = b_1 i + b_2 j + b_3 k$$

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Addition and Subtraction of Vectors

$$A + B = (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k$$

$$A - B = (a_1 - b_1)i + (a_2 - b_2)j + (a_3 - b_3)k$$

Dot or Scalar Product of Vectors

$$A \cdot B = a_x b_x + a_y b_y + a_z b_z = |A||B|\cos\theta = B \cdot A$$

where θ is the angle between vectors A and B.

Cross or Vector Product

$$A \times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)i + (a_3 b_1 - a_1 b_3)j + (a_1 b_2 - a_2 b_1)k$$

Triple Product

$$A(B \times C) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Matrices

A matrix is a rectangular array of numbers with m rows and n columns. Element a_{ij} is in row I and column j.

Multiplication

If matrix $A = (a_{ik})$ in an $m \times n$ matrix and $B = (b_{kj})$ is an $n \times s$ matrix, then the matrix product AB is an $m \times s$ matrix.

$$C = (c_{ij}) = \left[\sum_{l=1}^{n} a_{il} b_{lj}\right]$$

where n is the common integer representing the number of columns of A and the number of rows of b (I and $k = 1, 2, \ldots, n$).

Addition

If $A = (a_{ij})$ and $B = (b_{ij})$ are two matrices of the same size $m \times n$, the sum of $A + B$ is an $m \times n$ matrix $C = (c_{ij})$, where $c_{ij} = a_{ij} + b_{ij}$.

Identity

The matrix $I = (a_{ij})$ is a square $n \times n$ identity matrix when $a_{ii} = 1$ for $I = 1, 2, \ldots, n$ and $a_{ij} = 0$ for $i \neq j$.

Transpose

The matrix B is the transpose of the matrix A if each entry b_{ji} in B is the same as the entry a_{ij} in A, that is, $B = A^T$.

Inverse

The inverse B of a square $n \times n$ matrix A is $B = A^{-1}$ such that $AB = I$.

Determinants

A determinant of order n consists of n^2 numbers arranged in n rows and n columns and enclosed by two vertical lines.

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For a second-order determinant:

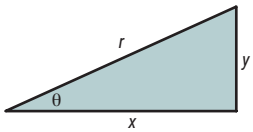
$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

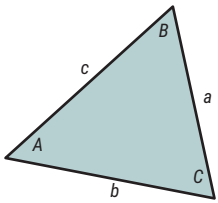
For a third-order determinant:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

TRIGONOMETRY

Trigonometric functions are defined using a right triangle.





$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}, \cot\theta = \frac{x}{y}$$

$$\csc\theta = \frac{r}{y}, \sec\theta = \frac{r}{x}$$

For any triangle, the side lengths and opposite angles are related as follows.

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Trigonometric Identities

$$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right) = -\sin\left(\theta - \frac{\pi}{2}\right)$$

$$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right) = -\cos\left(\theta + \frac{\pi}{2}\right)$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

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DERIVATIVES

Following are mathematical derivatives.

The Derivative

For any function $y = f(x)$, the derivative $= D_x y = dy/dx = y'$

$$y' = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

y' = the slope of the curve $f(x)$

Test for a Maximum

$y = f(x)$ is a maximum for $x = a$, if $f'(a) = 0$ and $f''(a) < 0$.

Test for a Minimum

$y = f(x)$ is a minimum for $x = a$, if $f'(a) = 0$ and $f''(a) > 0$.

Test for a Point of Inflection

$y = f(x)$ is a point of inflection at $x = a$, if $f''(a) = 0$ and if $f''(x)$ changes sign as x increases through $x = a$.

Rules of Differentiation

In the following relations, u, v, and w represent functions of x; a, b, and c are constants; n is an integer constant; and angles are in radians.

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(cx) = c$$

$$\frac{d}{dx}(cx^n) = ncx^{n-1}$$

$$\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$$

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \text{ (Chain rule)}$$

$$\frac{d[f(u)]}{dx} = \frac{d[f(u)]}{du}\frac{du}{dx}$$

$$\frac{du}{dx} = \frac{1}{dx/du}$$

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$$\frac{dy}{dx} = \frac{dy/du}{dx/du}$$

$$\text{Second derivative} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = f''(x) = y''$$

$$\text{Third derivative} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3} = f'''(x) = y'''$$

$$n\text{th derivative} = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right) = \frac{d^ny}{dx^n} = f^{(n)}(x) = y^{(n)}$$

Derivatives of Common Functions

$$\frac{d(\log_a u)}{dx} = (\log_a e)\frac{1}{u}\frac{du}{dx}$$

$$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d(a^u)}{dx} = (\ln a)\, a^u \frac{du}{dx}$$

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

$$\frac{d(u^v)}{dx} = vu^{v-1}\frac{du}{dx} + (\ln u)\, u^v \frac{dv}{dx}$$

$$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$$

$$\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$$

$$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$$

$$\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$$

$$\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$$

$$\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$$

Taylor's Series

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

INTEGRALS

The fundamental theorem of integral calculus is

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$\Delta x_i \to 0$ for all i.

$$\int df(x) = f(x)$$

$$\int dx = x$$

$$\int a\, f(x)dx = a \int f(x)dx$$

$$\int [u(x) \pm v(x)]dx = \int u(x)dx \pm \int v(x)dx$$

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$$\int x^m dx = \frac{x^{m+1}}{m+1} \ (m \neq -1)$$

$$\int u(x)\, dv(x) = u(x)v(x) - \int v(x)du(x)$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$$

$$\int a^x dx = \frac{a^x}{\ln a}$$

$$\int \sin x\, dx = \ -\cos x$$

$$\int \cos x\, dx = \sin x$$

$$\int \tan x\, dx = \ -\ln|\cos x| = \ln|\sec x|$$

$$\int \cot x\, dx = \ -\ln|\csc x| = \ln|\sin x|$$

$$\int e^{ax} dx = (1/a)e^{ax}$$

$$\int x\, e^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int \ln x\, dx = x[\ln(x) - 1] \ for \ (x > 0)$$

Numerical Integration

$$\Delta x = \frac{(b-a)}{n}$$

where n is the number of intervals.

Euler's Rule

$$\int_a^b f(x)dx \approx \Delta x \sum_{k=0}^{n-1} f(a + k\Delta x)$$

Trapezoidal Rule

For $n = 1$

$$\int_a^b f(x)dx \approx \Delta x\left[\frac{f(a)+f(b)}{2}\right]$$

For $n > 1$

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\left[f(a) + 2\sum_{k=1}^{n-1} f(a+k\Delta x) + f(b)\right]$$

Simpson's Rule

n must equal an even integer.

For $n = 2$,

$$\int_a^b f(x)dx \approx \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

For $n \geq 4$,

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(a) + 2\sum_{k=2,\,4,\,6\ldots}^{n-2} f(a+k\Delta x) + 4\sum_{k=1,\,3,\,5\ldots}^{n-1} f(a+k\Delta x) + f(b)\right]$$

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DIFFERENTIAL EQUATIONS

Following are first- and second-order differential equations.

First-Order Linear Homogeneous Differential Equation with Constant Coefficients

The form of this differential equation is

$$y' + ay = 0$$

where a is a real constant. The solution is

$$y = Ce^{-at}$$

where C is a constant that satisfies the initial conditions.

First-Order Linear Nonhomogeneous Differential Equation with Constant Coefficients

The form of this differential equation is

$$\tau\frac{dy}{dt} + y = Kx(t)$$

where $x(t) = At$ for $t < 0$ and $x(t) = Bt$ for $t > 0$

The initial condition at $t = 0$ is

$$y(0) = KA$$

τ is the time constant and K is the gain.

$$y(t) = KA + (KB - KA)\left(1 - \exp\left(\frac{-t}{\tau}\right)\right) \text{ or } \frac{t}{\tau} = \ln\left[\frac{KB - KA}{KB - y}\right]$$

Second-Order Linear Homogeneous Differential Equation with Constant Coefficients

The form of this differential equation is

$$y'' + ay' + by = 0$$

The solution to this differential equation has the form

$$y = Ce^{rx}$$

Substituting this solution into the differential equation gives

$$(r^2 + ar + b)Ce^{rx} = 0$$

The roots of the characteristic equation are

$$r_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

If $a^2 > 4b$, the solution is overdamped and of the form

$$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

where C_1 and C_2 are found from the boundary or initial conditions.

If $a^2 = 4b$, the solution is critically damped and of the form

$$y = (C_1 + C_2 x)\, e^{r_1 x}$$

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If $a^2 < 4b$, the solution is underdamped and of the form

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

where

$$\alpha = -\frac{a}{2} \text{ and } \beta = \frac{\sqrt{4b - a^2}}{2}$$

Numerical Solution of Ordinary Differential Equations

For the numerical solution of ordinary differential equations, use Euler's approximation.

Euler's Approximation

Given

$$\frac{dx}{dt} = f(x,t) \text{ and } x(0) = x_0$$

$$x[(k+1)\Delta t] \cong x(k\Delta t) + \Delta t f[x(k\Delta t), k\Delta t]$$

PROBABILITY AND STATISTICS

Following are probability and statistics laws and equations.

Permutations and Combinations

A *permutation* is a particular ordered sequence from a given set of objects. A *combination* is the set itself without reference to order. The number of different permutations of *n* objects taken *r* at a time is

$$P(n,r) = \frac{n!}{(n-r)!}$$

The number of different combinations of *n* objects taken *r* at a time is

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{[r!(n-r)!]}$$

Laws of Probability

Following are laws of probability for use in statistics.

Probability of an Event

$P(E)$ is a real number in the range 0 to 1. The probability of an impossible event is 0 and that of a certain event is 1.

Law of Total Probability

$$P(A+B) = P(A) + P(B) - P(A,B)$$

where

$P(A+B)$ = probability that either A or B occur alone or that both occur together
$P(A)$ = probability that A occurs
$P(B)$ = probability that B occurs
$P(A,B)$ = probability that both A and B occur simultaneously

Law of Compound or Joint Probability

If neither $P(A)$ nor $P(B)$ is zero, then

$$P(A,B) = P(A)P(B \mid A) = P(B)P(A \mid B)$$

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where

$P(B \mid A)$ = probability that B occurs given that A has already occurred

$P(A \mid B)$ = probability that A occurs given that B has already occurred

If either $P(A)$ or $P(B)$ is zero, then

$$P(A,B) = 0$$

Mean, Median, Mode, Variance, and Standard Deviation

If $X_1, X_2, \ldots, X_n$ represent the values of n items or observations from a population, the means of these items or observations are as follows.

Arithmetic Mean

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$\overline{X} \rightarrow \mu$, where μ is the population mean for sufficiently large n.

Weighted Arithmetic Mean

$$\overline{X}_w = \frac{\sum w_i X_i}{\sum w_i}$$

where X_i is the observed value and w_i is the weight applied to the X_i value.

Geometric Mean

$$\overline{G} = (X_1 X_2 \cdots X_n)^{\frac{1}{n}}$$

where ($X_k > 0$, $k = 1, 2, \ldots, n$)

Harmonic Mean

$$\frac{1}{\overline{H}} = \frac{1}{n}\left(\frac{1}{X_1} + \frac{1}{X_2} + \cdots + \frac{1}{X_n}\right)$$

where ($X_k > 0$, $k = 1, 2, \ldots, n$). The variance of the observations is the arithmetic mean of the squared deviations from the population mean μ.

Population Variance

The variance of the population is the arithmetic mean of the squared deviations from the population mean. If μ is the arithmetic mean of a discrete population of size N, the population variance is defined by

$$\sigma^2 = \frac{1}{N}\left[(X_1 - \mu)^2 + (X_2 - \mu)^2 + \cdots + (X_N - \mu)^2\right] = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2$$

Standard Deviation of Population

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2}$$

Sample Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

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Sample Standard Deviation

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2}$$

Sample Coefficient of Variation

$$CV = s/\overline{X}$$

Median

The discrete data a_i are arranged in increasing order. If n is odd, the median is the value of the $\left(\frac{n+1}{2}\right)^{\text{th}}$ item. If n is even, the median is the average of the $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2}+1\right)^{\text{th}}$ items.

Mode

The mode of the discrete data set a_i is the value that occurs with the greatest frequency.

Sample Range

Sample range R is the largest sample value (maximum) minus the smallest sample value (minimum).

PROBABILITY DISTRIBUTION FUNCTIONS

Table 3.1 shows the major probability density functions and their means and variances.

TABLE 3.1 Major probability density functions and their means and variances

Name	Density Function	Mean	Variance
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Poisson	$f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ
Exponential	$f(x) = \frac{1}{\beta} e^{-x/\beta}$	β	β^2
Weibull	$f(x) = \frac{\alpha}{\beta} x^{\alpha-1} e^{-x^{\alpha}/\beta}$	$\beta^{1/\alpha}\Gamma[(\alpha+1)/\alpha]$	$\beta^{2/\alpha}\left[\Gamma\left(\frac{\alpha+1}{\alpha}\right) - \Gamma^2\left(\frac{\alpha+1}{\alpha}\right)\right]$
Gamma	$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$ $\alpha > 0, \beta > 0$	$\alpha\beta$	$\alpha\beta^2$
Uniform	$f(x) = \frac{1}{(b-a)}$	$\frac{(a+b)}{2}$	$\frac{(b-a)^2}{12}$

Adapted from NCEES 2013

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Binomial Distribution

$P(x)$ is the probability that x successes will occur in n trials.

$$P_n(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

where

$x = 0, 1, 2, \ldots, n$
p = probability of success
q = probability of failure = $1 - p$

The variance for this distribution is $\sigma^2 = npq$

Normal Distribution

The probability density function for the normal (or Gaussian) distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

μ = population mean
σ = standard deviation of the population

$$-\infty \leq x \leq \infty$$

The standardized or unit normal distribution when $\mu = 0$ and $\sigma = 1$ is

$$Z(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The area under the unit normal distribution from $-\infty$ to x is

$$P(x) = \int_{-\infty}^{x} Z(t)dt$$

The area under the unit normal distribution from x to ∞ is

$$Q(x) = \int_{x}^{\infty} Z(t)dt$$

The area under the unit normal distribution from $-x$ to x is

$$A(x) = \int_{-x}^{x} Z(t)dt$$

$$P(x) + Q(x) = 1$$

$$P(-x) = Q(x)$$

$$A(x) = 2P(x) - 1$$

Table 3.2 is a unit normal distribution table giving values for P from which values of Q and A can be determined.

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TABLE 3.2 Normal probability distribution*

x	*P(x)*	*x*	*P(x)*	*x*	*P(x)*	*x*	*P(x)*
0.00	0.50000	0.90	0.81594	1.80	0.96407	2.70	0.99653
0.02	0.50798	0.92	0.82121	1.82	0.96562	2.72	0.99674
0.04	0.51595	0.94	0.82639	1.84	0.96712	2.74	0.99693
0.06	0.52392	0.96	0.83147	1.86	0.96856	2.76	0.99711
0.08	0.53188	0.98	0.83646	1.88	0.96995	2.78	0.99728
0.10	0.53983	1.00	0.84134	1.90	0.97128	2.80	0.99744
0.12	0.54776	1.02	0.84614	1.92	0.97257	2.82	0.99760
0.14	0.55567	1.04	0.85083	1.94	0.97381	2.84	0.99774
0.16	0.56356	1.06	0.85543	1.96	0.97500	2.86	0.99788
0.18	0.57142	1.08	0.85993	1.98	0.97615	2.88	0.99801
0.20	0.57926	1.10	0.86433	2.00	0.97725	2.90	0.99813
0.22	0.58706	1.12	0.86864	2.02	0.97831	2.92	0.99825
0.24	0.59483	1.14	0.87286	2.04	0.97932	2.94	0.99836
0.26	0.60257	1.16	0.87698	2.06	0.98030	2.96	0.99846
0.28	0.61026	1.18	0.88100	2.08	0.98124	2.98	0.99856
0.30	0.61791	1.20	0.88493	2.10	0.98214	3.00	0.99865
0.32	0.62552	1.22	0.88877	2.12	0.98300	3.05	0.99886
0.34	0.63307	1.24	0.89251	2.14	0.98382	3.10	0.99903
0.36	0.64058	1.26	0.89617	2.16	0.98461	3.15	0.99918
0.38	0.64803	1.28	0.89973	2.18	0.98537	3.20	0.99931
0.40	0.65542	1.30	0.90320	2.20	0.98610	3.25	0.99942
0.42	0.66276	1.32	0.90658	2.22	0.98679	3.30	0.99952
0.44	0.67003	1.34	0.90988	2.24	0.98745	3.35	0.99960
0.46	0.67724	1.36	0.91309	2.26	0.98809	3.40	0.99966
0.48	0.68439	1.38	0.91621	2.28	0.98870	3.45	0.99972
0.50	0.69146	1.40	0.91924	2.30	0.98928	3.50	0.99977
0.52	0.69847	1.42	0.92220	2.32	0.98983	3.55	0.99981
0.54	0.70540	1.44	0.92507	2.34	0.99036	3.60	0.99984
0.56	0.71226	1.46	0.92785	2.36	0.99086	3.65	0.99987
0.58	0.71904	1.48	0.93056	2.38	0.99134	3.70	0.99989
0.60	0.72575	1.50	0.93319	2.40	0.99180	3.75	0.99991
0.62	0.73237	1.52	0.93574	2.42	0.99224	3.80	0.99993
0.64	0.73891	1.54	0.93822	2.44	0.99266	3.85	0.99994
0.66	0.74537	1.56	0.94062	2.46	0.99305	3.90	0.99995
0.68	0.75175	1.58	0.94295	2.48	0.99343	3.95	0.99996
0.70	0.75804	1.60	0.94520	2.50	0.99379	4.00	0.99997
0.72	0.76424	1.62	0.94738	2.52	0.99413		
0.74	0.77035	1.64	0.94950	2.54	0.99446		
0.76	0.77637	1.66	0.95154	2.56	0.99477		
0.78	0.78230	1.68	0.95352	2.58	0.99506		
0.80	0.78814	1.70	0.95543	2.60	0.99534		
0.82	0.79389	1.72	0.95728	2.62	0.99560		
0.84	0.79955	1.74	0.95907	2.64	0.99585		
0.86	0.80511	1.76	0.96080	2.66	0.99609		
0.88	0.81057	1.78	0.96246	2.68	0.99632		

* $P(x) = \int_{-\infty}^{x} Z(t)dt$

Source: Sheppard 1903

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Student's *t* Distribution

The student's *t* distribution is the sampling distribution of the mean. $\overline{X}$ is the mean and *s* is the sample standard deviation of a random sample of size *n* from a normal distribution with population mean μ and variance σ^2.

$t = \overline{X} = \mu / s / \sqrt{\nu}$ is the value of a random variable having the student's *t* distribution where the number of degrees of freedom is $\nu = n - 1$. The student's *t* distribution is similar in form to the normal distribution and approaches the normal distribution as $\nu \rightarrow \infty$.

α is the area under the curve of the student's *t* distribution to the right of t_α and is the probability that *t* will exceed t_α. Table 3.3 gives values for t_α and various values of ν and α.

TABLE 3.3 Percentage points of student's *t* distribution*

CI†	20%	50%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
$\nu\backslash\alpha$	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.10	22.33	31.60
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.223	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.090	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

* In terms of α and ν.

Adapted from Pearson and Hartley 1954

† CI = confidence interval.

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Chi-Square Distribution

Chi-square distribution is the sampling distribution of the variance. s^2 is the sample variance of a random sample of size n from a normal population with variance σ^2. $\chi^2 = (n-1)s^2/\sigma^2$ is the value of a random variable having the chi-square distribution where the number of degrees of freedom is $\nu = n - 1$. The chi-square distribution is related to the gamma distribution (Γ) with the parameters $\alpha = \nu/2$ and $\beta = 2$. α is the area under the curve of the chi-square distribution to the right of χ^2_α and is the probability that χ^2 will exceed χ^2_α. Table 3.4 gives values χ^2_α for various values of ν and α.

TABLE 3.4 Percentage points of χ^2 distribution*

$\nu\backslash\alpha$	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	4.60517	5.99147	7.37778	9.21034	10.5966
3	0.07172	0.11483	0.21580	0.35185	0.58438	6.25139	7.81473	9.34840	11.3449	12.8381
4	0.20699	0.29711	0.48442	0.71072	1.06362	7.77944	9.48773	11.1433	13.2767	14.8602
5	0.41174	0.55430	0.83121	1.14548	1.61031	9.23635	11.0705	12.8325	15.0863	16.7496
6	0.67573	0.87209	1.23735	1.63549	2.20413	10.6446	12.5916	14.4494	16.8119	18.5476
7	0.98927	1.23904	1.68987	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.64648	2.17973	2.73264	3.48954	13.3616	15.5073	17.5346	20.0902	21.9550
9	1.7349	2.08791	2.70039	3.32511	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.1559	2.55821	3.24697	3.94030	4.86518	15.9871	18.3070	20.4831	23.2093	25.1882
11	2.6032	3.05347	3.81575	4.57481	5.57779	17.2750	19.6751	21.9200	24.7250	26.7569
12	3.0738	3.57056	4.40379	5.22603	6.30380	18.5494	21.0261	23.3367	26.2170	28.2995
13	3.5650	4.10691	5.00874	5.89186	7.04150	19.8119	22.3621	24.7356	27.6883	29.8194
14	4.0747	4.66043	5.62872	6.57063	7.78953	21.0642	23.6848	26.1190	29.1413	31.3193
15	4.6009	5.22935	6.26214	7.26094	8.54675	22.3072	24.9958	27.4884	30.5779	32.8013
16	5.1422	5.81221	6.90766	7.96164	9.31223	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.6972	6.40776	7.56418	8.67176	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.2648	7.01491	8.23075	9.39046	10.8649	25.9894	28.8693	31.5264	34.8053	37.1564
19	6.8440	7.63273	8.90655	10.1170	11.6509	27.2036	30.1435	32.8523	36.1908	38.5822
20	7.4339	8.26040	9.59083	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.0337	8.89720	10.2829	11.5913	13.2396	29.6151	32.6705	35.4789	38.9321	41.4010
22	8.6427	9.54249	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7956
23	9.2604	10.1957	11.6885	13.0905	14.8479	32.0069	35.1725	38.0757	41.6384	44.1813
24	9.8862	10.8564	12.4011	13.8484	15.6587	33.1963	36.4151	39.3641	42.9798	45.5585
25	10.520	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9278
26	11.160	12.1981	13.8439	15.3791	17.2919	35.5631	38.8852	41.9232	45.6417	48.2899
27	11.808	12.8786	14.5733	16.1513	18.1138	36.7412	40.1133	43.1944	46.9630	49.6449
28	12.461	13.5648	15.3079	16.9279	18.9392	37.9159	41.3372	44.4607	48.2782	50.9933
29	13.121	14.2565	16.0471	17.7083	19.7677	39.0875	42.5569	45.7222	49.5879	52.3356
30	13.787	14.9535	16.7908	18.4926	20.5992	40.2560	43.7729	46.9792	50.8922	53.6720
40	20.707	22.1643	24.4331	26.5093	29.0505	51.8050	55.7585	59.3417	63.6907	66.7659
50	27.991	29.7067	32.3574	34.7642	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900
60	35.535	37.4848	40.4817	43.1879	46.4589	74.3970	79.0819	83.2976	88.3794	91.9517
70	43.275	45.4418	48.7576	51.7393	55.3290	85.5271	90.5312	95.0231	100.425	104.215
80	51.172	53.5400	57.1532	60.3915	64.2778	96.5782	101.879	106.629	112.329	116.321
90	59.196	61.7541	65.6466	69.1260	73.2912	107.565	113.145	118.136	124.116	128.299
100	67.328	70.0648	74.2219	77.9295	82.3581	118.498	124.342	129.561	135.807	140.169

* In terms of α and ν.

Source: Thompson et al. 1941

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F (Variance Ratio) Distribution

F distribution is the sampling distribution for the ratio of variances. s_1^2 *and* s_2^2 are the sample variances of independent random samples of size n_1 and n_2, respectively, from two normal populations with the same variance σ^2.

$F = s_1^2 / s_2^2$ is the value of a random variable having the *F* distribution where the number of degrees of freedom are $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$, respectively.

α is the area under the curve of the *F* (variance ratio) distribution to the right of F_α and is the probability that *F* will exceed F_α. Table 3.5 gives values for F_α and various values of ν and $\alpha = 0.05$ (95% confidence level).

TABLE 3.5 Percentage points of *F* distribution*

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	243.9	245.9	248.0	250.1	252.2	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.43	19.45	19.46	19.48	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.85	8.74	8.70	8.66	8.62	8.57	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.86	5.80	5.75	5.69	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.62	4.56	4.50	4.43	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.94	3.87	3.81	3.74	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.51	3.44	3.38	3.30	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.22	3.15	3.08	3.01	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	3.01	2.94	2.86	2.79	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.85	2.77	2.70	2.62	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.72	2.65	2.57	2.49	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.85	2.69	2.62	2.54	2.47	2.38	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.77	2.60	2.53	2.46	2.38	2.30	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.46	2.39	2.31	2.22	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.40	2.33	2.25	2.16	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.35	2.28	2.19	2.11	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.31	2.23	2.15	2.06	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.27	2.19	2.11	2.02	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.23	2.16	2.07	1.98	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.20	2.12	2.04	1.95	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.42	2.25	2.18	2.10	2.01	1.92	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.15	2.07	1.98	1.89	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.37	2.20	2.13	2.05	1.96	1.86	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	2.11	2.03	1.94	1.84	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.34	2.16	2.09	2.01	1.92	1.82	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.32	2.15	2.07	1.99	1.90	1.80	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.31	2.13	2.06	1.97	1.88	1.79	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.29	2.12	2.04	1.96	1.87	1.77	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.28	2.10	2.03	1.94	1.85	1.75	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	2.01	1.93	1.84	1.74	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.92	1.84	1.74	1.64	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.10	1.92	1.84	1.75	1.65	1.53	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.83	1.75	1.66	1.55	1.43	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	1.94	1.75	1.67	1.57	1.46	1.32	1.00

* In terms of ν_1 and ν_2 – $\alpha = 0.05$.

Adapted from Pearson and Hartley 1954, as appears in Merrington and Thompson 1943

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HYPOTHESIS TESTING

The null hypothesis is $H_0 : \mu = \mu_0$. An alternative hypothesis is $H_1 : \mu = \mu_1$. Rejecting H_0 when it is true is called a *type I error.* α is the probability of a type I error. Accepting H_0 when it is wrong is called a *type II error.* β is the probability of a type II error. α, probability of a type I error, is known as the *level of significance* of the test.

Tables 3.6, 3.7, and 3.8 show tests on means of normal distributions with variance known and unknown and mean unknown, respectively.

TABLE 3.6 Tests on means of normal distribution—variance known

Hypothesis	Test Statistic	Criteria for Rejection
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$Z_0 \equiv \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$	$\lvert Z_0 \rvert > Z_{\alpha/2}$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$		$Z_0 < -Z_\alpha$
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$Z_0 > Z_\alpha$
$H_0: \mu_1 - \mu_2 = \gamma$ $H_1: \mu_1 - \mu_2 \neq \gamma$	$Z_0 \equiv \dfrac{\overline{X}_1 - \overline{X}_2 - \gamma}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$\lvert Z_0 \rvert > Z_{\alpha/2}$
$H_0: \mu_1 - \mu_2 = \gamma$ $H_1: \mu_1 - \mu_2 < \gamma$		$Z_0 < -Z_\alpha$
$H_0: \mu_1 - \mu_2 = \gamma$ $H_1: \mu_1 - \mu_2 > \gamma$		$Z_0 > Z_\alpha$

Adapted from NCEES 2013

TABLE 3.7 Tests on means of normal distribution—variance unknown

Hypothesis	Test Statistic	Criteria for Rejection
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$t_0 \equiv \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$	$\lvert t_0 \rvert > t_{\alpha/2,\, n-1}$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$		$t_0 < -t_{\alpha,\, n-1}$
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$t_0 > t_{\alpha,\, n-1}$
$H_0: \mu_1 - \mu_2 = \gamma$ $H_1: \mu_1 - \mu_2 \neq \gamma$	Variances equal $t_0 \equiv \dfrac{\overline{X}_1 - \overline{X}_2 - \gamma}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$	$\lvert t_0 \rvert > t_{\alpha/2,\, \nu}$
$H_0: \mu_1 - \mu_2 = \gamma$ $H_1: \mu_1 - \mu_2 < \gamma$	$S_p^2 = \left[(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2\right]/\nu$ $\nu = n_1 + n_2 - 2$	$t_0 < -t_{\alpha,\, \nu}$
$H_0: \mu_1 - \mu_2 = \gamma$ $H_1: \mu_1 - \mu_2 > \gamma$	Variances unequal $t_0 \equiv \dfrac{\overline{X}_1 - \overline{X}_2 - \gamma}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$ $\nu = \dfrac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{\dfrac{(S_1^2/n_1)^2}{n_1 - 1} + \dfrac{(S_2^2/n_2)^2}{n_2 - 1}}$	$t_0 > t_{\alpha,\, \nu}$

Adapted from NCEES 2013

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TABLE 3.8 Tests on variances of normal distribution—mean unknown

Hypothesis	Test Statistic	Criteria for Rejection
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$	$\chi_0^2 > \chi^2_{\alpha/2,\,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,\,n-1}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 < \sigma_0^2$		$\chi_0^2 < \chi^2_{1-\alpha/2,\,n-1}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 > \sigma_0^2$		$\chi_0^2 > \chi^2_{\alpha,\,n-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 \neq \sigma_2^2$	$F_0 = \frac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha/2,n_1-1,n_2-1}$ $F_0 < F_{1-\alpha/2,n_1-1,n_2-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 < \sigma_2^2$	$F_0 = \frac{S_2^2}{S_1^2}$	$F_0 > F_{\alpha,n_2-1,n_1-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 > \sigma_2^2$	$F_0 = \frac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha,n_1-1,n_2-1}$

Adapted from NCEES 2013

CONFIDENCE INTERVAL AND SAMPLE SIZE

See Table 3.9 for confidence interval percentages and the values of $Z_{\alpha/2}$.

TABLE 3.9 Values of $Z_{\alpha/2}$

Confidence Interval, %	$Z_{\alpha/2}$
80	1.2816
90	1.6449
95	1.9600
98	2.3263
99	2.5758

Adapted from NCEES 2013

Confidence Interval for the Mean μ of a Normal Distribution

Standard deviation of the population σ known

$$\overline{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Standard deviation of the population σ not known

$$\overline{X} - t_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ is for n −1 degrees of freedom.

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Confidence Interval for the Difference Between Two Means μ_1 and μ_2 of Normal Distributions

Standard deviations of the populations σ_1 and σ_2 known

$$\overline{X}_1 - \overline{X}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \overline{X}_1 - \overline{X}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

Standard deviations of the populations σ_1 and σ_2 not known

$$\overline{X}_1 - \overline{X}_2 - t_{\alpha/2}\sqrt{\frac{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\left[(n_1-1)S_1^2+(n_2-1)S_2^2\right]}{n_1+n_2-2}} \le \mu_1 - \mu_2$$
$$\le \overline{X}_1 - \overline{X}_2 + t_{\alpha/2}\sqrt{\frac{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\left[(n_1-1)S_1^2+(n_2-1)S_2^2\right]}{n_1+n_2-2}}$$

where $t_{\alpha/2}$ is for $n_1 + n_2 - 2$ degrees of freedom.

Confidence Interval for the Population Variance σ^2 of a Normal Distribution

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

Sample Size

$$n = \left[\frac{Z_{\alpha/2}\sigma}{\overline{x}-\mu}\right]^2$$

$$z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$$

LINEAR REGRESSION AND GOODNESS OF FIT

Given n pairs of points (x_i, y_i). Find $y = \hat{a} + \hat{b}x$

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$$

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2$$

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

where

n = sample size
S_{xx} = sum of squares of x
S_{yy} = sum of squares of y
S_{xy} = sum of x-y products

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Slope

$$\hat{b} = \frac{S_{xy}}{S_{xx}}$$

y Intercept

$$\hat{a} = \overline{y} - \hat{b}\overline{x}$$

Standard Error of Estimate

$$S_e^2 = \frac{S_{xx}S_{yy} - S_{xy}^2}{S_{xx}(n-2)} = \text{mean squared error}$$

Confidence Interval for *y* Intercept

$$\hat{a} \pm t_{\alpha/2,n-2}\sqrt{\left(\frac{1}{n} + \frac{\overline{x}^2}{S_{xx}}\right)S_e^2}$$

Confidence Interval for Slope

$$\hat{b} = t_{\alpha/2,n-2}\sqrt{\frac{S_e^2}{S_{xx}}}$$

Sample Correlation Coefficient

$$R = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

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CHAPTER

4

Physical Science and Engineering

R. Karl Zipf Jr., P.E.

STATICS

Force, moments, equilibrium, centroids, and inertia are discussed in the following subsections.

Force

A force F is a vector quantity defined by (1) a point of application, (2) a magnitude, and (3) a direction. The magnitude and direction of the force resulting from two or more forces can be determined graphically using the parallelogram law or trigonometrically using the law of cosines and the law of sines.

The vector form of a force in three dimensions is

$$F = F_x i + F_y j + F_z k$$

In two dimensions, the resultant F of n forces with components $F_{x,i}$ and $F_{y,i}$ has magnitude

$$F = \sqrt{\left(\sum_{i=1}^{n} F_{x,i}\right)^2 + \left(\sum_{i=1}^{n} F_{y,i}\right)^2}$$

The direction of the force with respect to the x-axis is

$$\theta = \arctan\left(\frac{\sum_{i=1}^{n} F_{y,i}}{\sum_{i=1}^{n} F_{x,i}}\right)$$

Components of a Force

Any force acting on a particle or rigid body can be resolved into two or more components that have the same effect on the body. If θ_x, θ_y, and θ_z are the angles that F forms with the x, y, and z coordinate axes, then the rectangular components of F are

$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z$$

The direction cosines are

$$\cos\theta_x = \frac{F_x}{F} \qquad \cos\theta_y = \frac{F_y}{F} \qquad \cos\theta_z = \frac{F_z}{F}$$

The resultant force is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Moments (Couples)

A *couple* is a system of two forces that are equal in magnitude, opposite in direction, parallel to each other, and separated by a perpendicular distance. *Moment* is a vector quantity defined as the cross product of the radius vector r with components r_x, r_y, and r_z and the force F with components F_x, F_y, and F_z.

$$M = rF$$

$$M_x = r_y F_z - r_z F_y$$

$$M_y = r_z F_x - r_x F_z$$

$$M_z = r_x F_y - r_y F_x$$

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Equilibrium

By resolving each force and each moment into its rectangular components, the necessary and sufficient conditions for equilibrium of a particle or rigid body are the following six scalar equations.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

In two dimensions, the preceding six equations reduce to three as follows:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

where the moment is taken about an arbitrary point A in the plane of the structure.

Centroids of Lines, Areas, Volumes, and Masses

The centroid of a line is

$$x_{lc} = \frac{\sum x_n l_n}{L} \qquad y_{lc} = \frac{\sum y_n l_n}{L} \qquad z_{lc} = \frac{\sum z_n l_n}{L}$$

where

$L = \sum l_n$
l_n = length of a line segment
x_n, y_n, and z_n = distance from x, y, and z axes, respectively

The centroid of an area is

$$x_{ac} = \frac{\sum x_n a_n}{A} \qquad y_{ac} = \frac{\sum y_n a_n}{A} \qquad z_{ac} = \frac{\sum z_n a_n}{A}$$

where

$A = \sum a_n$
a_n = elemental area
x_n, y_n, and z_n = distance from x, y, and z axes, respectively

The centroid of a volume is

$$x_{vc} = \frac{\sum x_n v_n}{V} \qquad y_{vc} = \frac{\sum y_n v_n}{V} \qquad z_{vc} = \frac{\sum z_n v_n}{V}$$

where

$V = \sum v_n$
v_n = elemental volume
x_n, y_n, and z_n = distance from x, y, and z axes, respectively

The centroid of a mass is

$$r_c = \frac{\sum m_n r_n}{\sum m_n}$$

where

r_c = radius vector from reference point to center of mass
m_n = mass of each particle in the system
r_n = radius vector from reference point to each particle

The moment of area M_a is defined as

$M_{ay} = \sum x_n a_n$ with respect to the y-axis

$M_{ax} = \sum y_n a_n$ with respect to the x-axis

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Moment of Inertia

The moment of inertia or second moment of area is

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

The polar moment of inertia is

$$J_O = \int r^2 dA$$

where r is the distance from O to the element of area dA.

Because $r^2 = x^2 + y^2$, then $J_O = I_x + I_y$, where I_x, I_y, and J_O are defined by the preceding equations.

The radius of gyration, r_O, r_x, and r_y, is the distance from a reference axis to a point where all of the area is imagined to be concentrated to produce the moment of inertia.

$$r_O = \sqrt{\frac{J_O}{A}} \qquad r_x = \sqrt{\frac{I_x}{A}} \qquad r_y = \sqrt{\frac{I_y}{A}}$$

Moment of Inertia Parallel-Axis Theorem

The moment of inertia of an area about any axis parallel to a centroidal axis is

$$I'_x = I_{xc} + d^2 A \qquad I'_y = I_{yc} + d^2 A$$

where d is the distance from the centroidal axis to the other axis, and I_{xc} and I_{yc} are the moments of inertia about the centroidal axes.

The product of inertia with respect to a particular coordinate system is defined as

$$I_{xy} = \int xydA \qquad I_{xz} = \int xzdA \qquad I_{yz} = \int yzdA$$

Area, Centroid, Moment of Inertia, Section Modulus, and Radius of Gyration for Selected Shapes

Following are the equations and figures for rectangular, circular, pipe, H, and channel sections with specifics for area, centroid, moment of inertia, section modulus, and radius of gyration for each.

Rectangular Section

For the following equations, see Figure 4.1.

Area

$$A = bh$$

Centroid

$$x_c = \frac{b}{2} \qquad y_c = \frac{h}{2}$$

Moment of Inertia

$$I = \frac{bh^3}{12}$$

Section Modulus

$$\frac{I}{c} = \frac{bh^2}{6}$$

Radius of Gyration

$$r_O = \frac{h}{\sqrt{12}}$$

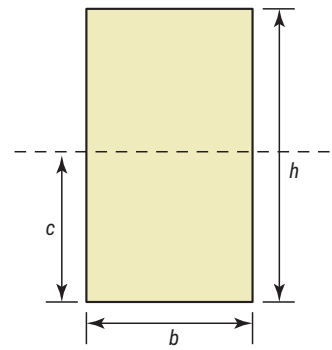

FIGURE 4.1 Rectangular section

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Circular Section

For the following equations, see Figure 4.2.

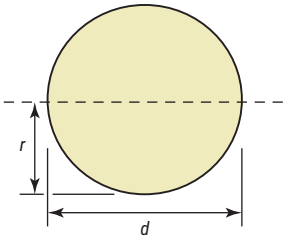

FIGURE 4.2 Circular section

Area

$$A = \frac{\pi}{4}d^2 = \pi r^2$$

Centroid

$$x_c = r$$

$$y_c = r$$

Moment of Inertia

$$I = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$$

Section Modulus

$$\frac{I}{c} = \frac{\pi d^3}{32} = \frac{\pi r^3}{4}$$

Radius of Gyration

$$r_0 = \frac{r}{2}$$

Pipe Section

For the following equations, see Figure 4.3.

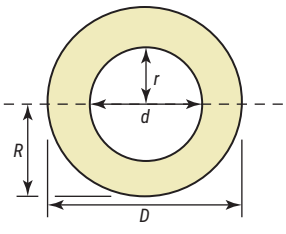

FIGURE 4.3 Pipe section

Area

$$A = \frac{\pi}{4}(D^2 - d^2) = \pi(R^2 - r^2)$$

Centroid

$$x_c = r$$

$$y_c = r$$

Moment of Inertia

$$I = \frac{(D^4 - d^4)}{64} = \frac{\pi(R^4 - r^4)}{4}$$

Section Modulus

$$\frac{I}{c} = \frac{(D^4 - d^4)}{32D} = \frac{\pi(R^4 - r^4)}{4R}$$

Radius of Gyration

$$r_0 = \frac{\sqrt{D^2 + d^2}}{4} = \frac{\sqrt{R^2 + r^2}}{2}$$

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H Section 1

For the following equations, see Figure 4.4.

Area

$$A = HB + bh$$

Centroid

$$x_c = \frac{(B + b)}{2}$$

$$y_c = \frac{H}{2}$$

Moment of Inertia

$$I = \frac{BH^3 + bh^3}{12}$$

Section Modulus

$$\frac{I}{c} = \frac{BH^3 + bh^3}{6H}$$

Radius of Gyration

$$r_O = \sqrt{\frac{BH^3 + bh^3}{12(BH + bh)}}$$

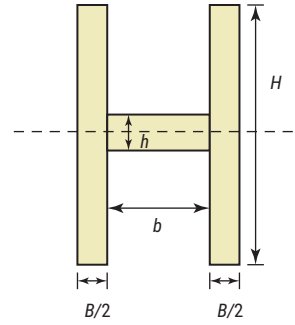

FIGURE 4.4 H section 1

H Section 2

For the following equations, see Figure 4.5.

Area

$$A = HB - bh$$

Centroid

$$x_c = \frac{B}{2}$$

$$y_c = \frac{H}{2}$$

Moment of Inertia

$$I = \frac{BH^3 - bh^3}{12}$$

Section Modulus

$$\frac{I}{c} = \frac{BH^3 - bh^3}{6H}$$

Radius of Gyration

$$r_O = \sqrt{\frac{BH^3 - bh^3}{12(BH - bh)}}$$

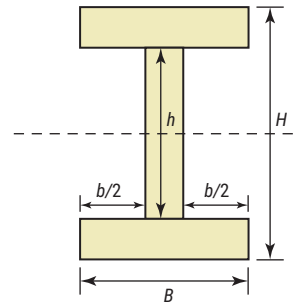

FIGURE 4.5 H section 2

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Channel Section 1

For the following equations, see Figure 4.6.

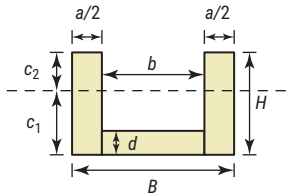

FIGURE 4.6 Channel section 1

Area

$$A = HB - b(H - d)$$

Centroid

$$x_c = B/2$$

$$y_c = \frac{aH^2 + (B - a)d^2}{HB - b(H - d)}$$

Moment of Inertia

$$I = \frac{(Bc_1^3 - bh^3 + ac_2^3)}{3}$$

Section Modulus

$$\frac{I}{c} = \frac{(Bc_1^3 - bh^3 + ac_2^3)}{3c}$$

$$c = c_1 \text{ or } c_2$$

$$c_1 = \frac{aH^2 + bd^2}{2(aH + bd)}$$

$$c_2 = H - c_1$$

Radius of Gyration

$$r_0 = \sqrt{\frac{I}{Bd + a(H - d)}}$$

Channel Section 2

For the following equations, see Figure 4.7.

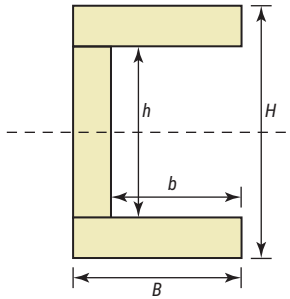

FIGURE 4.7 Channel section 2

Area

$$A = HB - bh$$

Centroid

$$x_c = \frac{B^2(H - h) + (B - b)^2 h}{2(HB - hb)}$$

$$y_c = H/2$$

Moment of Inertia

$$I = \frac{BH^3 - bh^3}{12}$$

Section Modulus

$$\frac{I}{c} = \frac{BH^3 - bh^3}{6H}$$

Radius of Gyration

$$r_0 = \sqrt{\frac{BH^3 - bh^3}{12(BH - bh)}}$$

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Friction

Plane and belt friction is discussed in the following sections.

Plane Friction

The largest friction force possible on a surface before an object starts to move is called the *limiting friction* given by

$$F_s = \mu_s N$$

where

F_s = static friction force
μ_s = coefficient of static friction
N = normal force between the surfaces in contact

Once motion begins, the magnitude of F may decrease to a lower value given by

$$F_k = \mu_k N$$

where

F_k = kinetic friction force
μ_k = coefficient of kinetic friction
N = normal force between the surfaces in contact

Belt Friction

$$F_1 = F_2 e^{\mu\theta}$$

where

F_1 = force applied in the direction of impending motion
F_2 = force applied to resist impending motion
μ = coefficient of static friction
θ = angle of contact between surfaces in radians

Analysis of Trusses

A truss is a rigid framework satisfying the following conditions:

- All members lie in a plane.
- All members are connected at the end with frictionless pins.
- All applied loads lie in the plane of the truss.

Statically Determinant and Indeterminant Trusses

- m is the number of members in a truss.
- r is the number of independent reaction components (a plane truss has three).
- j is the number of joints or pin connections.

$m + r < 2j$	The truss is unstable.
$m + r = 2j$	The truss is stable and statically determinant.
$m + r > 2j$	The truss is stable and statically indeterminant.

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Method of Joints

The method of joints begins by solving for all support reactions (Figure 4.8). Beginning at one of the support joints, apply the equilibrium equations at each joint in the truss and solve for the member forces. F_{xy} designates the force in segment *xy*, and R_{xd} designates the reaction force at joint *x* in direction *d*, either horizontal *H* or vertical *V*.

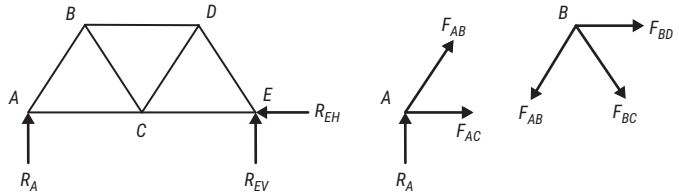

FIGURE 4.8 Method of joints

Method of Sections

The method of sections begins by solving for all support reactions (Figure 4.9). F_{xy} designates the force in segment *xy*, and R_{xd} designates the reaction force at joint *x* in direction *d*, either horizontal *H* or vertical *V*. Make an imaginary cut through any portion of the truss so that the unknown forces in particular truss members are exposed as external forces. Apply the equilibrium equations at each joint along the imaginary cut and solve for the member forces.

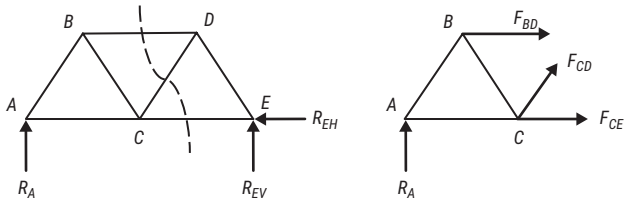

FIGURE 4.9 Method of sections

Analysis of Beams

First solve for the reaction forces on the beam (Figure 4.10). F_x designates the force at point *x*, and R_x designates the reaction force at support *x*. Make an imaginary cut at a particular cross section through the beam. Apply the equilibrium equations at this cut to calculate the shear force *V* and bending moment *M*.

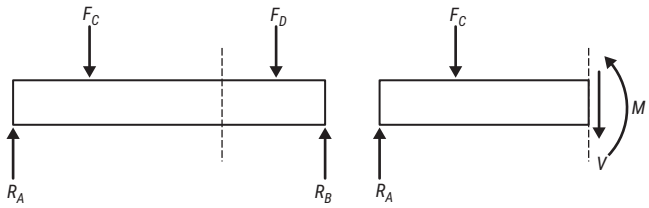

FIGURE 4.10 Method of beams

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DYNAMICS

Following are the designations used in dynamics.

Nomenclature

- t is time.
- s is position.
- v is velocity.
- a is acceleration.
- a_n is normal acceleration.
- a_t is tangential acceleration.
- θ is angle.
- ω is angular velocity.
- α is angular acceleration.
- Ω is the angular velocity of the x, y, z reference axes.
- Ω' is the angular acceleration of the reference axes.
- $r_{A/B}$ is the relative position of A with respect to B.
- $v_{A/B}$ is the relative velocity of A with respect to B.
- $\alpha_{A/B}$ is the relative acceleration of A with respect to B.

Kinematics

$r(t)$ is the position vector of a particle as a function of time. The instantaneous velocity is

$$v(t) = \frac{dr}{dt}$$

The instantaneous acceleration is

$$a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$a(t) = v\frac{dv}{dr}$$

Straight-Line Motion—Constant Acceleration

$$a(t) = a_0$$

$$v(t) = \int_{t_0}^{t} a_0 dt = v_0 + a_0(t - t_0)$$

$$s(t) = \int_{t_0}^{t} v(t)dt = \int_{t_0}^{t} (v_0 + a_0(t - t_0))dt = s_0 + v_0(t - t_0) + \frac{1}{2}a_0(t - t_0)^2$$

where s_0 is the initial position, and v_0 is the initial velocity at time $t = 0$.

By eliminating $(t - t_0)$ from the two preceding equations, another equation for velocity is obtained as

$$v^2 = v_0^2 + 2a_0(s - s_0)$$

For a free-falling body, $a_0 = -g$, which is the acceleration due to gravity (32.17 ft/s^2 or 9.8 m/s^2).

Projectile Motion

The initial velocity for a projectile is v_0 at an angle θ with respect to the x-axis. The components of acceleration in the x and y directions are constant. The equations for projectile motion are

$$a_x = 0$$

$$v_x = v_0 \cos\theta$$

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$$s_x = s_{x0} + v_0 \cos\theta t$$

$$a_y = -g$$

$$v_y = -gt + v_0 \sin\theta$$

$$s_y = s_{y0} + v_0 \sin\theta t - \frac{1}{2} g t^2$$

Circular Motion—Constant Acceleration

$$\alpha(t) = \alpha_0$$

$$\omega(t) = \int_{t_0}^{t} \alpha_0 dt = \omega_0 + \alpha_0 (t - t_0)$$

$$\theta(t) = \int_{t_0}^{t} \omega(t) dt = \int_{t_0}^{t} (\omega_0 + \alpha_0 (t - t_0)) dt = \theta_0 + \omega_0 (t - t_0) + \frac{1}{2} \alpha_0 (t - t_0)^2$$

where θ_0 is the initial angular displacement, and ω_0 is the initial angular velocity at time $t = 0$.

By eliminating $(t - t_0)$ from the two preceding equations, another equation for velocity is obtained as

$$\omega^2 = \omega_0^2 + 2\alpha_0 (\theta - \theta_0)$$

Nonconstant Acceleration

For nonconstant acceleration $a(t)$

$$v(t) = \int_{t_0}^{t} a(t) dt + v_{t_0}$$

$$s(t) = \int_{t_0}^{t} v(t) dt + s_{t_0}$$

For nonconstant angular acceleration $\alpha(t)$

$$\omega(t) = \int_{t_0}^{t} \alpha(t) dt + \omega_{t_0}$$

$$\theta(t) = \int_{t_0}^{t} \omega(t) dt + \theta_{t_0}$$

Weight

Weight, denoted by W, is a force caused by the acceleration due to gravity.

$$W = mg$$

where m is the mass, and g is the local acceleration due to gravity.

Newton's Second Law of Motion

The sum of the forces acting on a body is

$$\sum F = \frac{d(mv)}{dt}$$

where m is the mass, and v is the velocity vector.

For constant mass

$$\sum F = m \frac{dv}{dt} = ma$$

where a is the acceleration vector.

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Impulse and Momentum

Assuming mass is constant, the equation of motion can be rewritten as

$$mdv = Fdt$$

which may be expanded to

$$\sum m_i (v_i)_{t_2} = \sum m_i (v_i)_{t_1} + \sum \int_{t_1}^{t_2} F_i dt$$

The term on the left side of the equation is the linear momentum at time t_2. The first term on the right side of the equation is the linear momentum at time t_1. The second term on the right side is the impulse of the force F from time t_1 to t_2. F and v are vector quantities. When motion exists in only one dimension, say x, the preceding vector equation reduces to the following scalar equation:

$$m(v_x)_{t_2} - m(v_x)_{t_1} = \int_{t_1}^{t_2} F_x dt$$

Impact

When two bodies collide, momentum is conserved, but energy may or may not be conserved. An expression for conservation of momentum for two bodies is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

where

m_1 and m_2 = masses of the two bodies
v_1 and v_2 = velocity vectors before impact
v'_1 and v'_2 = velocity vectors after impact

When energy dissipation occurs, the relative velocities normal to the plane of impact before and after impact is given by

$$e = \frac{(v'_2)_n - (v'_1)_n}{(v_1)_n - (v_2)_n}$$

where

e = coefficient of restitution for the materials
superscript $'$ = velocity after impact
subscript n = component normal to the plane of impact

$$0 \leq e \leq 1$$

For $e = 0$, the collision is perfectly plastic and no rebound occurs. For $e = 1$, the collision is perfectly elastic and energy is conserved. Knowing e, the velocities after impact are

$$(v'_1)_n = \frac{m_2 (v_2)_n (1 + e) + (m_1 - em_2)(v_1)_n}{m_1 + m_2}$$

$$(v'_2)_n = \frac{m_1 (v_1)_n (1 + e) - (em_1 - m_2)(v_2)_n}{m_1 + m_2}$$

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Work and Energy

Work, denoted by W, is a scalar quantity defined as the integral of the product of the force vector and the force's displacement vector dr.

$$W = \int F \cdot dr$$

The kinetic energy, denoted by KE, of a particle is the work done accelerating the particle from rest to velocity v.

$$\text{KE} = \frac{1}{2}mv^2$$

The change in kinetic energy in going from v_1 to v_2 is

$$\text{KE}_2 - \text{KE}_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The potential energy, denoted by PE, is the work done by a force acting within a conservative field such as gravity. In a gravitational field, the potential energy is

$$\text{PE} = Wh = mgh$$

where h is the distance above an arbitrary datum.

Elastic potential energy is the recoverable strain energy stored in an elastic body. For a linear elastic spring with spring constant k, the force F_s in the spring as a function of deformation x is

$$F_s = kx$$

The elastic potential energy, denoted by U, stored in the spring as it is deformed from 0 to x is

$$U = \int_0^x F_s dx = \int_0^x kx\, dx = \frac{1}{2}kx^2$$

The change in elastic potential energy in deforming the spring from position x_1 to x_2 is

$$U_2 - U_1 = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

Conservation of Energy

If PE_i and KE_i are the potential and kinetic energy at state i, then the conservation of energy for a conservative system without energy dissipation is

$$\text{PE}_1 + \text{KE}_1 = \text{PE}_2 + \text{KE}_2$$

If friction is present, then the work done by friction W_f is included in the conservation of energy equation as follows

$$\text{PE}_1 + \text{KE}_1 + W_f = \text{PE}_2 + \text{KE}_2$$

Because frictional forces dissipate energy, W_f is negative.

In a nonconservative system, the work done by nonconservative forces is $W_{1\to 2}$. The conservation of energy equation is

$$\text{PE}_2 + \text{KE}_2 = \text{PE}_1 + \text{KE}_1 + W_{1\to 2}$$

Because nonconservative forces dissipate energy, $W_{1\to 2}$ is negative.

Friction

The friction force F is

$$F \le \mu N$$

where N is the normal force on the contact surface, and μ is the coefficient of friction.

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$F < \mu_s N$	No slip is occurring.
$F = \mu_s N$	Slip is impending.
$F = \mu_k N$	Slip is occurring.

where μ_s is the coefficient of static friction, and μ_k is the coefficient of dynamic friction.

Vibration—Single Degree-of-Freedom System

The simplest form equation of motion for a single degree-of-freedom system is

$m\ddot{x} + kx = 0$

where

m = lumped mass of the system
k = stiffness
$\ddot{x}$ = acceleration
x = displacement

The solution to this differential equation is

$$x(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t)$$

where $\omega_n = \sqrt{k/m}$ is the undamped natural frequency of the system, and C_1 and C_2 are integration constants to be determined from the boundary conditions.

If the initial conditions are $x(0) = x_0$ and $\dot{x}(0) = v_0$, then the solution becomes

$$x(t) = x_0 \cos(\omega_n t) + \left(\frac{v_0}{\omega_n}\right) \sin(\omega_n t)$$

The undamped natural period is

$$\tau_n = 2\pi/\omega_n$$

MECHANICS OF MATERIALS

Following are the basic mechanical properties of stress and strain.

Stress

If a force vector ΔP acts on an area ΔA, then stress σ is defined as

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A}$$

Strain

Strain is defined as change in length per unit length. Engineering strain is

$$\varepsilon = \frac{\Delta L}{L_0}$$

where ΔL is change in length, and L_0 is the initial length.

True strain is

$$\varepsilon_T = \frac{dL}{L_0}$$

where dL is the differential change in length.

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Stress–Strain Curves

A typical stress–strain curve for a ductile material in tension is shown Figure 4.11. The curve shown is for mild steel and is typical for many ductile materials such as A36 structural steel, aluminum, and some polymers. The slope of the initial portion of the stress–strain curve is called the *modulus of elasticity* or *Young's modulus*. Ductile materials then reach their yield strength. In the yielding range, deformation occurs with little or no increase in stress. For mild steel, yielding occurs up to a strain of about 0.02 (or 2% strain). Some ductile materials, such as mild steel, may then exhibit strain hardening. In this range, continued deformation results in a further increase in stress. Mild steel then reaches its ultimate strength at a strain of about 0.20 (or 20% strain). With continued deformation, rupture eventually occurs at a strain of about 0.25 (or 25% strain) for mild steel.

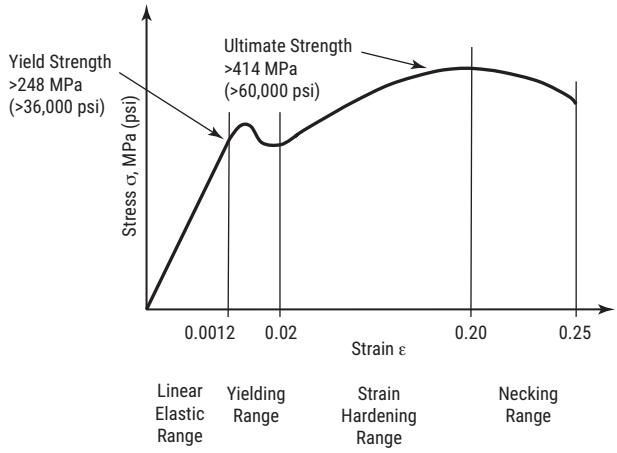

FIGURE 4.11 Stress–strain curve for mild steel in tension (ductile behavior)

Typical stress–strain curves for brittle materials in compression are shown in Figures 4.12 and 4.13. These curves are for concrete with compressive strengths ranging from 21 to 34 MPa (3,000 to 5,000 psi) and for common rocks with compressive strengths ranging from approximately 70 to 340 MPa (10,000 to 50,000 psi). With brittle materials, the initial modulus of elasticity is usually linear, but then it decreases as the material approaches its ultimate strength.

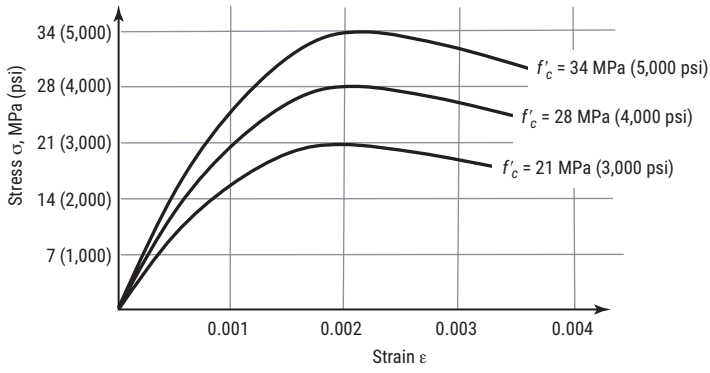

FIGURE 4.12 Stress–strain curves for concrete in compression (brittle behavior)

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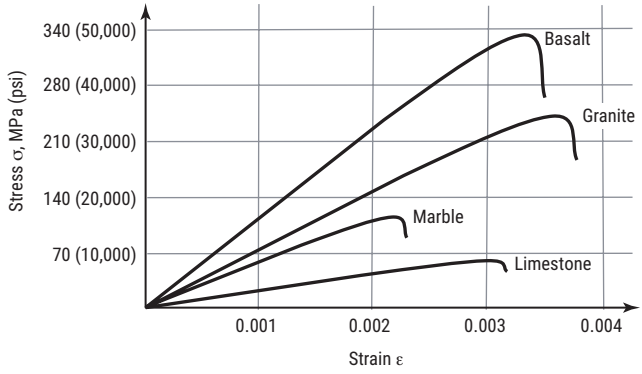

FIGURE 4.13 Stress–strain curves for rock in compression (brittle behavior)

Uniaxial Loading and Deformation

Given a body with length L and cross-section area A with an applied load P that acts normal to the plane A, the axial stress is

$$\sigma_a = \frac{P}{A}$$

The axial strain is

$$\varepsilon_a = \frac{\Delta L}{L}$$

where ΔL is change in length parallel to the applied load P, and L is the length of the body.

Stress is related to strain and vice versa through the modulus of elasticity E as

$$\sigma_a = E\varepsilon_a$$

or

$$\varepsilon_a = \frac{\sigma_a}{E}$$

The modulus of elasticity is thus

$$E = \frac{\sigma_a}{\varepsilon_a}$$

Axial deformation is

$$\delta_a = \varepsilon_a L = \frac{\sigma_a}{E} L = \frac{PL}{AE}$$

where

P = applied uniaxial load
L = length of the body
A = cross-section area
E = modulus of elasticity

Rearranging gives

$$P = \frac{AE}{L}\delta_a = k\delta_a$$

The term

$$k = \frac{AE}{L}$$

is called the *stiffness*.

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Tangent modulus is the differential change in stress divided by the differential change in strain at some value of stress or

$$E_T = \frac{d\sigma}{d\varepsilon}$$

Secant modulus E_S is the change in stress divided by the change in strain at some value of stress where the initial stress and strain are zero.

$$E_S = \frac{\Delta\sigma}{\Delta\varepsilon}$$

For linear elastic materials, the tangent modulus, the secant modulus, and modulus of elasticity are equal.

$$E_T = E_S = E$$

Lateral strain is

$$\varepsilon_l = \frac{\Delta L_l}{L}$$

where ΔL_l is change in length normal to the applied load P, and L is the length.

The Poisson ratio under uniaxial conditions is defined as

$$\nu = -\frac{\varepsilon_l}{\varepsilon_a}$$

where the negative sign arises because of the opposite sense of axial and lateral strains.

Lateral strain is thus

$$\varepsilon_l = -\nu\varepsilon_a = -\nu\frac{\sigma_a}{E}$$

Given a body with length L and cross-section area A with an applied load T that acts parallel to the plane of the area A, the shear stress is

$$\tau = \frac{T}{A}$$

The shear strain is

$$\gamma = \frac{\Delta L_T}{L}$$

where

ΔL_T = change in length in the direction of the applied shear load T
L = length of the body
γ = angular or rotational deformation of the body

Shear stress is related to shear strain and vice versa through the shear modulus G as

$$\tau = G\gamma$$

or

$$\gamma = \frac{\tau}{G}$$

The shear modulus G, the modulus of elasticity E, and the Poisson ratio are related as

$$G = \frac{E}{2(1+\nu)}$$

The bulk modulus K relates hydrostatic compressive stress to decrease in volume. For linear elastic isotropic solids, the bulk modulus K, the modulus of elasticity E, and Poisson's ratio are related as

$$K = \frac{E}{3(1-2\nu)}$$

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Elastic Strain Energy

If strain is within the elastic limit, then the work done by loading and deforming a body is transformed into elastic strain energy and can be recovered. The elastic strain energy is

$$U = \frac{P\delta}{2}$$

where P is the final load, and δ is the deflection.

For the uniaxial case (one dimension), $P = \sigma A$ and $\delta = \varepsilon L = \frac{\sigma}{E}L$.

The strain energy per unit volume (strain energy density) is

$$u = \frac{U}{V} = \frac{U}{AL} = \frac{P\delta}{2AL} = \frac{\sigma A \sigma L}{2ALE} = \frac{\sigma^2}{2E}$$

Failure Criteria

Criteria for the maximum normal stress and shear stress are discussed in the following sections.

Maximum Normal Stress Criterion

According to maximum normal stress criterion, failure occurs when one of the three principal stresses equals the strength of the material. The principal stresses are

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

Tensile failure occurs if

$$\sigma_1 \geq S_t$$

where S_t is the tensile strength of the material.

Compressive failure occurs if

$$\sigma_3 \leq S_c$$

where S_c is the compressive strength of the material.

This material is typically applied to metals, concrete, and other manufactured material, but not soil and rock.

Maximum Shear Stress Criterion

The Mohr–Coulomb failure criterion is frequently applied to soil and rock that tend to fail in shear along planes of weakness. Shear failure occurs when the applied shear stress exceeds the shear strength as given by

$$\tau \geq c + \tan\phi$$

where

τ = applied shear stress
c = cohesion
ϕ = friction angle of the material

The Mohr–Coulomb failure criterion applies to both intact material and discontinuities such as fault surfaces or joint planes. When applied to intact rock, the cohesion c is the inherent shear strength, and ϕ is the internal angle of friction for the material. When applied to a discontinuity, c is the cohesion of the discontinuity surface, and ϕ is the angle of friction for that surface.

Table 4.1 gives typical values for the modulus of elasticity, yield stress, and ultimate strength for a variety of common construction materials.

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TABLE 4.1 Stress and strength of common materials

Material	Density, γ mg/m^3 (lb/in.^{3})	Modulus, E GPa (Mpsi)	Yield Strength, σ_y MPa (psi)	Ultimate Strength, σ_u MPa (psi)
Steel, structural ASTM A36	7.8 (0.284)	200.0 (29.0)	250 (36,000)	400 (58,000)
Steel, 1090 mild	7.58 (0.276)	—	247 (35,800)	841 (122,000)
Steel, high-strength alloy, ASTM A514	7.8 (0.284)	—	690 (100,000)	760 (110,000)
Steel AISI 4130 quenched and tempered	7.85 (0.286)	—	951 (138,000)	1,110 (161,000)
Steel, stainless AISI 302, cold-rolled	8.19 (0.298)	—	520 (75,000)	860 (125,000)
Steel, chrome-vanadium AISI 6150	7.8 (0.284)	—	620 (90,000)	940 (136,000)
Cast iron 4.5% carbon ASTM A48	7.3 (0.266)	—	130 (19,000)	200 (29,000)
Aluminum alloy 2014-T6	2.8 (0.102)	—	414 (60,000)	483 (70,000)
Aluminum alloy 6061-T6	2.7 (0.098)	69 (10)	270 (39,000)	310 (45,000)
Copper, 99.9% Cu	8.92 (0.325)	—	69 (10,000)	220 (32,000)
Cupronickel 10% Ni, 1.6% Fe, 1% Mn, balance CU	8.94 (0.326)	—	130 (19,000)	350 (51,000)
Brass	8.73 (0.318)	—	200 + (29,000 +)	500 (72,500)
Glass	2.53 (0.092)	50–90 (7.25–13.1)	—	33 (4,800)
Concrete, 3,000 psi	2.32 (0.084)	21.5 (3.1)	—	21 (3,000)
Concrete, 5,000 psi	2.32 (0.084)	27.8 (4.0)	—	34 (5,000)
Wood, Douglas fir	0.471 (0.017)	11.0 (1.60)	—	26 (3,800)
Rock, shale	2.40 (0.087)	1.30 (0.20)	—	34 (5,000)
Limestone	2.70 (0.098)	3.0 (0.44)	—	62 (9,000)
Granite	2.65 (0.096)	6.0 (0.87)	—	234 (34,000)
Basalt	2.80 (0.101)	7.0 (1.02)	—	331 (48,000)

Stress and Strain in Three Dimensions

Figure 4.14 shows the nine components of stress at a point. Three components, designated by σ, act in orthogonal directions normal to the point and are called the *normal stress components*. Six components, designated by γ, act in orthogonal directions parallel to the point and are called *shear stress components*.

The nine components of stress at a point are represented by a 3 × 3 matrix as

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Equilibrium requires that

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{xz} = \tau_{zx}$$

$$\tau_{yz} = \tau_{zy}$$

Therefore, the 3 × 3 stress matrix is symmetric.

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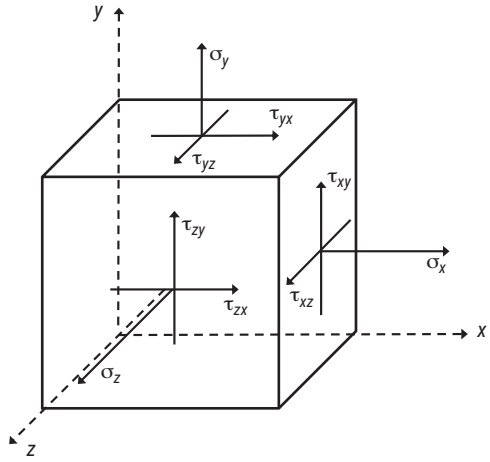

FIGURE 4.14 Nine components of stress at a point

Similar to stress, the nine components of strain at a point are represented by a symmetric 3 × 3 matrix as

$$\begin{bmatrix} \varepsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_z \end{bmatrix}$$

For a linear elastic isotropic solid, strain is related to applied stress as

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

Special Case 1—Plane Stress

In the special case of a thin plate, all stresses in the *z* direction are zero:

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

The preceding general three-dimensional stress–strain relations become

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

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An alternative form of the plane stress relations is

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{1-\nu^2}\begin{pmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{pmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

Special Case 2—Plane Strain

In this special case (plane strain), all strain and shear stress components in the *z* direction are zero:

$$\varepsilon_z = \tau_{xz} = \tau_{yz} = 0$$

The preceding general three-dimensional stress–strain relations become

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$0 = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

An alternative form of the plane strain relations is

$$\sigma_x = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\varepsilon_x + \left(\frac{\nu}{1-\nu}\right)\varepsilon_y\right)$$

$$\sigma_y = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\left(\frac{\nu}{1-\nu}\right)\varepsilon_x + \varepsilon_y\right)$$

$$\sigma_z = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\frac{\nu}{1-\nu}\right)(\varepsilon_x + \varepsilon_y)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Special Case 3—Uniaxial Stress

In this special case (uniaxial stress), normal stresses in the *y* and *z* directions are zero, and all shear stresses are zero:

$$\sigma_y = \sigma_z = \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

As expected, the preceding general three-dimensional stress–strain relations become

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E}$$

Stresses in Cylindrical Pressure Vessel

A hollow cylinder has an inside radius r_i and outside radius r_o. When subjected to an inside pressure P_i, the tangential hoop stress σ_t and the radial stress σ_r within the cylinder wall are

$$\sigma_t = P_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$

$$\sigma_r = -P_i$$

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When subjected to an outside pressure P_o, the tangential hoop stress σ_t and the radial stress σ_r within the cylinder wall are

$$\sigma_t = -P_o\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$

$$\sigma_r = -P_o$$

If the hollow cylinder is capped along the ends, then the axial stress σ_a parallel to the axis of the cylinder is

$$\sigma_a = P_i\left(\frac{r_i^2}{r_o^2 - r_i^2}\right)$$

The thickness of the hollow cylinder is $t = r_o - r_i$. A thin-wall cylinder has $t < (1/10)\, r_i$.

When subjected to an inside pressure P_i, the tangential hoop stress σ_t and the radial stress σ_r for a thin-wall cylinder wall are

$$\sigma_t = P_i\left(\frac{r}{t}\right)$$

$$\sigma_r = -P_i$$

where

$$r = \frac{r_i + r_o}{2}$$

Torsional Stress

For a hollow, thin-wall shaft or a solid shaft, the shear strain is maximum at the shaft radius c. The maximum shear stress in the shaft is

$$\tau_{max} = \frac{Tc}{J}$$

where

T = applied torque or moment
c = shaft radius
J = polar moment of inertia

For a solid shaft, the polar moment of inertia is

$$J = \frac{\pi c^4}{2}$$

For a hollow, thin-wall shaft, the polar moment of inertia is

$$J = \frac{\pi c^3}{t}$$

where t is the wall thickness.

Stress Transformations in Two Dimensions

The initial stress state σ_x, σ_y, and τ_{xy} is with respect to an initial coordinate system (Figure 4.15A). The new coordinate system is at an angle θ' counterclockwise from the initial coordinate system (Figure 4.15B).

The new stress state $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ is

$$\sigma_{x'} = \sigma_x \cos^2\theta' + \sigma_y \sin^2\theta' + \tau_{xy} \sin 2\theta'$$

$$\sigma_{y'} = \sigma_x \sin^2\theta' + \sigma_y \cos^2\theta' - \tau_{xy} \sin 2\theta'$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta' + \tau_{xy} \cos 2\theta'$$

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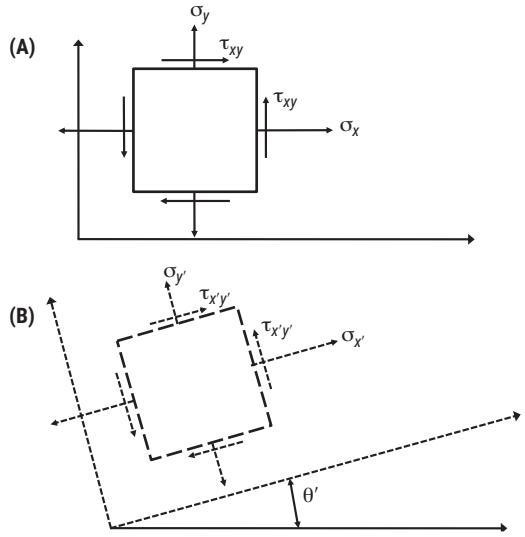

Adapted from engAPPLETS, n.d.

FIGURE 4.15 Stress transformation in two dimensions

Principal Stresses in Two Dimensions

A stress transformation angle θ exists such that $\tau_{x'y'} = 0$. This angle is computed as

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

The principal stresses are

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Mohr Circle for Stress Transformations in Two Dimensions

A Mohr circle is a convenient way to compute principal stresses or to graphically transform stresses from one coordinate system to another (Figure 4.16). Given the stress state σ_x, σ_y, and τ_{xy}, the center of a Mohr circle is at the point $(\sigma_x + \sigma_y)/2$. The radius of a Mohr circle is

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The points σ_x, τ_{xy} and σ_y, $-\tau_{xy}$ lie on the Mohr circle. The principal stresses σ_1 and σ_2 and the orientation of those stresses θ are graphically found. Given the orientation of a new coordinate system θ', the stresses in the new coordinate system $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ are also graphically found.

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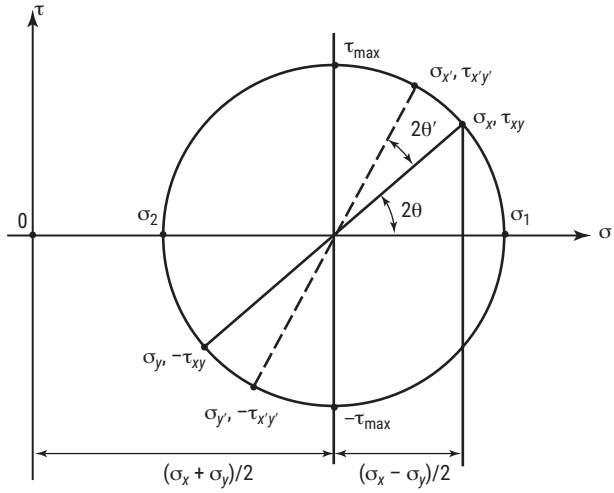

Source: Popov 1968

FIGURE 4.16 Mohr's circle of stress

Fracture Mechanics

Fracture mechanics considers inherent flaws or cracks in materials and whether those cracks will grow and lead to failure under a given stress. The fracture toughness of a material is a measure of resistance to crack growth that leads to failure.

The stress intensity factor K is defined as

$$K = Y\sigma\sqrt{\pi a}$$

where

σ = applied tensile stress
a = crack length
Y = geometry factor

Failure by fracture occurs when

$$K \geq K_{IC}$$

where K_{IC} is the critical stress intensity factor, which is a material property.

The geometry factor for two common fracture scenarios is given in Figure 4.17.

Table 4.2 gives values for the critical stress intensity factor at which catastrophic crack propagation occurs for several common materials.

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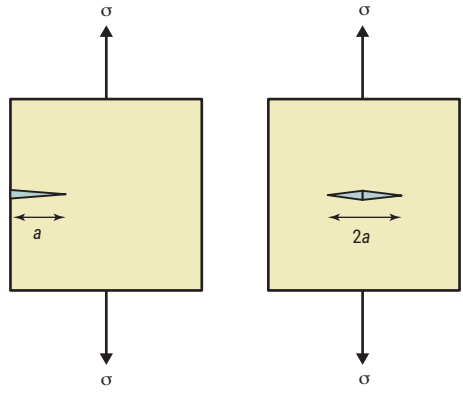

Exterior Crack Y = 1.1 Interior Crack Y = 1.0

Adapted from NCEES 2013

FIGURE 4.17 Geometry factors for two common fractures

TABLE 4.2 Critical stress intensity for common materials

Material	Yield or Ultimate Strength		Critical Stress Intensity Factor K_{IC}	
	MPa	psi	MPa-$m^{1/2}$	Ksi-$in.^{1/2}$
Carbon steel	248	36,000	220	200
Steel alloy (4340)	1,641	238,000	50.0	45.5
Maraging steel (200 grade)	1,669	242,000	175	159
Aluminum alloy (7075)	497	72,000	24	22
Aluminum alloy (2024-T3)	345	50,000	44	40
Concrete	28	4,000	0.15–0.35	0.14–0.32
Glass, soda lime	–	–	0.70–0.80	0.64–0.73
Coal	21	3,000	0.063	0.057
Limestone	62	9,000	1.0	0.91
Granite	234	34,000	2.0	1.82
Basalt	331	48,000	2.5	2.3

Analysis of Beams

Bending moment is positive if it deforms the beam concave upward and causes downward deflection. The top fibers are in compression, and the bottom fibers are in tension.

Shearing force is positive if the right portion of a beam tends to move downward with respect to the left portion. (See Figure 4.18.)

The section of beam shown in Figure 4.19 is subjected to a positive bending moment. The upper beam fibers are in compression, and the lower beam fibers are in tension. The neutral axis separates the tensile and compressive fibers and is the centroid of the beam.

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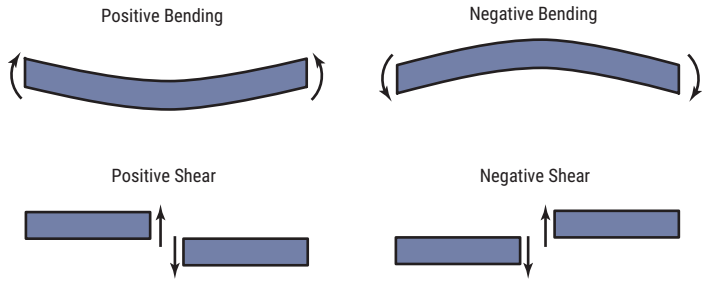

Adapted from Timoshenko and MacCullough 1949

FIGURE 4.18 Bending moment and shearing in beams

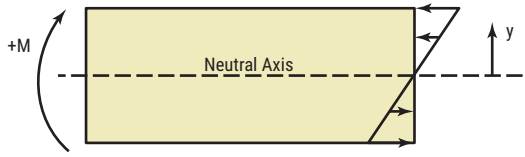

FIGURE 4.19 Elastic beam in pure flexure

Normal bending stresses in the beam are

$$\sigma_x = -\frac{My}{I}$$

where

M = moment at the beam section
y = distance above (+) or below (–) the neutral axis or beam centroid
I = moment of inertia for the cross section

Maximum bending stress occurs where

$$|y| = y_{max} = c$$

$$\sigma_x = -\frac{Mc}{I}$$

Maximum shear stress occurs along the neutral axis of a beam and is

$$\tau_{max} = \frac{3V}{2A}$$

where V is the shear force at the beam section, and A is the total area of the section.

The load w, the shear V, and the moment M for a beam are related as

$$w(x) = -\frac{dV(x)}{dx}$$

$$V(x) = \frac{dM(x)}{dx}$$

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If ρ is the radius of curvature for an elastic beam under load, then the radius of curvature is related to the applied moment on the beam as

$$\frac{1}{\rho} = \frac{M}{EI}$$

where

M = applied moment
E = modulus of elasticity
I = moment of inertia

If v is the deflection of the beam at a point x along the beam, then from analytic geometry

$$\frac{1}{\rho} \approx \frac{d^2v}{dx^2} = \frac{M}{EI}$$

and

$$EI\frac{d^2v}{dx^2} = M(x)$$

is the differential equation for the elastic deflection curve of the beam.

$$EI\frac{d^3v}{dx^3} = \frac{dM(x)}{dx} = V(x)$$

$$EI\frac{d^4v}{dx^4} = \frac{dV(x)}{dx} = -w(x)$$

The deflection curve equation is found by double integration of the differential equation for the elastic deflection curve of the beam.

$$EI\frac{dv}{dx} = \int M(x)dx$$

$$EIv = \iint M(x)dx$$

The integration constants are found knowing the beam geometry at its ends. For a free end, $M = 0$. For a simply supported end, $M = 0$, and $v = 0$. For a fixed end, $v = 0$, and $v' = 0$.

The equations for the elastic curve, maximum moment, shear, and deflection, and their locations follow for several common beam geometries and loading conditions.

Fixed One End, Free on One End, Concentrated Load at Free End

For the following equations, see Figure 4.20.

Elastic Curve

$$v = -\frac{Wx^2}{6EI}(3L - x)$$

Maximum Shear and Location

$$V_{max} = W$$

at all x.

Maximum Moment and Location

$$M_{max} = -WL$$

at $x = 0$.

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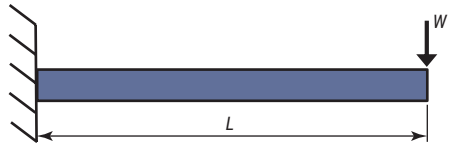


Adapted from Carvill 1993

FIGURE 4.20 Beam fixed on one end, free on the other, with load at the free end

Maximum Slope and Location

$$\theta_{max} = -\frac{WL^2}{2EI}$$

at $x = L$.

Maximum Deflection and Location

$$v_{max} = -\frac{WL^3}{3EI}$$

at $x = L$.

Fixed One End, Free on One End, Uniform Distributed Load

For the following equations, see Figure 4.21.

Elastic Curve

$$v = -\frac{wx^2}{24EI}(x^2 - 4Lx + 6L^2)$$

Maximum Shear and Location

$$V_{max} = wL$$

at $x = 0$.

Maximum Moment and Location

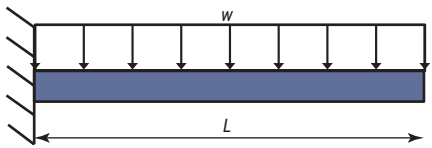
$$M_{max} = -\frac{wL^2}{2}$$

at $x = 0$.

Maximum Slope and Location

$$\theta_{max} = -\frac{wL^3}{6EI}$$

at $x = L$.



Adapted from Carvill 1993

FIGURE 4.21 Beam fixed on one end, free on the other, with distributed load

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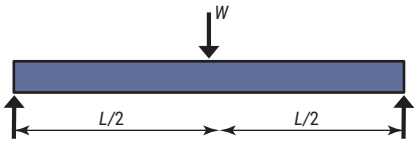

Adapted from Carvill 1993

FIGURE 4.22 Beam supported on both ends with mid-span load

Maximum Deflection and Location

$$v_{max} = -\frac{wL^4}{8EI}$$

at $x = L$.

Simply Supported on Both Ends, Concentrated Load at Mid Span

For the following equations, see Figure 4.22.

Elastic Curve

$$v = -\frac{WL^3}{48EI}x(3 - 4x^2)$$

Maximum Shear and Location

$$V_{max} = W/2$$

at all x.

Maximum Moment and Location

$$M_{max} = -WL/4$$

at $x = L/2$.

Maximum Slope and Location

$$\theta_{max} = -\frac{WL^2}{16EI}$$

at $x = 0$ and $x = L$.

Maximum Deflection and Location

$$v_{max} = -\frac{WL^3}{48EI}$$

at $x = L/2$.

Simply Supported on Both Ends, Concentrated Load at Variable Location

For the following equations, see Figure 4.23.

Elastic Curve

For $x < kL$,

$$v = -\frac{WL^3}{6EI}(1 - k)x(2k - k^2 - x^2)$$

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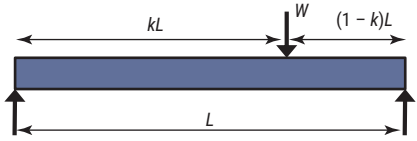

Adapted from Carvill 1993

FIGURE 4.23 Beam supported on both ends with variable location load

For $x > kL$

$$v = -\frac{WL^3}{6EI}kx(2k - k^2 - x^2)$$

Maximum Shear and Location

$$V_{max} = kW$$

or

$$V_{max} = (1 - k)W$$

at $x = 0$ or $x = L$.

Maximum Moment and Location

$$M_{max} = -k(1 - k)WL$$

at $x = kL$.

Maximum Slope and Location

For $x < kL$,

$$\theta_{max} = -\frac{WL^2}{6EI}k(1 - k)(2 - k)$$

or for $x > kL$,

$$\theta_{max} = -\frac{WL^2}{6EI}k(1 - k^2)$$

Maximum Deflection and Location

$$v_{max} = -\frac{WL^3}{3EI}k\left(\frac{1 - k^2}{3}\right)^{3/2}$$

at

$$x = L\sqrt{\frac{1 - k^2}{3}}$$

Simply Supported on Both Ends, Uniform Distributed Load

For the following equations, see Figure 4.24.

Elastic Curve

$$v = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$$

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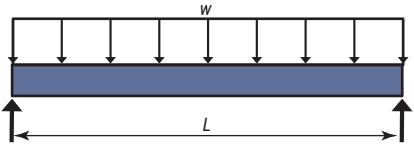


Adapted from Carvill 1993

FIGURE 4.24 Beam supported on both ends with uniformly distributed load

Maximum Shear and Location

$$V_{max} = wL/2$$

at $x = 0$ or $x = L$.

Maximum Moment and Location

$$M_{max} = -\frac{wL^2}{8}$$

at $x = L/2$.

Maximum Slope and Location

$$\theta_{max} = -\frac{wL^3}{24EI}$$

at $x = 0$ and $x = L$.

Maximum Deflection and Location

$$v_{max} = -\frac{5wL^4}{384EI}$$

at $x = L/2$.

Fixed at Both Ends, Concentrated Load at Mid Span

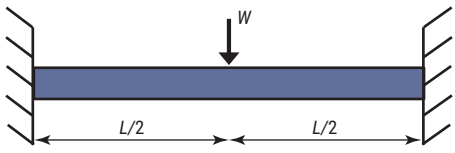
For the following equations, see Figure 4.25.

There is no analytic solution available for elastic curve when the beam is fixed on both ends and the load is concentrated at mid span.

Maximum Shear and Location

$$V_{max} = W/2$$

at all x.



Adapted from Carvill 1993

FIGURE 4.25 Beam fixed at both ends with mid-span concentrated load

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Maximum Moment and Location

$$M_{max} = -WL/8$$

at $x = L/2$, and $x = 0$, and $x = L$.

Maximum Slope and Location

$$\theta_{max} = -\frac{WL^2}{64EI}$$

at $x = 0$ and $x = L$.

Maximum Deflection and Location

$$v_{max} = -\frac{WL^3}{192EI}$$

at $x = L/2$.

Fixed at Both Ends, Uniform Distributed Load

For the following equations, see Figure 4.26.

There is no analytic solution available for elastic curve when the beam is fixed at both ends and the load is uniformly distributed.

Maximum Shear and Location

$$V_{max} = wL/2$$

at $x = 0$ or $x = L$.

Maximum Moment and Location

$$M_{max} = -\frac{wL^2}{12}$$

at $x = 0$ or $x = L$.

Maximum Slope and Location

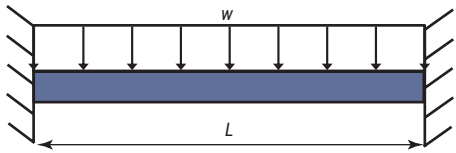
$$\theta_{max} = -0.00803\frac{wL^3}{EI}$$

at $x = 0.211L$ or $x = 0.789L$.

Maximum Deflection and Location

$$v_{max} = -\frac{wL^4}{384EI}$$

at $x = L/2$.



Adapted from Carvill 1993

FIGURE 4.26 Beam fixed at both ends with uniformly distributed load

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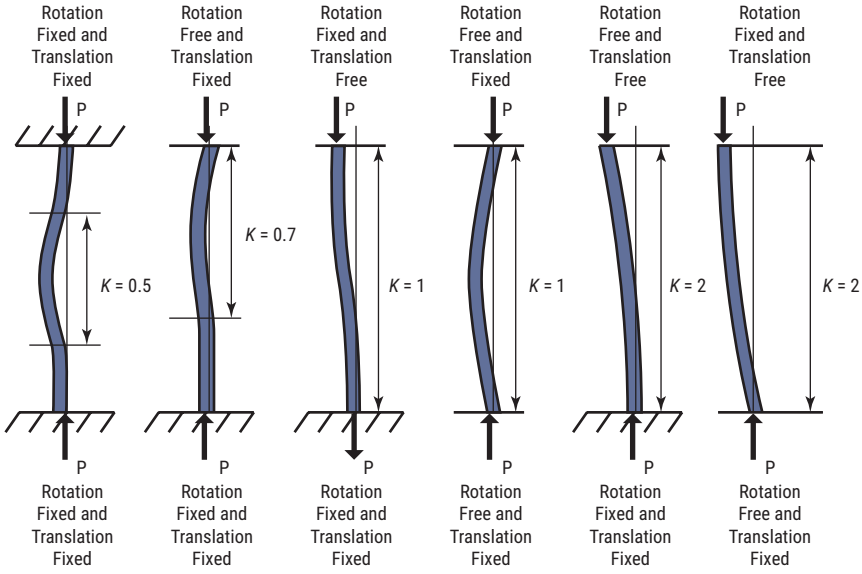
Analysis of Columns

The critical buckling load for an axially loaded column pinned at both ends is given by the Euler formula as

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

where L is the unsupported column length and K is the effective length factor. Figure 4.27 shows the effective column lengths for various end support conditions, and Table 4.3 gives the effective length factors K for those end support conditions.



Adapted from Herbert 2006

FIGURE 4.27 Effective lengths of columns with different restraints

TABLE 4.3 Values for *K*

End Condition, Top	End Condition, Bottom	Theoretical *K* Value	Recommended *K* Value
Rotation fixed and translation fixed	Rotation fixed and translation fixed	0.5	0.65
Rotation free and translation fixed	Rotation fixed and translation fixed	0.7	0.80
Rotation fixed and translation free	Rotation fixed and translation fixed	1.0	1.2
Rotation free and translation fixed	Rotation free and translation fixed	1.0	1.0
Rotation free and translation free	Rotation fixed and translation fixed	2.0	2.1
Rotation fixed and translation free	Rotation free and translation fixed	2.0	1.0

Adapted from Herbert 2006

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FLUID MECHANICS

A list of the definitions for fluid mechanics is presented first with concepts following.

Definitions and Concepts

- Mass density is ρ = mass/volume.
- Specific volume is $\nu = 1/\rho$.
- Specific weight is $\gamma = \rho g$, where $g = 9.810\ m/s^2 = 32.2\ ft/s^2$.
- Mass density of water at standard conditions is $\rho_w = 1{,}000\ kg/m^3 = 62.4\ lb/ft^3$.
- Specific gravity is $SG = \rho/\rho_w$.
- *P* is the static pressure at a point.
- τ is the shear stress at a point parallel to a boundary.
- *V* is the velocity parallel to a boundary.
- *dv* is the differential velocity parallel to a boundary.
- *y* is the perpendicular distance from a boundary.
- *dy* is the differential distance perpendicular to a boundary.

The shear stress τ is related to the rate of shearing strain $\frac{dV}{dy}$ as

$$\tau = \mu \frac{dV}{dy}$$

where μ is the absolute or dynamic viscosity with units newton-second per square meter or pound-mass per foot per second.

The kinematic viscosity is defined as

$$\nu = \frac{\mu}{\rho}$$

with units of square meter per second or square foot per second.

Typical values for specific weight, density, absolute viscosity, and kinematic viscosity for water are given in Tables 4.4 and 4.5

TABLE 4.4 Properties of water in SI metric units

Temperature, °C	Specific Weight γ, kN/m^3	Density ρ, kg/m^3	Absolute Viscosity μ, Pa·s	Kinematic Viscosity ν, m^2/s	Vapor Pressure P_v, kPa
0	9.805	999.9	0.001781	0.000001785	0.61
5	9.807	1000.0	0.001518	0.000001518	0.87
10	9.804	999.7	0.001307	0.000001306	1.23
15	9.798	999.1	0.001139	0.000001139	1.70
20	9.789	998.2	0.001002	0.000001003	2.34
25	9.777	997.0	0.000890	0.000000893	3.17
30	9.764	995.7	0.000798	0.000000800	4.24
40	9.730	992.2	0.000653	0.000000658	7.38
50	9.689	988.0	0.000547	0.000000553	12.33
60	9.642	983.2	0.000466	0.000000474	19.92
70	9.589	977.8	0.000404	0.000000413	31.16
80	9.530	971.8	0.000354	0.000000364	47.34
90	9.466	965.3	0.000315	0.000000326	70.10
100	9.399	958.4	0.000282	0.000000294	101.33

Source: NCEES 2013

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TABLE 4.5 Properties of water in English units

Temperature, °F	Specific Weight γ, lbf/ft^3	Density ρ, lbf·sec^2/ft^4	Absolute Viscosity μ ($\times 10^{-5}$), lbf·s/ft^2	Kinematic Viscosity ν ($\times 10^{-5}$), ft^2/s	Vapor Pressure P_v, psi
32	62.42	1.940	3.746	1.931	0.09
40	62.43	1.940	3.229	1.664	0.12
50	62.41	1.940	2.735	1.410	0.18
60	62.37	1.938	2.359	1.217	0.26
70	62.30	1.936	2.050	1.059	0.36
80	62.22	1.934	1.799	0.930	0.51
90	62.11	1.931	1.595	0.826	0.70
100	62.00	1.927	1.424	0.739	0.95
110	61.86	1.923	1.284	0.667	1.24
120	61.71	1.918	1.168	0.609	1.69
130	61.55	1.913	1.069	0.558	2.22
140	61.38	1.908	0.981	0.514	2.89
150	61.20	1.902	0.905	0.476	3.72
160	61.00	1.896	0.838	0.442	4.74
170	60.80	1.890	0.780	0.413	5.99
180	60.58	1.883	0.726	0.385	7.51
190	60.36	1.876	0.678	0.362	9.34
200	60.12	1.868	0.637	0.341	11.52
212	59.83	1.860	0.593	0.319	14.70

Source: NCEES 2013

Flow Characterization

The Reynolds number, denoted by Re, is a dimensionless number that is the ratio between inertial forces to viscous forces in a moving fluid. The Reynolds number distinguishes laminar flow from turbulent flow and characterizes friction losses in a moving fluid. For Re < 2,000, the flow is laminar. For Re > 20,000, the flow is fully turbulent. For 2,000 < Re < 20,000, the flow is in the transition zone. The Reynolds number is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

where

ρ = mass density
V = fluid velocity
D = pipe diameter
μ = absolute viscosity
ν = kinematic viscosity

The Froude number, denoted by Fr, is a dimensionless number that is the ratio between inertial forces to gravitational forces. The Froude number describes the flow regime in open channel flow. For Fr < 1, the flow is subcritical. For Fr > 1, the flow is supercritical. In an ordinary open channel, the flow is usually designed to be subcritical (Fr < 1). The Froude number is

$$\mathrm{Fr} = \frac{V}{\sqrt{gl}}$$

where

V = fluid velocity
g = acceleration due to gravity
l = characteristic length, such as the fluid depth

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The Mach number, denoted by Ma, is a dimensionless number that is the ratio between the mean fluid velocity and the local speed of sound in the fluid. The Mach number characterizes the flow regime for compressible flow. For Ma < 1, the flow is subsonic. For Ma > 1, the flow is supersonic. The Mach number is

$$\text{Ma} = \frac{V}{c}$$

where V is the fluid velocity, and c is the local speed of sound in the fluid.

For an ideal gas, the local speed of sound is

$$c = \sqrt{kRT}$$

where

k = ratio of specific heats
R = gas constant
T = temperature

The ratio of specific heats is

$$k = \frac{c_p}{c_v}$$

where c_p is the specific heat at constant pressure, and c_v is the specific heat at constant volume.

For dry air at 20°C (293.15 K), the specific heat ratio is 1.4, and the gas constant is 287.06 J/(kg·K). The local sound speed c is 343.4 m/s (1,127 ft/s).

Pressure in a Static Liquid

The difference in pressure P between two points 1 and 2 (Figure 4.28) is

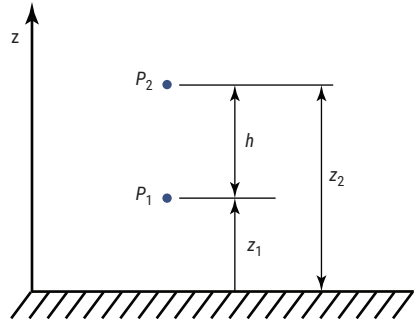
$$P_2 - P_1 = -\gamma(z_2 - z_1) = -\gamma h = -\rho g h$$

where

z = height above some datum
λ = specific weight
ρ = mass density
g = acceleration due to gravity

The pressure variation for an incompressible fluid at rest is

$$\frac{dP}{dh} = \gamma$$



Adapted from Bober and Kenyon 1980

FIGURE 4.28 Static liquid pressure

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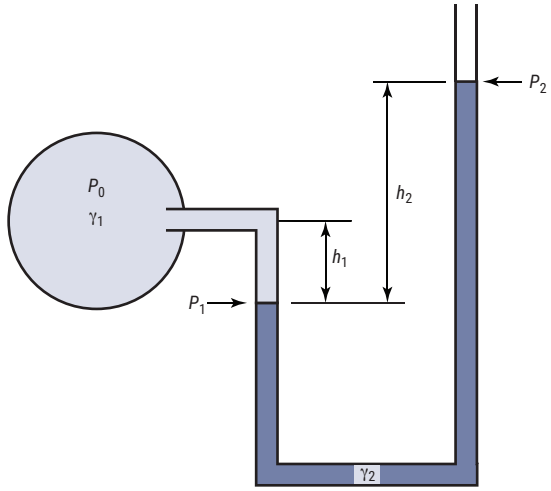

Adapted from Bober and Kenyon 1980

FIGURE 4.29 Pressure measurement inside a vessel

The pressure variation for a compressible fluid at rest is

$$P_2 = P_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

where

g = acceleration due to gravity
z = elevation
R = gas constant (286.7 J/kg·K for air)
T_0 = temperature, K

A manometer is the simplest device for pressure measurement inside a vessel (Figure 4.29). The vessel contains a fluid with specific weight γ_1 and pressure P_0. The manometer contains a fluid with specific weight γ_2. By measuring h_1 and h_2, the pressure P_0 inside the vessel is given by

$$P_0 = P_2 + \gamma_2 h_2 - \gamma_1 h_1 = P_2 + g(\rho_2 h_2 - \rho_1 h_1)$$

If $h_1 = h_2 = h$, then

$$P_0 = P_2 + (\gamma_2 - \gamma_1)h = P_2 + (\rho_2 - \rho_1)gh$$

The barometer is a device for measuring atmospheric pressure (Figure 4.30). The atmospheric pressure P_{atm} is

$$P_{atm} = P_v + \gamma h = P_v + \rho g h$$

where P_v is the vapor pressure of the fluid with specific weight γ.

The maximum pressure acting on a vertical surface extending to a depth h into a fluid (Figure 4.31) is

$$P_{max} = \gamma h$$

The resultant force per unit width of vertical surface is

$$F_R = \frac{1}{2}\gamma h^2$$

acting $h/3$ above the base.

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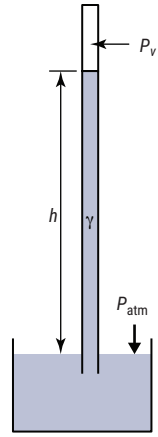

Adapted from Bober and Kenyon 1980

FIGURE 4.30 Atmospheric pressure measurement

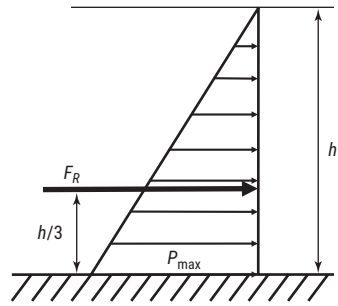

FIGURE 4.31 Vertical surface maximum pressure

Continuity Equation for Incompressible Flow

Conservation of mass for one-dimensional fluid flow requires that the mass flow rate passing any two points in a stream is constant.

$$\dot{m}_1 = \dot{m}_2$$

where $\dot{m}$ is the mass flow rate at points 1 and 2.

If the density of the fluid is ρ, then

$$\dot{m} = \rho Q = \rho A V$$

where

ρ = fluid density
Q = volumetric flow rate
A = cross-sectional area of the flow
V = average flow velocity

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If the fluid density ρ is constant, then

$$Q_1 = Q_2$$

and the continuity equation is

$$Q = A_1 V_1 = A_2 V_2$$

Momentum Equation for Incompressible Flow

The force of a flowing fluid in one dimension is found as

$$F = \dot{m}V = \rho QV$$

If fluid enters a control volume at point 1 and leaves the control volume at point 2, then the resultant force on the control volume equals the rate of change of fluid momentum or

$$\sum F = \rho_1 Q_1 V_1 - \rho_2 Q_2 V_2$$

Energy Equation for Incompressible Flow

The energy equation, also called the Bernoulli equation, shows incompressible flow in one dimension:

$$P + \frac{1}{2}\rho V^2 + \gamma z = \text{constant}$$

where

P = pressure
ρ = mass density
V = flow velocity
γ = specific weight (= ρg)
z = elevation above an arbitrary datum

If no work is done on or by the system by a pump or turbine, and if there are no energy losses caused by friction, then the energy along a streamline is constant.

An alternative form of the energy equation is

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

where

P/γ = pressure head
$V^2/2g$ = velocity head
z = elevation head

The Bernoulli equation between two points along a flow path with no friction losses and no work done on or by the system is

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Friction Losses

The Bernoulli equation between two points along a flow path with friction losses is

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

For pipe flow, the head loss h_f or pressure drop ΔP caused by friction loss is determined with the Darcy–Weisbach equation as

$$h_f = f\frac{l}{D}\frac{V^2}{2g}$$

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$$\Delta P = \rho g h_f = f \frac{l}{D} \frac{\rho V^2}{2}$$

where

f = friction factor
l = equivalent pipe length
D = pipe diameter
V = flow velocity
ρ = mass density
g = acceleration due to gravity

The friction factor f is found from a Moody diagram, and it depends on the Reynolds number of the flow and the relative roughness (ε/D) of the pipe. ε is the roughness of the pipe of diameter D and depends on the type of pipe and its condition. Figure 4.32 is a Moody diagram showing typical values of the roughness of pipes.

For turbulent flow with Re > 2,000, the Colebrook formula can be used to calculate the friction factor f. This formula expresses the entire turbulent flow portion of the Moody diagram.

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

For laminar flow with Re < 2,000, the Poiseuille equation applies.

$$Q = \frac{\pi D^4 \Delta P}{128 \mu l}$$

where

Q = volumetric flow rate
D = pipe diameter
ΔP = pressure drop
μ = absolute viscosity
l = pipe length

Minor Head Losses Caused by Pipe Fittings

The Bernoulli equation with friction losses can be expanded to include a term for friction losses caused by pipe fittings as

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f + h_{ff}$$

where h_f is the friction loss due to pipe friction from preceding Darcy–Weisbach equation and h_{ff} is the friction loss caused by pipe fittings.

The friction losses caused by pipe fittings are usually calculated using a head loss coefficient K such that

$$h_{ff} = K \frac{V^2}{2g}$$

Table 4.6 gives typical values for the head loss coefficient K.

Alternatively, the friction losses caused by pipe fittings can be expressed as an equivalent length of pipe.

$$f \frac{l_{eq}}{D} \frac{V^2}{2g} = K \frac{V^2}{2g}$$

$$l_{eq} = \frac{KD}{f}$$

From the total of the head loss coefficients for the system, an equivalent length of pipe is calculated with the preceding equation. This equivalent length of pipe is added to the actual length of pipe to determine the total head loss for the system.

The term *minor head losses due to pipe fittings* can be a misnomer. Frequently the head losses caused by pipe fittings exceed the head losses caused by pipe friction and dominate the behavior of the system.

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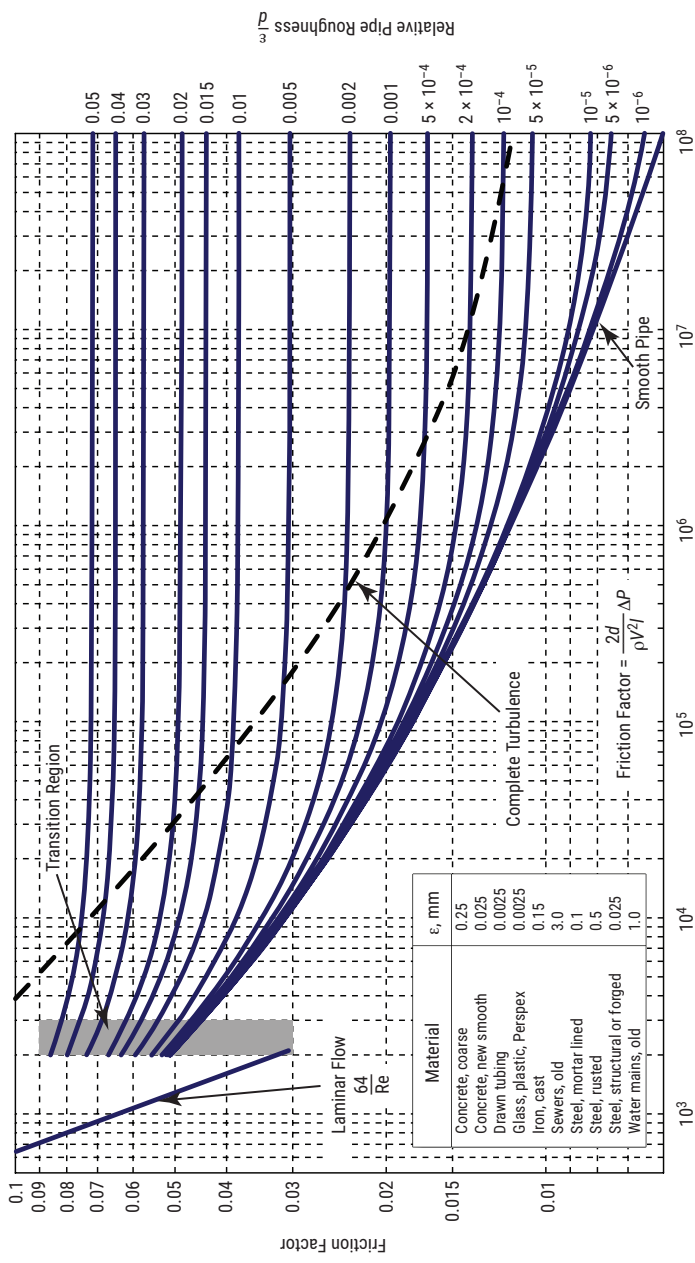

Source: Beck and Collins 2012/2016

FIGURE 4.32 Moody diagram (friction factor for pipes)

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TABLE 4.6 Head loss coefficient values

Type of Fitting or Configuration	*K*
Elbow, 45°	0.35
Elbow, 90°	0.75
Tee	1
Return bend	1.5
Coupling	0.04
Union	0.04
Gate valve (wide open)	0.17
Gate valve (half open)	4.5
Globe valve (wide open)	6
Globe valve (half open)	9.5
Angle valve (wide open)	2
Check valve (ball)	70
Check valve (swing)	2
Water meter (disk)	7
Rounded entrance to pipe	0.05
Sudden contraction	$0.45(1-B)$*
Sudden expansion	$(1/B-1)^2$
Expansion into infinite reservoir	1
Orifice	$2.7(1-B)(1-B^2)/B^2$

* B = (smaller cross-sectional area)/(larger cross-sectional area). Source: NCEES 2013

Flow in Noncircular Conduits

Flow through a conduit with a noncircular cross section is analyzed using the hydraulic radius R_H or the hydraulic diameter D_H. Hydraulic radius is defined as

$$R_H = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{D_H}{4}$$

Airway Resistance

See Chapter 15 on ventilation for more information. For water flow in closed conduits, the friction factor and the Darcy–Weisbach equation are used to compute head loss. For airflow in ducts, the resistance factor R is used to relate pressure loss and airflow quantity Q.

$$\Delta P = RQ^2 = \frac{RV^2}{A^2}$$

The Darcy–Weisbach equation is

$$\Delta P = \rho g h_f = f\frac{l}{D}\frac{\rho V^2}{2} = f\frac{l}{D}\frac{\rho Q^2}{2A^2}$$

The resistance factor for airflow in ducts is

$$R = f\frac{l}{D}\frac{\rho}{2A^2}$$

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Flow Networks—Multipath Pipe Flow

Pipes and airways in series: For two or more pipes or ducts in series, the flow quantity through each is

$$Q_1 = Q_2 = \cdots = Q_n$$

and the total head loss is

$$h_{f-\text{total}} = \sum_{i=1}^{n} h_{fi}$$

where n is the number of pipes or ducts in series.

Pipes and airways in parallel: For two or more pipes or ducts in parallel, the total flow quantity is

$$Q_{total} = \sum_{i=1}^{n} Q_i$$

And the head loss in each is

$$h_{f1} = h_{f2} = \cdots = h_{fn}$$

For two circular pipes in parallel, the head losses are the same in each pipe as

$$h_f = f_A \frac{l_A}{D_A} \frac{V_A^2}{2g} = f_B \frac{l_B}{D_B} \frac{V_B^2}{2g}$$

And the total flow quantity is

$$Q_{total} = \left(\frac{\pi D_A^2}{4}\right) V_A + \left(\frac{\pi D_B^2}{4}\right) V_B$$

The two preceding equations for two circular pipes in parallel are solved for the two unknowns V_A and V_B.

Drag Force

The drag force F_D on a body immersed in a flowing fluid or on a moving body in a stationary fluid is

$$F_D = \frac{C_D \rho V^2 A}{2}$$

where

C_D = drag coefficient
ρ = mass density
V = velocity of the flowing fluid or the moving body
A = projected area normal to the flow

For a flat plate oriented parallel to the flow, the drag coefficient for $10^4 < \text{Re} < 5 \times 10^5$ is

$$C_D = \frac{1.33}{\text{Re}^{0.5}}$$

and for $10^6 < \text{Re} < 10^9$, it is

$$C_D = \frac{0.031}{\text{Re}^{0.143}}$$

The characteristic length D for the Reynolds number calculation is the length of the plate parallel to the flow or the largest dimension perpendicular to the flow for blunt objects.

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Force on Pipe Bends

A moving fluid exerts a force on pipe bends, enlargements, and contractions. The momentum equation gives the magnitude of that force.

$$F = \dot{m}V = \rho QV$$

where

F = resultant force on the pipe bend
ρ = mass density of the fluid
Q = volumetric flow rate
V = flow velocity

Consider a section of pipe containing a bend with angle α as shown Figure 4.33. The force F exerted on the bend is computed in the following equation.

$$\sum F_x = P_1A_1 - P_2A_2\cos(\alpha) - F_x = \rho Q(V_2\cos(\alpha) - V_1)$$

$$\sum F_y = -W - P_2A_2\sin(\alpha) - F_y = \rho Q(V_2\sin(\alpha))$$

where

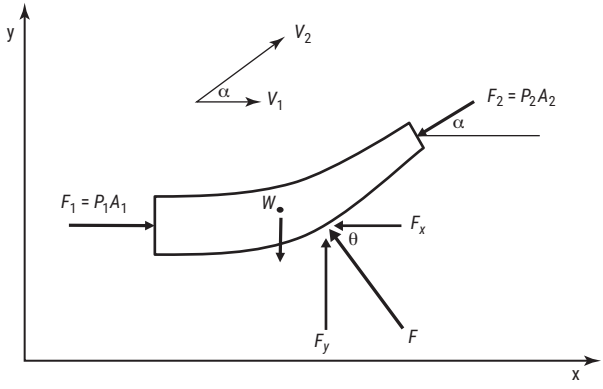
P = fluid pressure
A = cross-sectional area
α = bend angle with respect to the x-axis
F_x and F_y = force components acting on the bend
V = fluid velocity

The resultant force F is

$$F = \sqrt{F_x^2 + F_y^2}$$

This resultant force is the force necessary to restrain the pipe bend. The direction of this force with respect to the x-axis is

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$



Adapted from Vennard 1954

FIGURE 4.33 Pipe section with bend

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Centrifugal Pumps

A typical characteristic curve for a centrifugal pump is shown in Figure 4.34. As the flow rate Q increases, the head H delivered by the pump decreases, the power P increases, and the net positive suction head (NPSH) increases. As flow rate Q increases, the efficiency η increases at first, reaches a maximum, then decreases. With a pump, the mass density ρ is constant. Available net positive suction head $NPSH_A$ is

$$NPSH_A = \frac{P_{atm}}{\rho g} + h_s - h_f - \frac{V^2}{2g} - \frac{P_{vapor}}{\rho g}$$

where

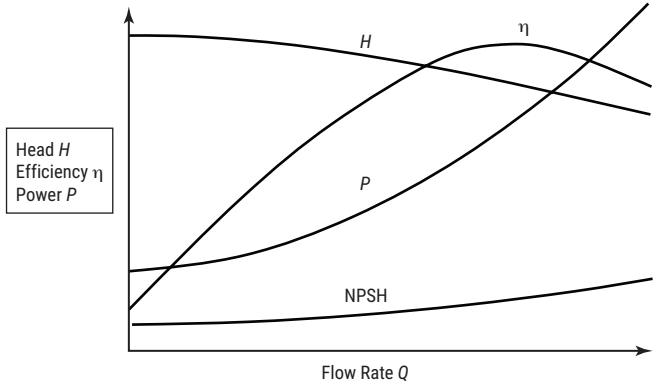
P_{atm} = atmospheric pressure at the fluid surface
h_s = static head at the pump inlet
h_f = friction loss from the pipe inlet to the pump inlet
V = fluid velocity in the pipe
P_{vapor} = vapor pressure of the fluid at the pump inlet
ρ = mass density
g = acceleration due to gravity

Fluid power is

$$\dot{W}_{fluid} = \rho g H Q$$

where

ρ = mass density
g = acceleration due to gravity
H = head increase developed by the pump
Q = volumetric flow rate



Adapted from NCEES 2013

FIGURE 4.34 Centrifugal pump curve

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Pump power is

$$\dot{W}_{pump} = \frac{\rho g H Q}{\eta_{pump}}$$

where η_{pump} is the pump efficiency found from the pump's characteristic curve.

Motor power is

$$\dot{W}_{motor} = \frac{\dot{W}_{pump}}{\eta_{motor}}$$

where η_{motor} is the motor efficiency found from its specifications.

Total power required is

$$\dot{W}_{total} = \frac{\rho g H Q}{\eta_{pump}\eta_{motor}}$$

Fans

A typical characteristic curve for a fan is shown in Figure 4.35. For these curves, the fan speed N, the fan diameter D, and the mass density ρ of the fluid are constant. As the flow rate Q increases, the pressure increase ΔP delivered by the fan increases at first then decreases, the power P increases, and the efficiency η increases at first, reaches a maximum, then decreases. The power requirement for a fan is

$$\dot{W} = \frac{\Delta P Q}{\eta_{fan}}$$

where

ΔP = pressure increase delivered by the fan
Q = volumetric flow rate
η_{fan} = fan efficiency found from the fan's characteristic curve

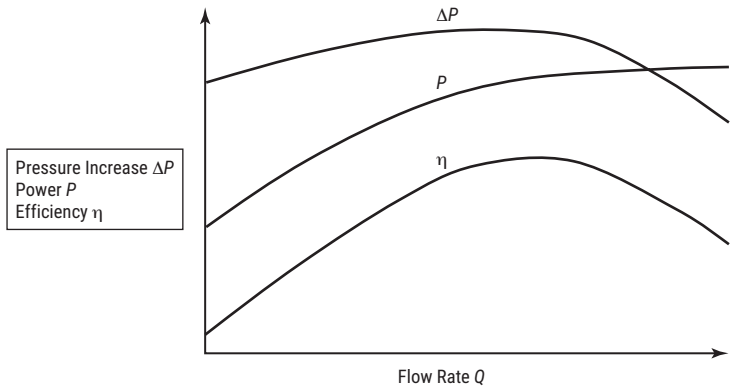

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FIGURE 4.35 Fan curve

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Compressors

Similar to a pump, a compressor adds energy to the working fluid; however, unlike a pump, the mass density ρ is variable. For an adiabatic compressor where no heat is added or removed and where the change in potential energy is zero and the kinetic energy is negligible, the work done by the compressor on the fluid is

$$\dot{W}_{\text{comp}} = -\dot{m}(h_e - h_i)$$

where

$\dot{m}$ = mass flow rate
h_e = fluid pressure (head) at the compressor outlet
h_i = fluid pressure (head) at the compressor inlet

For an ideal gas with constant specific heat, the work done by the compressor is

$$\dot{W}_{\text{comp}} = -\dot{m}c_p(T_e - T_i)$$

where

c_p = specific heat at constant pressure
T_e = fluid temperature at the compressor outlet
T_i = fluid temperature at the compressor inlet

The work done by the compressor per unit mass of fluid flow is

$$w_{\text{comp}} = \frac{\dot{W}_{\text{comp}}}{\dot{m}} = -c_p(T_e - T_i)$$

The isentropic efficiency of a compressor is

$$\eta_c = \frac{w_s}{w_a} = \frac{T_{es} - T_i}{T_e - T_i}$$

where

w_a = actual work done by the compressor per unit mass
w_s = isentropic or ideal work done by the compressor per unit mass
T_{es} = isentropic or ideal fluid temperature at the compressor outlet

If the change in kinetic energy is included, then the work done by the compressor on the fluid is

$$\dot{W}_{\text{comp}} = -\dot{m}\left(h_e - h_i + \frac{V_e^2 - V_i^2}{2}\right) = -\dot{m}\left(c_p(T_e - T_i) + \frac{V_e^2 - V_i^2}{2}\right)$$

where V_e and V_i are the exit and inlet velocities to the compressor.

For adiabatic compression with no heat addition or subtraction, the actual work done by the compressor is

$$\dot{W}_{\text{comp}} = \frac{\dot{m}P_i k}{(k-1)\rho_i \eta_c}\left[\left(\frac{P_e}{P_i}\right)^{1-1/k} - 1\right]$$

where

P_i = inlet pressure
P_e = exit pressure
k = specific heat ratio
ρ_i = mass density at the inlet
η_c = isentropic compressor efficiency

For isothermal compression, the actual work done by the compressor is

$$\dot{W}_{\text{comp}} = \frac{\bar{R}T_i}{M\eta_c}\ln\frac{P_e}{P_i}(\dot{m})$$

where

T_i = fluid temperature at the inlet (and exit)
$\bar{R}$ = universal gas constant
M = molecular weight of the gas

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Turbines

A turbine extracts energy from a fluid as useful work. The energy decrease results in a decrease in fluid pressure or head. For an adiabatic turbine where no heat is added or removed and where the change in potential energy is zero and the kinetic energy is negligible, the work produced by the turbine from the fluid is

$$\dot{W}_{\text{turb}} = \dot{m}(h_i - h_e)$$

where

$\dot{m}$ = mass flow rate
h_i = fluid pressure (head) at the turbine inlet
h_e = fluid pressure (head) at the turbine outlet

For an ideal gas with constant specific heat, the work produced by the turbine is

$$\dot{W}_{\text{turb}} = \dot{m}c_p(T_i - T_e)$$

where

c_p = specific heat at constant pressure
T_i = fluid temperature at the turbine inlet
T_e = fluid temperature at the turbine outlet

The work produced by the turbine per unit mass of fluid flow is

$$w_{\text{turb}} = \frac{\dot{W}_{\text{turb}}}{\dot{m}} = c_p(T_i - T_e)$$

The isentropic efficiency of a turbine is

$$\eta_T = \frac{w_a}{w_s} = \frac{T_i - T_e}{T_i - T_{es}}$$

where

w_a = actual work produced by the turbine per unit mass
w_s = isentropic or ideal work produced by the turbine per unit mass
T_{es} = isentropic or ideal fluid temperature at the turbine outlet

If the change in kinetic energy is included, then the work produced by the turbine from the fluid is

$$\dot{W}_{\text{turb}} = \dot{m}\left(h_e - h_i + \frac{V_e^2 - V_i^2}{2}\right) = \dot{m}\left(c_p(T_e - T_i) + \frac{V_e^2 - V_i^2}{2}\right)$$

where V_e and V_i are the exit and inlet velocities to the turbine.

Scaling Laws

$$\left(\frac{Q}{ND^3}\right)_2 = \left(\frac{Q}{ND^3}\right)_1 \qquad \left(\frac{P}{\rho N^2 D^2}\right)_2 = \left(\frac{P}{\rho N^2 D^2}\right)_1$$

$$\left(\frac{\dot{m}}{\rho ND^3}\right)_2 = \left(\frac{\dot{m}}{\rho ND^3}\right)_1 \qquad \left(\frac{\dot{W}}{\rho N^3 D^5}\right)_2 = \left(\frac{\dot{W}}{\rho N^3 D^5}\right)_1$$

$$\left(\frac{h}{N^2 D^2}\right)_2 = \left(\frac{h}{N^2 D^2}\right)_1$$

where

Q = volumetric flow rate
$\dot{m}$ = mass flow rate
h = head
P = pressure rise
$\dot{W}$ = power
ρ = mass density
N = rotational speed
D = impeller diameter

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Open Channel Flow or Gravity Flow in Pipes and Channels

The Manning equation gives the flow velocity for gravity-driven flow in pipes and channels. The velocity V is

$$V = \left(\frac{K}{n}\right) R_H^{2/3} S^{1/2}$$

where

K = 1 in the SI system and 1.49 in U.S. customary units
n = Manning coefficient (Tables 4.7 and 4.8)
R_H = hydraulic radius of the channel or pipe
S = slope of the pipe or channel

TABLE 4.7 Values for Manning coefficient *n* in pipe and channel flow

Pipe Material and Condition	Typical Value for *n*
Asbestos cement	0.011
Asphalt	0.016
Brass	0.011
Brick and cement mortar sewers	0.015
Canvas	0.012
Cast or ductile iron, new	0.012
Clay tile	0.014
Concrete, steel forms	0.011
(Cement), finished	0.012
Wooden forms	0.015
Centrifugally spun	0.013
Copper	0.011
Corrugated metal	0.022
Galvanized iron	0.016
Glass	0.010
Lead	0.011
Masonry	0.025
Plastic	0.009
Polyethylene (PE), smooth inner walls	0.009–0.015
Corrugated inner walls	0.018–0.025
Polyvinyl chloride (PVC), smooth inner walls	0.009–0.011
Rubble masonry	0.017–0.022
Steel, coal-tar enamel	0.010
Smooth	0.012
New unlined	0.011
Riveted	0.019
Vitrified clay sewer pipe	0.013–0.015
Wood, planed	0.012
Unplaned	0.013
Wood stave pipe, small diameter	0.011–0.012
Large diameter	0.012–0.013

Source: Engineering ToolBox 2004

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TABLE 4.8 Values for Manning coefficient *n* in channels

Natural and Manufactured Channels	Typical Value for *n*
Earth, smooth	0.018
Gravel, firm	0.023
Earth channel, clean	0.022
Gravelly	0.025
Weedy	0.030
Stony, cobbles	0.035
Natural streams, clean and straight	0.030
Major rivers	0.035
Sluggish with deep pools	0.040
Natural channels, very poor condition	0.060
Floodplains, pasture, farmland	0.035
Light brush	0.050
Heavy brush	0.075
Trees	0.15

Source: Engineering ToolBox 2004

The volumetric flow rate Q follows as

$$Q = VA$$

where A is the cross-sectional area of the flow.

The Hazen–Williams equation also gives the flow velocity for gravity-driven flow in pipes and channels. The velocity V is

$$V = KCR_H^{0.63} S^{0.54}$$

where

K = 0.849 in the SI system and 1.318 in U.S. customary units
C = roughness coefficient (Table 4.9)
R_H = hydraulic radius of the channel or pipe
S = slope of the pipe or channel

Fluid Flow Measurement with Pitot Tube

A pitot tube measures the stagnation pressure of a fluid from which fluid velocity is determined. The static pressure is P_s and the velocity is V at a point in the fluid upstream of the pitot tube. At the tip of the pitot tube, the velocity is zero, and the pressure is the stagnation pressure P_0. From the Bernoulli equation, the fluid velocity V is

$$V = \sqrt{\frac{2g(P_0 - P_s)}{\gamma}} = \sqrt{\frac{2(P_0 - P_s)}{\rho}}$$

where

g = acceleration due to gravity
P_0 = stagnation pressure
P_s = static pressure in the fluid
γ = specific weight (= ρg)
ρ = mass density of the fluid

For open channel flow as shown in Figure 4.36, measuring the height H of the fluid above the datum gives the fluid velocity. Because $P_s = \gamma D$ and $P_0 = \gamma(D + H)$, the preceding equation for velocity reduces to

$$V = \sqrt{2gH}$$

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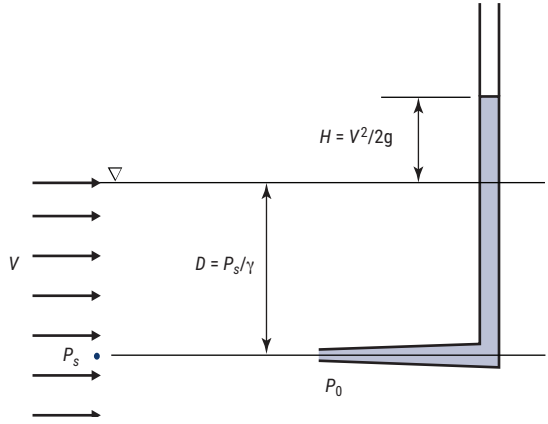
TABLE 4.9 Values for *C* for Hazen–Williams equation

Material	*C* Factor, Low Value	*C* Factor, High Value
Asbestos cement	140	140
Cement-mortar lined ductile iron pipe	140	140
Concrete	100	140
Copper	130	140
Galvanized iron	120	120
Steel	90	110
Cast iron, new	130	130
10 years	107	113
20 years	89	100
30 years	75	90
40 years	64	83
Fiber-reinforced plastic (FRP)	150	150
Polyethylene	140	140
Polyvinyl chloride (PVC)	150	150



Adapted from Vennard 1954

FIGURE 4.36 Fluid velocity in open channel flow

For air velocity measurements or pressurized flow in a closed conduit, the static pressure P_s is measured by turning the tip of the pitot tube perpendicular to the flow. The preceding relationship applies to compressible flow provided the Mach number is Ma < 0.3.

Fluid Flow Measurement with Venturi Meters

A venturi meter measures volumetric flow rate Q in pipes. It consists of a contraction followed by a gradual expansion along the flow. The static pressure P_1 is measured upstream of the contraction, and the static pressure P_2 is measured in a short straight section downstream of the contraction and upstream of the expansion (Figure 4.37). The volumetric flow rate is calculated as

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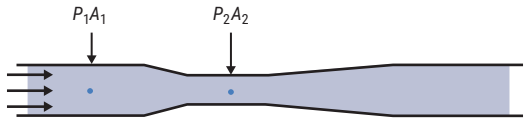

Adapted from Vennard 1954

FIGURE 4.37 Fluid flow using a venturi meter

$$Q = \frac{C_v A_2}{\sqrt{1-(A_2/A_1)^2}} \sqrt{2g\left(\frac{P_1}{\gamma} + z_1 - \frac{P_2}{\gamma} - z_2\right)}$$

where

Q = volumetric flow rate
C_v = coefficient specified by the venturi meter manufacturer
A_1 = upstream cross-sectional area
A_2 = venturi meter cross-sectional area
P_1 = upstream pressure measurement
P_2 = pressure measurement in the venturi meter
z_1 = elevation at cross-sectional area 1
z_2 = elevation at cross-sectional area 2
γ = specific weight of the fluid

Flow Measurement with a Weir

A weir is a common method to measure flow in open channels. The volumetric flow over a weir is

$$Q = CH^{5/2}$$

where

H = depth of discharge over the weir, m (ft)
C = 1.84 in SI units or 3.33 in U.S. customary units for a rectangular weir
C = 1.40 in SI units or 2.54 in U.S. customary units for a 90-degree V-notch weir

Flow in Porous Media

This section discusses Darcy's law, confined and unconfined aquifers, and flow nets.

Darcy's Law

Flow through a porous medium is

$$Q = -KA\left(\frac{dh}{dx}\right)$$

where

Q = discharge rate, m^3/s
K = hydraulic conductivity, m/s
A = cross-sectional area of flow, m^2
dh/dx = hydraulic gradient or change in head per unit distance, m/m

The specific discharge is

$$q = \frac{Q}{A} = -K\left(\frac{dh}{dx}\right)$$

where q has units of meter per second and is also called the *Darcy velocity* or *superficial velocity*.

The average seepage velocity is

$$v = \frac{q}{n} = -\frac{K}{n}\left(\frac{dh}{dx}\right)$$

where n is the effective porosity ($n < 1$).

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Unconfined Aquifer

The bottom of the aquifer is impermeable, and the top of the aquifer is the groundwater level (Figure 4.38).

For an unconfined aquifer, the Dupuit formula relates flow rate Q from a well to the piezometric surface as

$$Q = \frac{\pi k(h_2^2 - h_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

where

Q = flow rate from well, m^3/s
k = coefficient of permeability of aquifer, m/s
h_1 = height of piezometric surface above aquifer bottom at well radius r_1, m
h_2 = height of piezometric surface above aquifer bottom at radius r_2 from well centerline, m
r_1 = radius of well, m
r_2 = radius to location h_2, m

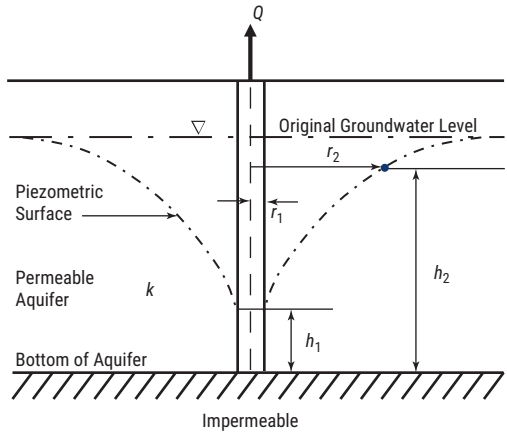

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FIGURE 4.38 Fluid flow in an unconfined aquifer

Confined Aquifer

Both the bottom and top of the aquifer are impermeable (Figure 4.39).

For a confined aquifer, the flow rate Q from a well and the piezometric surface are related as

$$Q = \frac{2\pi T(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

where

Q = flow rate from well, m^3/s
T = transmissivity of aquifer = KD, m^2/s
K = hydraulic conductivity, m/s
D = thickness of confined aquifer, m
h_1 = height of piezometric surface above aquifer bottom at radius r_1 from well centerline, m
h_2 = height of piezometric surface above aquifer bottom at radius r_2 from well centerline, m
r_1 = radius to location h_1, m
r_2 = radius to location h_2, m

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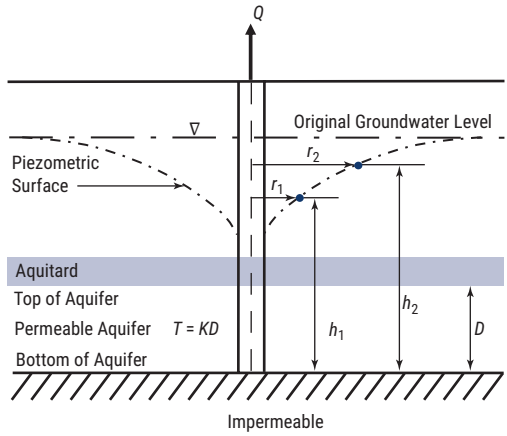

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FIGURE 4.39 Fluid flow in a confined aquifer

Flow Nets

The flow net is a method to estimate flow through porous media (Figure 4.40).

The flow Q is

$$Q = kH\left(\frac{N_f}{N_d}\right)$$

where

k = coefficient of permeability of aquifer, m/s
H = total hydraulic head differential, m
N_f = number of flow channels in the flow net
N_d = number of equipotential drops in the flow net

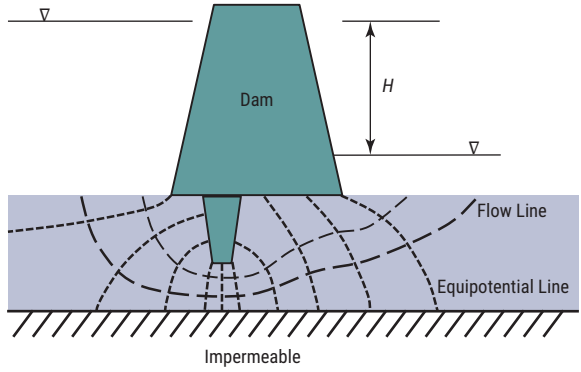

FIGURE 4.40 Flow net

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THERMODYNAMICS

Following are the basic thermodynamic properties and their definitions.

Basic Thermodynamic Properties

- P is absolute pressure in pascals (Pa) or pound-foot per square inch (lbf/in.^{2}).
- T is absolute temperature in Kelvin (K) or degrees Rankine (°R).
- V is volume in cubic meters (m^3) or cubic feet (ft^3).
- m is mass in kilograms (kg) or pound-mass (lbm).
- ν is specific volume, V/m, in cubic meters per kilogram (m^3/kg) or cubic feet per pound-mass (ft^3/lbm).
- U is internal energy in kilojoules (kJ) or British thermal units (Btu).
- u is specific internal energy, U/m, in kilojoules per kilogram (kJ/kg) or British thermal units per pound-mass (Btu/lbm).
- H is enthalpy in kilojoules (kJ) or British thermal units (Btu).
- h is specific enthalpy, $u + P\nu = H/m$, in kilojoules per kilogram (kJ/kg) or British thermal units per pound-mass (Btu/lbm).
- S is entropy in kilojoules per Kelvin (kJ·K) or British thermal units per degrees Rankine (Btu/°R).
- s is specific entropy, S/m, in kilojoules per kilogram-Kelvin [kJ/(kg·K)] or British thermal unit per pound-mass degrees Rankine [Btu/(lbm·°R)].

The heat capacity at constant pressure c_P is

$$c_P = \left(\frac{\partial h}{\partial T}\right)_P \text{ [Btu/(lbm·°R) or kJ/(kg·K)]}$$

The heat capacity at constant volume c_v is

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \text{ [Btu/(lbm·°R) or kJ/(kg·K)]}$$

For two-phase vapor–liquid systems, the quality x is the mass fraction of the vapor phase or

$$x = m_g/(m_g + m_l)$$

where m_g is the mass of the gas phase, and m_l is the mass of the liquid phase.

The specific volume for a two-phase system is

$$\nu = x\nu_g + (1 - x)\nu_l = \nu_l + x\nu_{lg}$$

where

ν_g = specific volume of saturated gas
ν_l = specific volume of saturated liquid
ν_{lg} = specific volume change on vaporization

$$\nu_{lg} = \nu_g - \nu_l$$

Analogous expressions exist for the specific internal energy u, the specific enthalpy h, and the specific entropy s for two-phase systems as

$$u = xu_g + (1 - x)u_l = u_l + xu_{lg}$$

$$h = xh_g + (1 - x)h_l = h_l + xh_{lg}$$

$$s = xs_g + (1 - x)s_l = s_l + xs_{lg}$$

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Pressure, Volume, and Temperature Relations

The ideal gas law is

$$P = \rho RT$$

or

$$P\nu = RT$$

or

$$PV = mRT$$

where

P = absolute pressure
ρ = mass density
R = gas constant
T = absolute temperature
ν = specific volume
V = volume
m = mass of gas

For an ideal gas, the pressure P, specific volume ν, and temperature T at two different states are related as

$$\frac{P_1\nu_1}{T_1} = \frac{P_2\nu_2}{T_2}$$

The gas constant R is specific to each gas and is found as

$$R = \frac{\overline{R}}{mw_g}$$

where the universal gas constant $\overline{R}$ is 1,545 ft·lbf/(lbmol·°R) or 8,314 J/(kmol·K), and mw_g is the molecular weight of the gas.

The molecular weight of air is approximately 28.964, therefore R for air is 53.352 ft·lbf/(lbm·°R) or 287.06 J/(kg·K).

From the ideal gas law, the following relationships are derived. At constant temperature (Boyle's law),

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

At constant pressure (Charles' law),

$$\frac{T_1}{T_2} = \frac{\rho_2}{\rho_1}$$

At constant volume (ρ = constant),

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

The gas constant R is related to the heat capacities as

$$c_P - c_v = R$$

The heat capacity at constant pressure c_P for air is c_P = 186.7 ft·lbf/(lbm·°R) or 1004.5 J/(kg·K). The heat capacity at constant volume c_v for air is c_v = 133.4 ft·lbf/(lbm·°R) or 717.5 J/(kg·K). These values of heat capacity for air are nearly constant over a wide range of ordinary temperatures.

The ratio of specific heats k is

$$k = \frac{c_P}{c_v}$$

For air, k is approximately 1.4 over a wide range of ordinary temperatures.

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For ideal gases at constant temperature, the following relations apply.

$$\left(\frac{\partial h}{\partial P}\right)_T = 0$$

$$\left(\frac{\partial u}{\partial \nu}\right)_T = 0$$

$$\Delta u = c_v \Delta T$$

$$\Delta h = c_P \Delta T$$

$$\Delta s = c_P \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$\Delta s = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{\nu_2}{\nu_1}\right)$$

For an isentropic process (constant entropy), $\Delta s = 0$ in the preceding relations, and the following relations apply.

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{\nu_1}{\nu_2}\right)^k$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{T_2}{T_1} = \left(\frac{\nu_1}{\nu_2}\right)^{k-1}$$

Compressible Isentropic Flow

For an ideal gas undergoing an isentropic (constant entropy) process, the static properties pressure P, temperature T, and mass density ρ at any two points 1 and 2 in the flow are related as

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{(k-1)}} = \left(\frac{\rho_2}{\rho_1}\right)^k$$

where k is the specific heat ratio (about 1.4 for air at standard temperature and pressure).

The stagnation temperature T_0 is related to the static temperature T and fluid velocity V as

$$T_0 = T + \frac{V^2}{2c_p}$$

where c_p is the specific heat of the gas at constant pressure.

The stagnation properties T_0, P_0, and ρ_0 are related to the static properties T, P, and ρ as

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{(k-1)}} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{k}{(k-1)}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{(k-1)}} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{1}{(k-1)}}$$

First Law of Thermodynamics

The first law of thermodynamics is the conservation of energy principle, which states that the net energy crossing a system boundary equals the change in energy within the system. Energy crosses the system boundary in two forms—heat Q and work W. Heat Q is positive if inward, that is, when heat is added to the system. Work W is positive if outward, that is, when work is done by the system.

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Closed Thermodynamic System

In a closed thermodynamic system, no mass crosses the system boundary. Only energy crosses the system boundary in the form of heat Q or work W. Conservation of energy is

$$Q - W = \Delta U + \Delta \text{KE} + \Delta \text{PE}$$

where

ΔU = change in internal energy
ΔKE = change in kinetic energy
ΔPE = change in potential energy

w_b is work done at the boundary by the system per unit mass. Reversible boundary work is

$$w_b = \int P dv$$

The following are special cases of closed thermodynamic systems with an ideal gas and no change in kinetic or potential energy (ΔKE = 0 and ΔPE = 0).

Constant Pressure Process (Charles' Law)

$$w_b = P\Delta\nu$$

$$\rho T = \frac{T}{\nu} = \text{constant}$$

Constant Volume Process

$$w_b = 0$$

$$\frac{T}{P} = \text{constant}$$

Constant Temperature Process (Boyle's Law)

$$P\nu = \frac{P}{\rho} = \text{constant}$$

$$w_b = RT \ln\left(\frac{\nu_2}{\nu_1}\right) = RT \ln\left(\frac{P_2}{P_1}\right)$$

Isentropic (Constant Entropy) Process

$$P\nu^k = \text{constant}$$

$$w = \frac{(P_2\nu_2 - P_1\nu_1)}{1-k} = R(T_2 - T_1)/(1-k)$$

Polytropic Process

$$P\nu^n = \text{constant}$$

$$w = \frac{(P_2\nu_2 - P_1\nu_1)}{1-n}$$

where $n \neq 1$.

The polytropic process is a generalization for all the processes just described.

- $n = 0$ is a constant pressure (isobaric) process.
- $n = 1$ is a constant temperature (isothermal) process.
- $n = k$ is a constant entropy (isentropic) process.
- $n \rightarrow \infty$ is a constant volume process.

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Open Thermodynamic System

In an open thermodynamic system, mass crosses the system boundary, and there is flow work $P\nu$ done by the mass entering the system. Reversible flow work is

$$w_{rev} = -\int \nu \, dP + \Delta KE + \Delta PE$$

The energy balance for an open thermodynamic system is

$$\sum \dot{m}_i \left[h_i + \frac{V_i^2}{2} + gZ_i \right] - \sum \dot{m}_e \left[h_e + \frac{V_e^2}{2} + gZ_e \right] + \dot{Q}_{in} - \dot{W}_{net} = \frac{d(m_s u_s)}{dt}$$

where

$\dot{m}_i$ = mass flow rate into the system
$\dot{m}_e$ = mass flow rate out of the system
h_i = specific enthalpy of mass entering the system
h_e = specific enthalpy of mass leaving the system
V_i = velocity of mass entering the system
V_e = velocity of mass leaving the system
g = acceleration due to gravity
Z_i = elevation of mass entering the system
Z_e = elevation of mass leaving the system
$\dot{Q}_{in}$ = rate of heat transfer into the system
$\dot{W}_{net}$ = rate of work done by the system
m_s = mass of fluid within the system
u_s = specific internal energy of the system

The following are special cases of open thermodynamic systems with an ideal gas and no change in kinetic or potential energy ($\Delta KE = 0$ and $\Delta PE = 0$).

Constant Pressure Process

$$w_{rev} = 0$$

Constant Volume Process

$$w_{rev} = -\nu(P_2 - P_1)$$

Constant Temperature Process

$$P\nu = \frac{P}{\rho} = \text{constant}$$

$$w_{rev} = RT \ln\left(\frac{\nu_2}{\nu_1}\right) = RT \ln\left(\frac{P_2}{P_1}\right)$$

Isentropic (Constant Entropy) Process

$$P\nu^k = \text{constant}$$

$$w_{rev} = \frac{k(P_2\nu_2 - P_1\nu_1)}{1-k} = kR\left(\frac{T_2 - T_1}{1-k}\right)$$

$$w_{rev} = \frac{k}{k-1} RT_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \right]$$

Polytropic Process

$$P\nu^n = \text{constant}$$

$$w_{rev} = \frac{n(P_2\nu_2 - P_1\nu_1)}{1-n}$$

where $n \neq 1$.

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Steady Flow of Open Thermodynamic Systems

In a steady-flow system, the system does not change state with time, and the mass flow rate into the system $\dot{m}_i$ equals the mass flow rate out of the system $\dot{m}_e$. The right-hand side of the conservation of energy equation, describing the time rate of change of the mass and specific internal energy of the system, is zero. The conservation of energy equation reduces to

$$\sum \dot{m}\left(h_i + \frac{V_i^2}{2} + gZ_i\right) - \sum \dot{m}\left(h_e + \frac{V_e^2}{2} + gZ_e\right) + \dot{Q}_{in} - \dot{W}_{net} = 0$$

Special cases of a steady-flow open thermodynamic systems with an ideal gas are turbines, pumps, and compressors. The flow can be considered adiabatic (no heat transfer). Velocity terms are negligible, and there is no elevation difference between the inlet and exit. The conservation of energy reduces to

$$h_i = h_e + w$$

where w is the specific work done by the system in watts per meter.

The isentropic efficiency of a turbine is

$$\eta_T = \frac{h_i - h_e}{h_i - h_{es}}$$

where

h_i = specific enthalpy of mass entering the system
h_e = actual specific enthalpy of mass leaving the system
h_{es} = ideal specific enthalpy of mass leaving the system

The isentropic efficiency of a pump or compressor is

$$\eta_C = \frac{h_{es} - h_i}{h_e - h_i}$$

For a pump where the flow is incompressible,

$$h_{es} - h_i = \nu_i (P_e - P_i)$$

Psychrometrics

Dry-bulb temperature T_{db} is the ordinary temperature of an air–water mixture. Wet-bulb temperature T_{wb} is the indicated temperature of a thermometer covered with a wick saturated with liquid water and in contact with moving air.

$$P = P_a + P_v$$

where

P = pressure of the air–water mixture, normally 1 atm
P_a = partial pressure of dry air
P_v = partial pressure of water vapor

Specific humidity, absolute humidity, or humidity ratio ω is

$$\omega = \frac{m_v}{m_a} = 0.622\frac{P_v}{P_a} = 0.622\frac{P_v}{P - P_v}$$

where m_v is the mass of water vapor, and m_a is the mass of dry air.

Relative humidity, designated by rh or ϕ, is

$$\phi = \frac{P_v}{P_{g'}}$$

where $P_{g'}$ is the saturation pressure of water at temperature T.

Enthalpy h for the air–water mixture is

$$h = h_a + \omega h_v$$

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Dew-point temperature T_{dp} is

$$T_{dp} = T_{sat} \text{ at } P_g = P_v$$

HEAT TRANSFER

Three main modes of heat transfer are conduction, convection, and radiation.

Conduction

The Fourier law of conduction is

$$\dot{Q} = -kA\left(\frac{dT}{dx}\right)$$

where

$\dot{Q}$ = rate of heat transfer, W
k = thermal conductivity, W/(m·K)
A = surface area perpendicular to the direction of heat transfer, m^2
dT = differential temperature, K
dx = differential distance along direction of heat transfer, m

Convection

Newton's law of cooling is

$$\dot{Q} = hA(T_w - T_\infty)$$

where

h = convective heat transfer coefficient, W/(m^2·K)
A = convection surface area, m^2
T_w = surface temperature of wall, K
T_∞ = bulk fluid temperature, K

Radiation

Radiant energy emitted by a body (radiant heat transfer) is given by

$$\dot{Q} = \varepsilon\sigma AT^4$$

where

ε = emissivity of the body
σ = Stefan–Boltzmann constant, 5.67×10^{-8} W/($m^2 \cdot K^4$)
A = surface area of body, m^2
T = absolute temperature, K

Conduction Through a Wall

For conduction through a wall, including heat transfer and thermal resistance, see Figure 4.41.

Conductive heat transfer through a plane wall is

$$\dot{Q} = -kA\left(\frac{T_2 - T_1}{L}\right) = -\frac{(T_2 - T_1)}{R}$$

where

T_2 = wall temperature on the outside or cold side of the wall, K
T_1 = wall temperature inside or on the hot side of the wall, K
L = wall thickness, m
R = conductive thermal resistance defined as

$$R = \frac{L}{kA}$$

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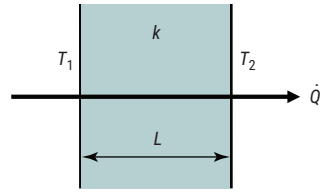

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Figure 4.41 Conduction through a wall

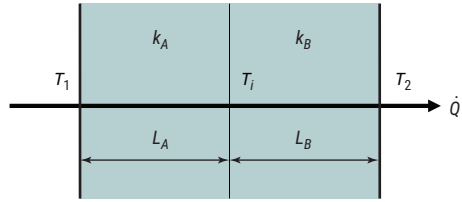

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Figure 4.42 Conduction through composite walls

For composite plane walls (Figure 4.42), resistances in series are added as

$$R = \sum R$$

The intermediate temperature T_i in a two-layer composite wall with resistances R_A and R_B is

$$\dot{Q} = \left(\frac{T_1 - T_i}{R_A}\right) = \left(\frac{T_i - T_2}{R_B}\right)$$

$$T_i = \frac{R_B T_1 + R_A T_3}{(R_A + R_B)}$$

Conduction Through a Cylindrical Wall

For conduction through a cylindrical wall, including heat transfer and thermal resistance, see Figure 4.43.

Conductive heat transfer through a cylindrical wall is

$$\dot{Q} = 2\pi kL\left(\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}\right)$$

where

L = cylinder length, m
T_1 = inside temperature, K
T_2 = outside temperature, K
r_1 = inside radius, m
r_2 = outside radius, m

The conductive thermal resistance of a cylinder is

$$\dot{Q} = \frac{\Delta T}{R}$$

where

$$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

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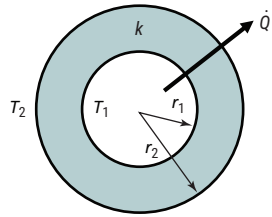

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Figure 4.43 Conduction through a cylindrical wall

Convection from a Cylinder

For convection from a cylinder, including heat transfer and thermal resistance, see Figure 4.44.

Convective heat transfer from a cylinder is

$$\dot{Q} = hA(T_w - T_\infty)$$

where $T_w = T_2$, which is the outside temperature of the cylinder wall.

The convective thermal resistance of a cylinder is

$$\dot{Q} = \frac{\Delta T}{R}$$

where

$$R = \frac{1}{hA}$$

The critical insulation radius is the insulation thickness where conductive heat transfer through the insulation balances convective heat transfer from the insulation surface.

$$r_{cr} = \frac{k_{insulation}}{h_\infty}$$

Natural (Free) Convection from a Vertical Wall in a Stationary Fluid

The average convection heat transfer coefficient of the fluid is

$$\overline{h} = C\left(\frac{k}{L}\right)\mathrm{Ra}_L^n$$

where

C = constant that depends on the Rayleigh number, denoted by Ra (Table 4.10)
k = thermal conductivity of the fluid, W/(m·K)
L = length of the vertical wall in the vertical direction, m
n = exponent (Table 4.10)

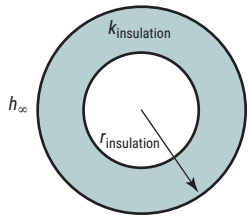

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Figure 4.44 Convection from a cylinder

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The Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$

where

g = acceleration due to gravity
β = coefficient of thermal expansion, 1/K
T_s = surface temperature of vertical wall, K
T_∞ = fluid temperature
ν = kinematic viscosity of the fluid, m^2/s
Pr = Prandtl number

The coefficient of thermal expansion β is approximately 1 if the fluid is water.

For an ideal gas,

$$\beta = \frac{2}{(T_s + T_\infty)}$$

The Prandtl number is

$$\mathrm{Pr} = \frac{\mu c_P}{k} = \frac{\rho \nu c_P}{k}$$

where

μ = absolute or dynamic viscosity of the fluid, kg/(s·m)
c_P = heat capacity at constant pressure, J/(kg·K)
k = thermal conductivity of the fluid, W/(m·K)
ρ = mass density of the fluid
ν = kinematic viscosity of the fluid, m^2/s

For air and most gases, Pr is approximately 0.7. For water, Pr is approximately 2–8, depending on temperature.

The coefficients *C* and *n* for the vertical wall are shown in Table 4.10.

TABLE 4.10 Vertical wall coefficient values

Ra_L	*C*	*n*
10^4–10^9	0.59	0.250
10^9–10^{13}	0.10	0.333

Source: NCEES 2013

Natural (Free) Convection from a Long Horizontal Cylinder in a Stationary Fluid

The average convection heat transfer coefficient of the fluid is

$$\overline{h} = C\left(\frac{k}{D}\right)\mathrm{Ra}_D^n$$

where *D* is the diameter of the cylinder.

The Rayleigh number is

$$\mathrm{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\mathrm{Pr}$$

The coefficients *C* and *n* for the long horizontal cylinder are shown in Table 4.11.

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TABLE 4.11 Long horizontal cylinder coefficient values

Ra_D	C	n
10^{-3}–10^{2}	1.02	0.148
10^{2}–10^{4}	0.850	0.188
10^{4}–10^{7}	0.480	0.250
10^{7}–10^{12}	0.125	0.333

Source: NCEES 2013

ELECTRICITY AND MAGNETISM

Resistors, capacitors, inductors and their definitions are discussed in the following sections.

Definitions

Basic units are charge quantity Q in coulombs, electric potential V in volts, current I in amperes, and resistance R and impedance in ohms.

Electromotive force E in volts is

$$E = \frac{W}{Q}$$

where W is the energy or work in joules, and Q is the charge quantity in coulombs.

Current I in amperes or charge quantity Q per unit time t in seconds is

$$I = \frac{Q}{t}$$

Resistance R in ohms is

$$R = \frac{\rho l}{A}$$

where

ρ = resistivity, which is a material property, Ω·m (ohm meter)
l = length of resistor along the direction of current flow, m
A = cross-sectional area perpendicular to the current flow

Voltage drop V through a resistor also known as Ohm's law is

$$V = IR$$

Power P or energy per unit time is

$$P = VI$$

Power dissipation in a resistor also known as Joule's law is

$$P = I^2R = \frac{V^2}{R}$$

Capacitance in farads is the energy stored in an electric field defined as

$$C = \frac{Q}{V}$$

Capacitance of a parallel-plate capacitor is

$$C = \frac{\varepsilon A}{d}$$

where

ε = permittivity of the medium, F/m (farads/meter)
A = cross-sectional area of the plates, m^2
D = distance between plates, m

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The energy stored in a capacitor is

$$W = \frac{v_c^2}{2} = \frac{q_c^2}{2C} = \frac{q_c v_c}{2}$$

The time required to charge a capacitor connected to a direct current source (called the *time constant*) is

$$t = RC$$

Inductance L in henrys is the energy stored in a magnetic field. The inductance of a coil is

$$L = \frac{N^2 \mu A}{l}$$

$$N\phi = LI$$

where

N = number of turns of wire wound around a core
μ = permeability of the medium, H/m (henrys/meter)
A = cross-sectional area of core, m^2
l = mean path length around the core, m
ϕ = average flux of each turn, Wb (weber = V·s)
I = current, A (ampere)

The energy stored in an inductor is

$$W = \frac{LI^2}{2}$$

The time constant for a resistor–inductor (RL) circuit is

$$t = \frac{L}{R}$$

Resistors, Capacitors, and Inductors in Series and Parallel

Following are the equations for resistors, capacitors, and inductors in series and in parallel.

Resistors in Series

$$R_T = \sum_{i=1}^{n} R_i$$

where R_T is the total resistance, and R_i is the resistance of the ith resistor in series.

Resistors in Parallel

$$\frac{1}{R_T} = \sum_{i=1}^{n} R_i$$

where R_i is the resistance of the ith resistor in parallel.

For two resistors in parallel, the total resistance is

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Capacitors in Series

$$\frac{1}{C_T} = \sum_{i=1}^{n} C_i$$

where C_T is the total capacitance, and C_i is the capacitance of the ith capacitor in series.

Capacitors in Parallel

$$C_T = \sum_{i=1}^{n} C_i$$

where C_i is the capacitance of the ith capacitor in parallel.

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Inductors in Series

$$L_T = \sum_{i=1}^{n} L_i$$

where L_T is the total inductance, and L_i is the inductance of the ith inductor in series.

Inductors in Parallel

$$\frac{1}{L_T} = \sum_{i=1}^{n} L_i$$

where L_i is the inductance of the ith inductor in parallel.

Kirchhoff's Laws

The algebraic sum of all currents entering a junction is zero.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

The algebraic sum of the potential rises and drops around any closed loop in a conductor network is zero.

$$\sum V_{\text{rises}} = \sum V_{\text{drops}}$$

Alternating Current Circuits

Frequency and period for a sinusoidal voltage or current are related as

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

where

f = frequency, Hz
T = time, s
ω = angular frequency, radians/s

X_{max} is the peak amplitude of the sinusoid. The average value of a full-wave rectified sine wave is

$$X_{\text{ave}} = \frac{2X_{\text{max}}}{\pi}$$

The average value of a half-wave rectified sinusoid is

$$X_{\text{ave}} = \frac{X_{\text{max}}}{\pi}$$

The root-mean-square, denoted by rms, or effective value of a full-wave rectified sine wave is

$$X_{\text{eff}} = X_{\text{rms}} = \frac{X_{\text{max}}}{\sqrt{2}}$$

The rms or effective value of a half-wave rectified sine wave is

$$X_{\text{eff}} = X_{\text{rms}} = \frac{X_{\text{max}}}{2}$$

In an alternating current (AC) circuit, the capacitive reactance X_C or reactance is

$$X_C = \frac{1}{2\pi fC}$$

where f is the frequency in hertz.

In an AC circuit, the inductive reactance X_L or impedance is

$$X_L = 2\pi fL$$

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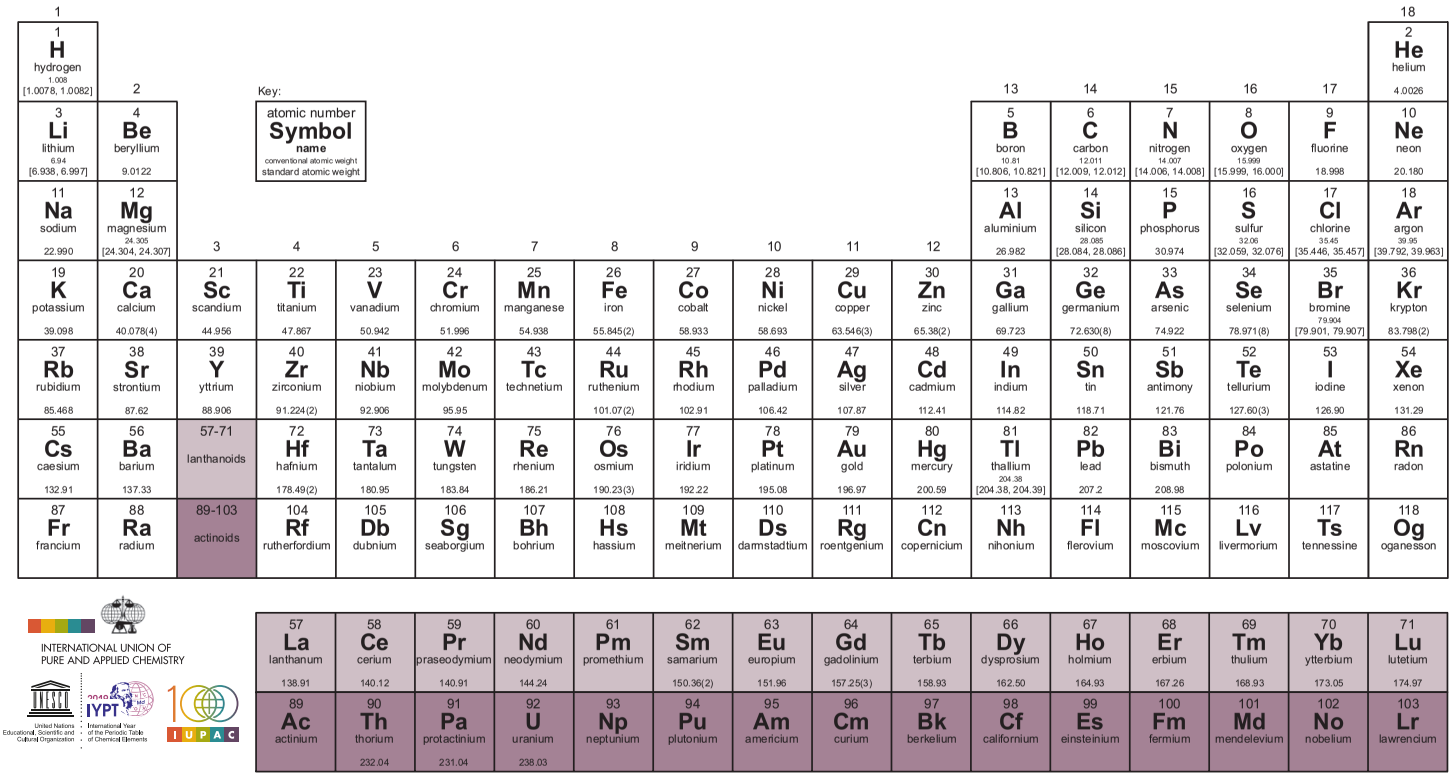

For notes and updates to this table, see www.iupac.org. This version is dated 1 December 2018.
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FIGURE 4.45 Periodic table

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Real power in an AC system is

$$P = \frac{1}{2} V_{max} I_{max} \cos\theta = V_{rms} I_{rms} \cos\theta$$

where θ is the angle measured from the voltage to the current sinusoids.

The lagging or leading power factor is

$$pf = \cos\theta$$

depending on whether the current *I* leads or lags the voltage *V*.

For resistors, $\theta = 0$, and real power is

$$P = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

CHEMISTRY

Figure 4.45 is the periodic table of elements, showing atomic number, electron configuration, and recurring chemical properties of all the known chemical elements.

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CHAPTER

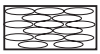
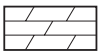
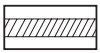
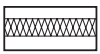
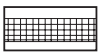
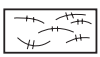
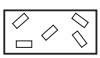
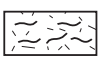
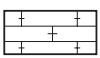
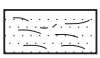
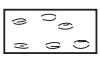
5

Geology and Geophysics

William K. Smith, P.E.

GEOLOGIC MAP SYMBOLS

Figure 5.1 shows commonly used symbols on geologic maps.

Lithological Symbols			
Breccias		Chert	
Conglomerate		Dolomite	
Sandstone, bedded		Gypsum	
Sandstone, calcareous		Ahydrite	
Sandstone, crossbedded		Salt	
Sandstone, shaly		Volcanics	
Siltstone		Tuffs, breccias	
Mudstone-claystone		Flows, basic	
Shale		Flows, other	
Coal		Granite	
Calcareous shale		Porphyritic	
Limestone, cherty		Schists	
Limestone, bedded		Gneiss	
Limestone, massive		Marble	
Limestone, sandy		Quartzite	
Limestone, shaly		Serpentine	

(continues)

FIGURE 5.1 Geologic map symbols

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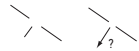
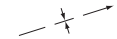
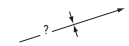
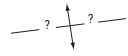
Geological Symbols*			
Strike and dip of bedding plane		Anticline overturned, limb dips, axial plunge	
Uncertain		Syncline axis and plunge	
Generalized		Syncline inferred	
Horizontal bedding		Syncline uncertain	
Vertical bedding		Syncline concealed	
Overturned bedding		Syncline overturned, limb dips	
Strike, dip, lineation plunge		Dome	
Contact		Lineation plunge	
Contact with dip and lithology		Lineation horizontal	
Contact inferred		Lineation plunge and bedding, magnitude unspecified	
Contact uncertain		Vertical beds	
Contact vertical		Vertical beds, horizontal lineation	
Contact concealed		Foliation showing dip	
Anticline axis and plunge		Foliation vertical	
Anticline uncertain		Foliation horizontal	
Anticline concealed		Foliation, horizontal lineation	
Cleavage		Thrust fault, teeth on upthrown rock	
Cleavage vertical		Fault, relative displacement	

(continues)

FIGURE 5.1 Geologic map symbols (continued)

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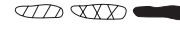
Geological Symbols (continued)*			
Cleavage horizontal		Fault, reverse	
Fault and dip		Fault zone	
Fault vertical		Joint with dip	
Fault inferred		Joint vertical	
Fault uncertain		Joint horizontal	
Fault concealed		Vein	
Fault, sense, amount of displacement, dip known		Vein attitude	
Fault, ball or hachure on downthrown block		Mineralization, ore bodies showing increasing intensity	

Sedimentological Symbols			
Laminae, plane parallel		Underclay slickenside	
Laminae, curved parallel		Clasts, nodules	
Laminae, plane nonparallel		Organic material	
Laminae, curved nonparallel		Marine fossils	
Laminae, wavy parallel		Plant fossils	
Laminae, wavy nonparallel		Brackish fossils	
Large cross-stratification		Pyrite	
Ripples		Load cast	
Flaser bedding		Flute cast	
Bioturbated		Scoured surface	
Rooted		Lag	
Claystone (seat rock) underclay			

(continues)

FIGURE 5.1 Geologic map symbols (continued)

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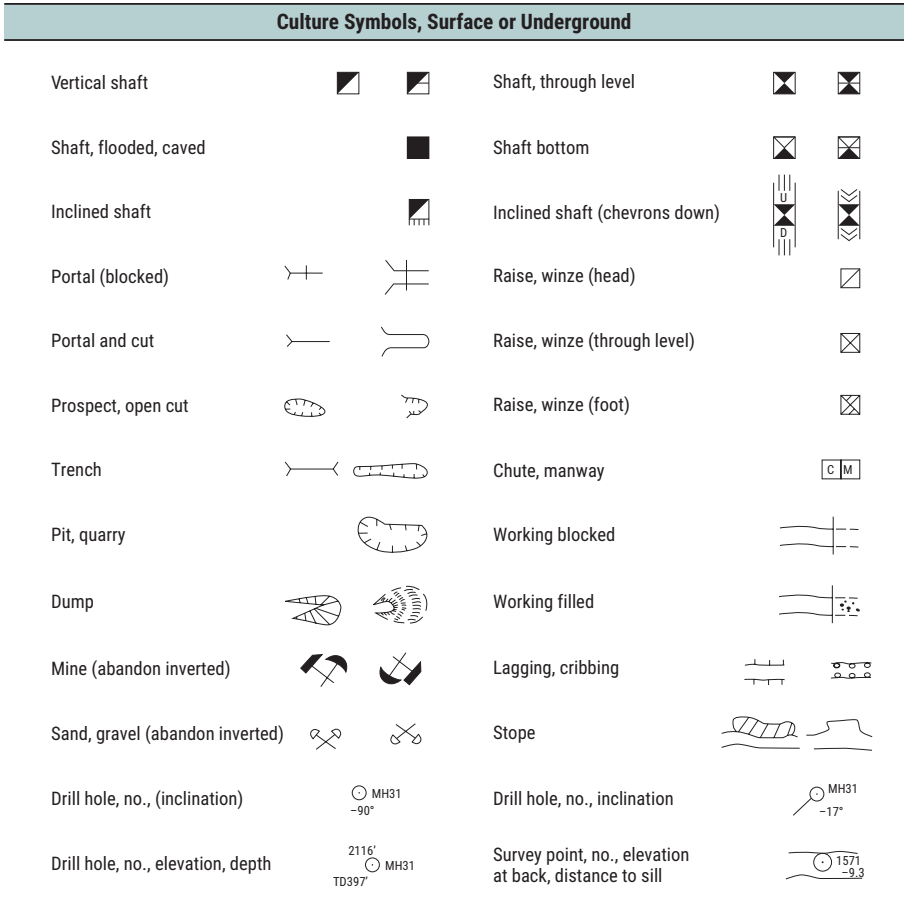

*Bedding dips show options with and without double arrowheads. The arrowhead prevents map confusion if the blocky limestone symbol is used. It is advisable to use a half arrow on veins and faults as it allows clear symbol identification and shows relationships where considerable detail is involved. Source: Erickson and Padgett 2011

FIGURE 5.1 Geologic map symbols (continued)

GEOLOGIC TIME

The following chart (Figure 5.2) shows the principal chronostratigraphic and geochronologic units. The relative duration of the time periods are depicted by the box heights; thus "different scaling factors are used for the Phanerozoic column on the left than for the column on the right, which represents a longer time period. Map symbols are in parentheses" (USGS 2018).

ROCK CLASSIFICATION

Tables 5.1, 5.2, and 5.3 show the classification of igneous, sedimentary, and metamorphic rocks, respectively.

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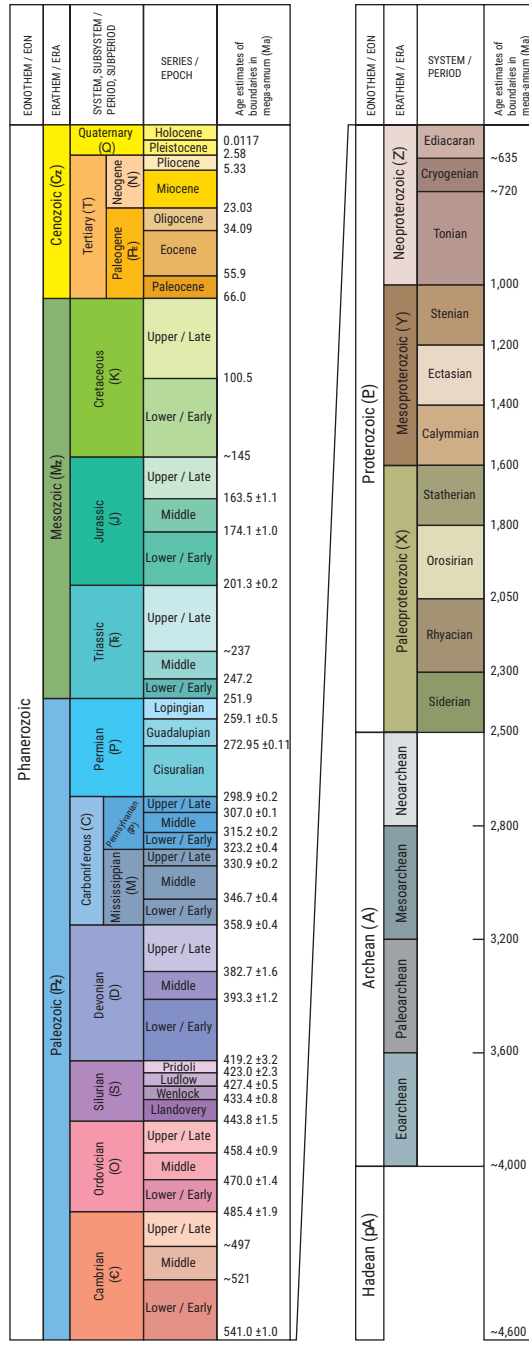

Source: USGS 2018

FIGURE 5.2 Geologic time divisions

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TABLE 5.1 Classification of igneous rocks

Color		Light				Dark					Special Types
						Plagioclase > 2/3 Total Feldspar					
Essential Minerals	Feldspar	K-Feldspar > 2/3 Total Feldspar		K-Feldspar 1/3–2/3 Total Feldspar		K-Feldspar > 10% Total Feldspar	Sodic Plagioclase		Calcic Plagioclase	Chiefly Pyroxene and/or Olivine	
	Quartz	>10%	<10%	>10%	<10%	>10%		<10%			
Chief Accessory Minerals		Hornblende Biotite Muscovite		Hornblende Biotite Pyroxene		Hornblende Biotite Pyroxene			Pyroxene Olivine	Serpentine Iron ore	Pegmatite—very coarse-grained, normally silicic dike rock, crystals 1 cm–1 m in size
Phaneritic >0.1 mm	Equigranular	Granite	Syenite	Quartz Monzonite	Monzonite	Granodiorite	Quartz Diorite	Diorite	Gabbro	Peridotite	
Porphyritic	Phaneritic Groundmass	Granite Porphyry	Syenite Porphyry	Quartz Monzonite Porphyry	Monzonite Porphyry	Granodiorite Porphyry	Quartz Diorite Porphyry	Diorite Porphyry	Gabbro Porphyry	Peridotite Porphyry	Aplite—fine-grained, sugary texture
	Aphanitic Groundmass	Rhyolite Porphyry	Trachyte Porphyry	Quartz Latite Porphyry	Latite Porphyry	Dacite Porphyry		Andesite Porphyry	Basalt Porphyry	(Rare)	
Aphanitic <0.1 mm	Microcrystalline	Rhyolite	Trachyte	Quartz Latite	Latite	Dacite		Andesite	Basalt		
	Glassy	Obsidian—dark colored Pitchstone—resinous Vitrophyre—porphyritic Perlite—concentric fractures Pumice—light colored, finely vesicular, floats in water Scoria—dark colored, coarsely vesicular									

Adapted from Travis 1955 and U.S. Bureau of Reclamation 1998

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TABLE 5.2 Classification of sedimentary rocks

Grain Size		<4 μm	4 μm–2 mm						>2 mm	
Texture		Crystalline, Clastic, or Amorphous	Crystalline, Clastic, Bioclastic, Oolitic, etc.		Clastic Size Grades: 4–62.5 μm: Silt; 62.5–125 μm: Very Fine Sand; 125–250 μm: Fine Sand; 0.25–0.5 mm: Medium Sand; 0.5–1.0 mm: Coarse Sand; 1–2 mm: Very Coarse Sand				Clastic 2–4 mm: Granules; 4–64 mm: Pebbles; 64–256 mm: Cobbles; >256 mm: Boulders	
Composition of Major Fraction		Clay Minerals or Clay-Size Materials (<4 μm)	Composition as Indicated in Left Column for Minor Fraction	Chiefly Calcite or Dolomite	Chiefly Quartz			Quartz with >25% Feldspar	Quartz, Feldspar, and Rock Fragments	Volcanic Ejecta	
					>90% Quartz	10%–25% Feldspar	>10% Rock Fragments				
Composition of Minor Fraction	<10% Minor Fraction			Limestone Dolomite	Quartz Sandstone	Feldspathic Sandstone	Lithic Sandstone	Arkose	Graywacke	Ash—Unconsolidated Fragments <4 mm Tuff—Consolidated Ash Volcanic Breccia—Angular Fragments >4 mm Agglomerate—>25% Volcanic Bombs	Sedimentary rocks with predominant grain size greater than 2 mm are generally classified as *Conglomerate*, if the fragments are rounded, or *breccia* if the fragments are angular. Attach appropriate modifiers, for example, Quartz Cobble Conglomerate or Limestone Pebble Breccia. Special types are Fanglomerate, indurated alluvial fan deposits, and Tillite, indurated glacial till.
	Clay Minerals or Clay-Size Materials (<4 μm)	Claystone, Siltstone, Mudstone — Non-Fissile; Shale — may include Much Silt Bentonite — Sodium Montmorillonite, Swells and Disaggregates in Water		Argillaceous Limestone	Argillaceous Quartz Sandstone	Argillaceous Feldspathic Sandstone	Argillaceous Lithic Sandstone	Argillaceous Arkose	Argillaceous Graywacke		
	Silica Opal Chalcedony Quartz Chert	Siliceous Shale Siliceous Claystone	Chert Diatomite Radiolarite	Siliceous Limestone Cherty Limestone	Orthoquartzite (Siliceous Quartz Sandstone)	Feldspathic Orthoquartzite	Siliceous Lithic Sandstone	Siliceous Arkose	Siliceous Graywacke		
	Calcite or Dolomite	Calcareous Shale Marlstone	Limestone Dolomite		Calcareous Quartz Sandstone	Calcareous Feldspathic Sandstone	Calcareous Lithic Sandstone	Calcareous Arkose	Calcareous Graywacke		
	Carbon	Coal Bituminous Anthracite	Carbonaceous Shale Oil Shale	Add appropriate modifier to rock names in vertical columns above, e.g., Carbonaceous Limestone or Bituminous Quartz Sandstone. Humus yields carbonaceous derivatives; sapropel yields bituminous derivatives.							
	Misc. Phosphate Evaporites Halite Anhydrite Gypsum	Phosphorite Rock Salt Anhydrite Gypsum	Phospahatic Shale, etc.	Add appropriate modifier to rock names in vertical columns above, e.g., Phosphatic Limestone.							

Adapted from Travis 1955 and U.S. Bureau of Reclamation 1998

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TABLE 5.3 Classification of metamorphic rocks*

Color	Chief Minerals	Accessory Minerals	Nondirectional Structure		Directional Structure (Lineate or Foliate)					
			Contact Metamorphism		Mechanical Metamorphism	Regional Metamorphism				Plutonic Metamorphism
						Highly Foliated		Less Foliated		
			Fine	Fine to Coarse	Cataclastic	Slaty	Phyllitic	Schistose	Gneissose	Migmatitic
						Alphanitic	Fine	Fine to Coarse		
Lighter	Feldspar	Actinolite Albite Andalusite Anthophyllite Biotite Chiastolite Chlorite Cordierite Diopside Enstatite Epidote Garnet Gluacophane Graphite Kyanite Muscovite Olivine Phlogopite Pyrophyllite Scapolite Sericite Serpentine Sillimanite Staurolite Tourmaline Tremolite Wollastonite	Hornfels	Metaquartzite	Formed by crushing and shearing with only minor recrystallization Cataclasite—nondirectional, fine-grained Crush breccia—nondirectional, coarse-grained Mylonite—finely ground, foliate Flaser granite, Flaser diorite, Flaser sandstone, etc. Flaser structure—lenses or layers of original or relatively unaltered granular minerals surrounded by matrix of highly sheared and crushed material Augen gneiss—Augen structure	Slate—Most slates are dark colored	Phyllite—Intermediate between slate and schist Recrystallization of micaceous minerals gives a sheen to the rock, but grains are too small for megascopic determination	Schist—Finely foliated because of parallel orientation of phaneritic flaky or lamellar minerals such as mica	Gneiss—Coarsely banded, alternating schistose and granulose layers Augen gneiss—Augen structure—lensoid grains in finer grained groundmass	Migmatite—These rocks have a gneissose, streaked, or irregular structure produced by intimate mixing of metamorphic and magmatic materials. They may originate by injection or by differential fusion. Many migmatites probably originate by particle granitization or by metamorphic differentiation. Migmatites are named by prefixing the rock name of the granitic material to the appropriate root, e.g., *granite migmatite*, *monzonite injection migmatite*.
	Quartz									
	Mica			Amphibolite						
Darker	Hornblende									
	Chlorite									
	Actinolite									
Lighter	Tremolite									
	Talc			Soapstone						
	Calcite or Dolomite			Marble						
	Calcsilicates			Skarn						
Darker	Serpentine			Serpentinite				Serpentinite		

* Naming a metamorphic rock mainly consists of prefixing the appropriate structural term with mineral names or an appropriate rock name. The rock name indicates either the original rock, if recognizable, or the new mineral composition. The prefix "meta," as in *metagabbro*, *metasandstone*, *metatuff*, is applied to rocks that have undergone considerable recrystallization but have largely retained their original fabric. Most of the minerals listed as accessories are genetically important and if present should be included in the rock name regardless of their quantity.

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Mohs Hardness Scale

Following is the Mohs scale of mineral hardness, delineating the scratch resistance of various minerals. Minerals are listed from softest to hardest.

Minerals

1. Talc
2. Gypsum
3. Calcite
4. Fluorite
5. Apatite
6. Orthoclase
7. Quartz
8. Topaz
9. Corundum
10. Diamond

Common Materials

- Fingernail—2.5
- U.S. bronze cent (pre-1982)—3.0
- Window glass—5.5
- Knife blade—6.0
- Hardened steel file—6.5

STRUCTURAL GEOLOGY

The following section includes fault classification and terminology and apparent dip, as well as equations for determining the strike and dip of a plane, the intersection of two planes, and the strike and dip from two apparent dips.

Fault Classification and Terminology

- Dip is the inclination of a plane, measured from the horizontal. Dip direction is always perpendicular to the strike.
- Dip slip is a component of fault motion in the direction of dip, measured in the plane of the fault (*cb* in Figure 5.3C).
- Footwall is the lower block of an inclined fault.
- Hade is the complement of the dip angle (obsolete term).
- Hanging wall is the upper block of an inclined fault.
- Heave is the horizontal component of the dip slip, measured in a vertical section perpendicular to the strike.
- Net slip is the total displacement on a fault, measured in the plane of the fault (*ab* in Figures 5.3C and 5.3E).
- Normal fault is when the hanging wall moves down relative to the footwall. It is also called a *gravity fault* (indicates extension of the earth's crust; see Figure 5.3A).
- Rake is the angle between a line in a plane and a horizontal line in that plane. Rake is sometimes called *pitch* (angle *abc* in Figure 5.3E is the rake of the net slip).
- Reverse fault is when the hanging wall moves up relative to the footwall. A reverse fault is also (usually) called a *thrust fault*, especially if the dip is less than 45 degrees (indicates shortening of the earth's crust; see Figure 5.3D).
- Right lateral and left lateral are descriptions applied to the strike-slip component of the fault movement; right and left refer to the apparent movement of the block opposite the observer (Figure 5.3B is left lateral).
- Strike is the bearing of a horizontal line in a plane.
- Strike slip is the horizontal component of fault movement (*ac* in Figure 5.3C).
- Strike-slip fault is when blocks move horizontally with respect to each other with little or no vertical component of movement (see Figure 5.3B).
- Throw is the vertical component of the net slip and dip slip, measured in the vertical section perpendicular to the strike.

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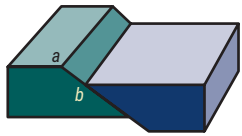
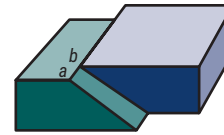
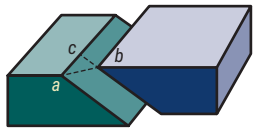


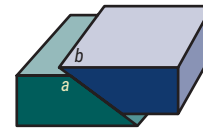
A. *ab* = net slip = dip slip; strike slip is zero



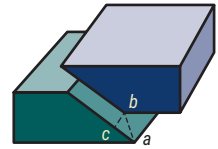
B. *ab* = net slip = strike slip; dip slip is zero



C. *ab* = net slip; *cb* = dip slip; *ac* = strike slip



D. *ab* = net slip = dip slip; strike slip is zero



E. *ab* = net slip, *bc* = strike slip, *ac* = dip slip

Adapted from Billings 1954

FIGURE 5.3 Fault terminology: net slip, dip slip, and strike slip

Apparent Dip

In a vertical cross-section drawn at other than a right angle to the strike of a planar feature such as a stratigraphic bed, fault, or vein, the apparent dip in the line of section will be less than the true dip of the plane. The apparent dip is given by

$$\tan \alpha = \tan \delta \sin \theta \qquad \textbf{(EQ 5.1)}$$

where

α = apparent dip
δ = true dip
θ = angle between strike of plane and line of section

NOTE: Equation 5.1 does not apply to projection of linear features such as fault intersections or boreholes, in which the apparent inclination is greater than the true inclination.

Determining the Strike and Dip of a Plane (Three-Point Problem)

To determine the strike and dip of a plane given the location and elevation of three noncolinear points on the plane, proceed as follows (Figure 5.4):

1. Select a suitable scale and plot a plan view of the three points (*A*, *B*, *C*). Label the points with their elevations.
2. Connect the high and the low points with a straight line (*AC*).
3. Compute the proportion $\Delta h_{AB} / \Delta h_{AC}$ or $\Delta h_{BC} / \Delta h_{AC}$, where Δh is the elevation difference between the points indicated by the subscripts.
4. Lay off the proportionate distance of *AC* calculated in step 3 from the appropriate end (*A* or *C*) to locate point *D*. Line *BD* is the strike of the plane.

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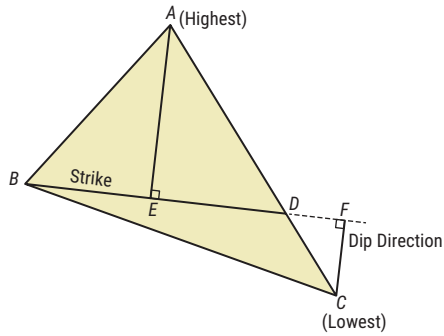

FIGURE 5.4 Determining strike and dip of a plane

5. To determine the dip, draw a line *AE* or *CF* perpendicular to *BD*. Scale the horizontal distance. Using the elevation difference Δh_{AB} or Δh_{BC}, where δ is the angle of dip, calculate the dip from the following equation. The direction of dip can be determined by inspection.

$$\tan \delta = \Delta h_{AB}/AE = \Delta h_{BC}/CF \quad \textbf{(EQ 5.2)}$$

Determining the Intersection of Two Planes

To determine the bearing and plunge of the intersection of two planes given their strikes and dips, proceed as follows (Figure 5.5):

1. Plot the strikes of the two planes so that they intersect at a convenient point (*A*).
2. Select a suitable scale and a suitable elevation drop Δh (e.g., 100 ft or 100 m) for constructing structure contours on each plane.
3. For each plane, calculate the horizontal distance x corresponding to the selected elevation drop Δh , where δ is the dip of the respective plane, from

$$x = \Delta h/\tan \delta \quad \textbf{(EQ 5.3)}$$

4. Plot lines *DE* and *FG* parallel to *AB* and *AC*, respectively, at the respective horizontal distances x in the down-dip directions. Label the intersection of *DE* and *FG* as point *H*. Line *AH* gives the bearing of the line of intersection of the two planes.
5. Scale the horizontal distance *AH*. Calculate the plunge ρ of the intersection from

$$\tan \rho = \Delta h/AH \quad \textbf{(EQ 5.4)}$$

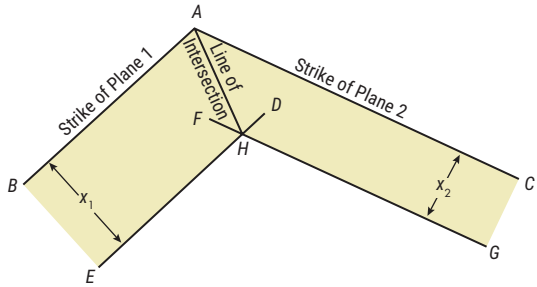

FIGURE 5.5 Determining the intersection of two planes, graphical method

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If a stereonet is available, the bearing and plunge can be determined with a quicker method. Equal-angle (Wulff) nets are preferred to equal-area nets for this purpose. Proceed as follows (Figure 5.6):

1. Plot the strikes and lower hemisphere traces of the two planes on tracing paper laid over the stereonet. The vector *OA* from the center of the net to the intersection of the lower hemisphere traces represents the intersection of the two planes.
2. Rotate the paper to align the north indexes and read the bearing on the outer circle of the net.
3. Rotate the paper so that *OA* lies on the east–west line of the stereonet. Determine the plunge by counting the meridian lines inward from the outer circle of the net to point *A*.

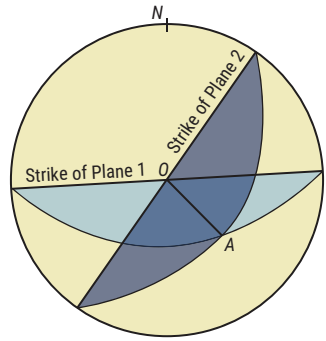

FIGURE 5.6 Determining the intersection of two planes, stereonet method

Determination of Strike and Dip from Two Apparent Dips

To determine the strike and dip of a plane given apparent dips in two directions, proceed as follows (Figure 5.7):

1. Plot the apparent dip directions intersecting at point *O*.
2. Select a suitable scale and elevation drop Δh, such as 100 ft or 100 m.
3. Calculate and plot horizontal distances *OA* and *OB* , where α is the respective apparent dip, using

$$x = \Delta h / \tan \alpha \qquad \text{(EQ 5.5)}$$

4. Line *AB* is the strike of the plane.

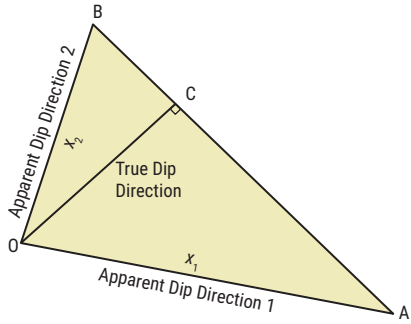

FIGURE 5.7 Strike and dip from two apparent dips, graphical method

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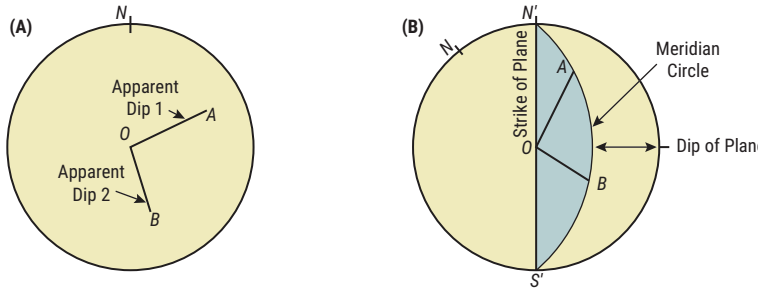

FIGURE 5.8 Strike and dip from two apparent dips, stereonet method

5. Plot line *OC* perpendicular to *AB* and scale horizontal distance *OC*. Calculate the dip, where δ is the dip of the plane, from the following equation. The directed line *OC* is the dip direction.

$$\tan \delta = \Delta h / OC \qquad \text{(EQ 5.6)}$$

The resolution of strike and dip from two apparent dips can also be solved using a stereonet, as follows (Figure 5.8):

1. Plot the apparent dip vectors *OA* and *OB* on the tracing paper from the center of the net.
2. Rotate the tracing paper about *O* until points *A* and *B* lie on the same meridian line on the net.
3. Plot the strike line of the plane through the north and south poles of the net and trace the meridian line *N-A-B-S*.
4. Measure the dip of the plane by counting from the outer circle of the net to the trace of the plane.
5. Rotate the tracing paper about *O* to align the north index on the paper with the north pole of the net.
6. Read the strike of the plane on the outer circle of the net.

GEOPHYSICS

The following section includes plate tectonics, geophysical exploration methods, and the interpretation of seismic refraction surveys.

Plate Tectonics

Figure 5.9 shows a generalized map of the Earth's major tectonic plates. Arrows indicate the approximate direction of spreading for these plates, standardly measured in centimeters per year.

Geophysical Exploration Methods

Ground-based, aerial, and borehole methods have all been used in mineral exploration (Sumner 1992; Erickson 1992).

Ground-Based Methods

See Sumner (1992).

- Gravity
- Magnetic
- Electrical
  - Telluric
  - Magnetotelluric
  - Self-potential
  - Applied potential
  - Resistivity
  - Induced-polarization
  - Electromagnetic
- Seismic
  - Reflection seismology
  - Refraction seismology
- Radiometric

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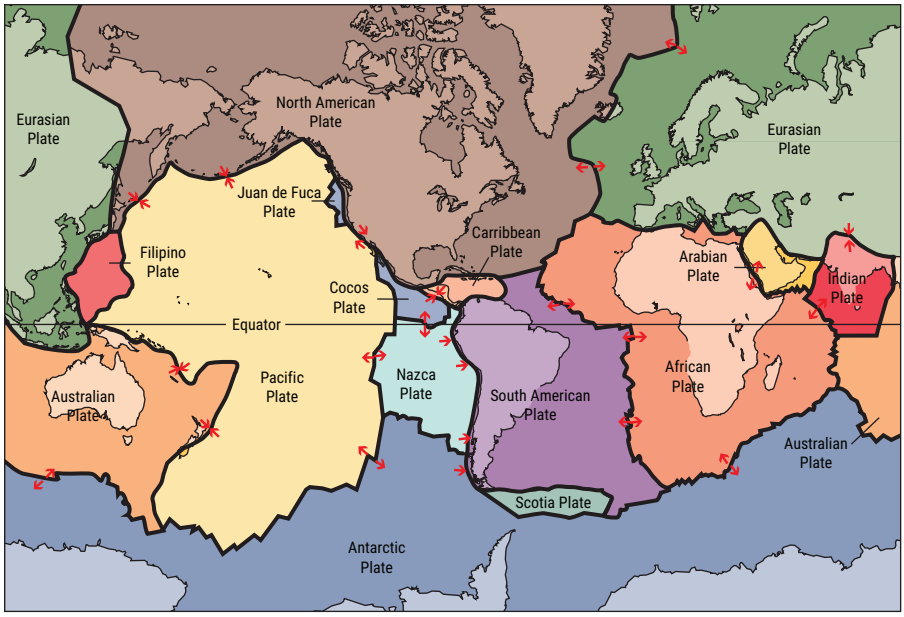

Courtesy of U.S. Geological Survey

FIGURE 5.9 Plate tectonic maps of the world

Aerial Methods

See Sumner (1992).

- Aeromagnetic
- Radiometric

Borehole Methods

See Erickson (1992).

- Spontaneous-potential
- Natural gamma
- Bulk density (gamma-gamma)
- Neutron density
- Sonic velocity
- Resistivity

Gravity surveys can be used to locate high-density ores, such as hematite and barite, or low-density features, such as buried stream channels. They may also be used to locate shallow buried structures such as the edges of pediments under alluvium (Sumner 1992). Detailed gravity surveys require stations to be located to within 30 m (~100 ft) horizontally and elevations determined to within 30 mm (~0.1 ft) (Dobrin 1976). Horizontal positions can be readily determined with handheld GPS instruments, but elevations must still be determined by traditional surveying methods.

The various electrical methods use either natural electrical currents within the earth or artificially generated currents. Induced polarization surveys have been highly effective in exploration for disseminated sulfide deposits. Radiometric surveys, however, can be either ground-based or aircraft-based and are used for exploration for uranium. And seismic methods are only occasionally applicable to mineral exploration problems. Seismic refraction can be used to aid interpretation of other methods by determining the depth to bedrock (Sumner 1992). Finally, borehole geophysical logging is not commonly used in hard-rock mineral exploration, but is essential in coal and uranium resource evaluation (Erickson, 1992).

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Interpretation of Seismic Refraction Surveys

On a plot of arrival times versus distance from the shot point (Figure 5.10), the seismic velocity of successive layers is the inverse slope of the time–distance plot (slope = 1/V). For the two-horizontal-layer case, the depth to the interface is given by the following equation (Dobrin 1976):

$$z = \frac{x_c}{2}\sqrt{\frac{V_1 - V_0}{V_1 + V_0}} \qquad \text{(EQ 5.7)}$$

where

z = depth to interface
V_0 = seismic velocity of upper layer
V_1 = seismic velocity of lower layer
x_c = critical distance (the distance to the break in slope)

NOTE: A low-velocity layer underlying a higher velocity layer will not be detected by the seismic refraction method and, if present, will introduce errors into depth calculations of deeper layers.

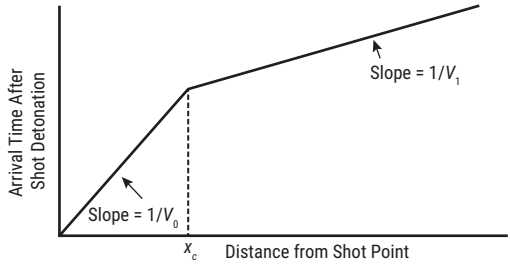

FIGURE 5.10 Idealized time–distance plot from seismic refraction survey

REMOTE SENSING IMAGERY

Aerial photography and remote sensing imagery are used in mineral exploration at a range of scales (local or regional). Although the U.S. Geological Survey no longer supplies paper copies or film transparencies of its aerial photos, high-resolution digital scans can be downloaded, many at no cost. Detailed color satellite imagery of most of the world is available for free on the internet. The advent of drones has made collection of aerial photography more accessible.

The four basic types of film used for aerial photography are black and white panchromatic (BW), black and white infrared (BIR), natural color (C), and color infrared (CIR). BW film is sensitive to light, approximately in the range of human vision. BIR film extends the range to the near infrared; water features and wet areas show up as a darker black than with BW. Natural color is probably the best choice for mineral exploration work. Although more expensive than BW, the added cost of color is usually a small percentage of the overall cost of a photo mission. CIR gives a false-color image of the area. Vegetation shows up as shades of red, healthier vegetation being more intense red. This could be useful if differences in the health of vegetation reflect differences in the soil chemistry or underlying geologic conditions.

The scale of a vertical photograph from the air is given by

$$S = f/H \qquad \text{(EQ 5.8)}$$

where

S = the scale of the air photograph at a given elevation
f = the focal length of the aerial camera
H = the flying height above the given elevation

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Some remote sensing techniques are generally of lower resolution than traditional photography. The following are such techniques:

- Multispectral scanner (MSS) and thematic mapper (TM) images from Landsat satellites are used to map regional structural, lithologic, and alteration features (Lee-Moreno 2011).
- Side-looking airborne radar (SLAR) imagery penetrates vegetation and several to many meters of unconsolidated materials to show structural features of the bedrock and buried alluvial channels (Sumner 1992).
- Lidar (from light detection and ranging) collects three-dimensional (3-D) point clouds of the Earth's surface. This technology is being used for a wide range of applications including high-resolution topographic mapping and 3-D surface modeling (USGS, n.d.).
- Photogrammetry uses multiple photographs to recover exact locations of surface points. The only equipment necessary to conduct this analysis is a digital camera (preferably one with a fixed focal length lens) and a GPS station. Photogrammetry is being used for many applications because it is relatively low cost. The use of photogrammetry methods in conjunction with drones (with onboard GPS capability) allows low-cost analysis of spatial data in a wide variety of environments and can specifically be useful for otherwise inaccessible or dangerous locations. Some examples for use in mining applications include mapping out underground tunnel fractures, stockpile growth monitoring, and surveying pit slope highwalls to estimate creep.

DRILLING

See Chapter 6, which covers sampling and analysis, for information on drilling, including commonly used wireline core drilling sizes and the spacing of drill holes in coalfields.

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CHAPTER

6

Sampling and Analysis

Marcus A. Wiley, P.E.

SAMPLING

Sampling is a technique for obtaining objective, reliable information about a population.

Sampling Theory

Because it is usually impractical to measure the entire population, samples representing the entire population must be selected. The sample contains only the facts that are available on the subject slice of the population. Therefore, any conclusions drawn about the total population are merely inferences and are subject to verification. Conclusions about a population are only as good as the data on which they are based.

Geologic Data Collection and Recording

Table 6.1 shows the key features of geological data collection.

TABLE 6.1 Geologic data collection

Location Data
Sample, map, mine, or drill location on each sheet. May include geographic data such as state, county, section, township, range, latitude, longitude, coordinates, elevation, mining district, mine, pit, bench, level, working, claim, claim corner, or any and all information that will clearly identify the unique location of the geologic data points. Data cannot be used if the geologist does not know where data originated.
Lithologic Data
Typical data to describe rock, sample, or unit. Should include color, texture, mineralogic characteristics, lithology, and rock type. Appropriate descriptive modifiers; stratigraphic information (if known); top and bottom data; age relationships; and general gross features such as hardness, competency, and bedding characteristics. Subjective generic terms should be avoided unless well-established or qualified to distinguish inference from observable facts. Primary sedimentary structure and sedimentological features such as bedding, laminations, casts, soft-sediment deformation, graded bedding, burrows, bioturbation, fossil content, or banding, foliation, and lineation (with appropriate attitudes) should be noted where possible. When describing important lithology information, best practice is to start with the most important features (e.g., rock type) and end with the less important ones.
Structural Data
Secondary structural features that post-date rock formation. Should include clear description and attitudes of joints; fractures; faults; and breccias with quantitative description of selvages, gouge zones, fragment size, and healed or recemented character of breccias. Folds, drag folds, crenulations, lineations, and foliation should be noted. Age relationships, mineralization association, and overall effect on rock mass are important. Weathering and oxidation intensity data are usually critical and commonly related to structure but may be included with lithologic data. Where possible, quantification of structural data is extremely important because these data can play a key role in determining minability of a deposit.
Alteration Data
Nature, mineralogy, intensity, and distribution of features. Should include color, texture, mineralogy, intensity, pervasiveness, fracture control, stages, mineralization association, and overall effect on rock mass. Weathering and oxidation intensity are important but may be included with lithologic data. Where possible, quantification is extremely useful, as is description of age relationships between various alteration features.
Mineralization Data
Nature, intensity, mineralogy, and distribution of the desired resource. Should include primary and secondary classification; estimates of specific and total quantity of various minerals; intensity; character of veinlet, vein, or disseminations; supergene features; weathering and oxidation intensity; and associated gangue mineralogy. Vein age relationships tied to mineralogy, alteration, or lithology offer important data for understanding both zoning and grade estimates and overall deposit genesis.

(continues)

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TABLE 6.1 Geologic data collection (continued)

Coal Data
Logging of any and all features that aid in correlation and understanding the distribution of sedimentary facies, as well as construction of a depositional model of the coalbed(s) and coal-bearing sequence (in addition to the preceding standard lithological and structural data). Critical to include detailed descriptions of horizons immediately above and below the roof and floor and accurate measurements of depths and thickness of all units associated with the coal. Key features include slickenside in roof or floor rocks, abundance and type of marine or freshwater fossils, the presence of roots representing old soil horizons, pyrite bands, nodules or streaks, siderite or ironstone nodules, and plant debris. Description of individual coalbeds, either the banded or nonbanded groups, requires careful measurement or estimates of the banded lithotypes, vitrain, clarain, durain, and fusain content (Stopes 1919; Ward 1984). A more practical system (Schopf 1960) describes the thickness and amount or concentration of vitrain and fusain bands in a matrix of attrital coal. The latter is described by five luster levels that range from bright to dull. Nonbanded sapropelic coals, boghead, and cannel end-members descriptions rely on identification of these massive, faintly banded, fine-grained accumulations of algae or spores and usually require a microscope for adequate description. The nature of cleats, partings, bone, and shale layers needs description and careful thickness measurements to separate *net* from *gross* coalbed thicknesses. Coalbed description, although straightforward, requires some supervised training to ensure adequate data recording.
Other Features
Other features that may supply extremely important information with direct bearings on mining and/or metallurgy. This may be reasonably objective, such as fracture frequency, rock quality determination measurements, and longest and shortest unbroken core recovered in a run, or more subjective, such as an overall estimate of rock strength, friability, or competency. Metallurgically significant features such as hardness (which affects grindability), grain size (which controls grinding for particle liberation), or oxidation intensity (which affects flotation recoveries) should be noted. Added testing is almost always needed here; however, geologic data collections should indicate these and other potential problem areas requiring specialized study.

Adapted from Erickson and Padgett 2011

Size, Frequency, or Density of Sample

For a sample to be representative of the whole population, it must be of the proper size and frequency. The larger the sample, and the greater the number of increments taken, the more likely that the sample will be representative of the whole. The following tables offer guidelines to assist in obtaining a representative sample.

Table 6.2 lists equipment that is useful in collecting geologic data. For example, drill-hole size is an important consideration in obtaining a representative sample. A smaller drill-hole diameter can be used if the deposit is expected to be relatively homogeneous. A 12-in. blasthole sample will contain 50 times the amount of sample that would be obtained in a BX-size core hole. Table 6.3 furnishes diameter size data for commonly used wireline drills.

Sufficient material must be obtained and submitted to the testing laboratory to permit proper execution of all desired tests. The quantities given in Table 6.4 generally provide adequate material for routine grading and quality analysis. A rule of thumb for sample size is to obtain a 50-lb sample for each inch diameter of the coarsest particle size. A sample that is too large can be reduced in the laboratory, but a sample that is too small is of no value and represents wasted effort.

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TABLE 6.2 Equipment used for geologic data collection

Equipment routinely used in day-to-day geologic work:

- Compass
- Geologist's pick
- Hand lens
- Pocket tape measure
- Several cloth survey tapes or chains (nonmetallic for safety)
- Knife
- Protractor or 152-mm (6-in.) plastic scales with imprinted protractor
- Dilute hydrochloric acid (HCl)
- Magnet and nail
- Survey spads
- Cement nails
- Sample bags and tags, plus a sample ring for holding sample bags during chip sampling
- Marking pens and aluminum sheet holder (preferably end-opening with an end-extractable pencil holder attached), pencils, pens, mapping sheets, or logging forms
- Camera
- GPS unit
- Mine and topographic maps
- Reference materials such as core guide, grain size chart, and nearby core information

In addition, a field vest (with large pockets for samples) and clipboard are helpful. Access to brushes and water buckets is important for core logging, and spray bottles help. Extra core blocks are useful, as is a good binocular microscope.

Adapted from Erickson 1992

TABLE 6.3 Commonly used wireline core drilling sizes

Company	Designation	Core Diameter, in.	Hole Diameter, in.
Boyles Bros.*	AX	1.067	1.890
	BX	1.432	2.360
	BX		2.400
	BX		2.440
	NX	1.875	2.980
	NX		3.032
	NX		3.125
	NX		3.250
	HX	2.400	3.650
	HX		3.750
	HX		3.937
	CP	3.345	4.827
CBC†	AXE	1.015	1.852
	BXE	1.437	2.385
	NXE	2.000	3.005
	NCE	2.406	3.685
	3-in.	3.000	3.915
Longyear‡	AQ	1.062	1.890
	BQ	1.432	2.360
	NQ	1.875	2.980
	NQ	2.500	3.782
	PQ	3.345	4.827

* C. Hirschi, Boyles Bros., Reno, NV, personal communication.

† N. Trujillo, CBC Drilling, Tucson, AZ, personal communication.

‡ A.O. Krause, Longyear Company, Peoria, AZ, personal communication.

Source: Metz 1992

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TABLE 6.4 Required size of coal samples

Mechanically Cleaned Coal			
Top size	16 mm (⅝ in.)	50 mm (2 in.)	150 mm (6 in.)
Minimum number of increments	15	15	15
Minimum weight per increment	1 kg (2 lb)	3 kg (6 lb)	7 kg (15 lb)
Total sample weight	15 kg (30 lb)	45 kg (90 lb)	105 kg (225 lb)
Raw Uncleaned Coal			
Top size	16 mm (⅝ in.)	50 mm (2 in.)	150 mm (6 in.)
Minimum number of increments	35	35	35
Minimum weight per increment	1 kg (2 lb)	3 kg (6 lb)	7 kg (15 lb)
Total sample weight	35 kg (70 lb)	105 kg (210 lb)	245 kg (525 lb)

Source: ASTM D2234/D2234M-17

TABLE 6.5 Spacing of drill holes in exploration of coalfields

Type of Deposit	Spacing, ft*	
	Measured Reserves	Indicated Reserves
Horizontal or Gently Sloping		
Uniform beds	3,000	6,000
Fairly consistent beds	1,500	3,000
Inconsistent beds	750	1,500
Deposits with Simple First-Order Folding		
Uniform beds	1,000	2,000
Fairly consistent beds	750	1,500
Inconsistent beds	500	1,000
Deposits with Complex Folding and Consequent Faulting		
Uniform beds	750	1,500
Fairly consistent beds	400	750
Inconsistent beds	Explored during mining	—

* Grid dimensions for horizontal deposits; distances between exploratory profiles for folded and complex deposits.

Adapted from Misaqi 1973

For all in situ and non–in situ samples, the top size to which any sample is crushed is important for determining the weight of the sample required. And the spacing of the drill holes or other exploratory openings must be considered if a representative sample is to be obtained. Table 6.5 contains general guidelines for a horizontal deposit such as coal. A deposit with a complex structure would require spacing with a greater density.

The concept of error can be divided into two components that are referred to as precision (or repeatability) and accuracy (or lack of bias). Figure 6.1 illustrates the concepts of precision and accuracy by considering four targets. The shooting results shown on target A are both inaccurate and imprecise. Target B is imprecise but accurate. Target C is precise but inaccurate. And target D is both accurate and precise (Springett 1984).

The quality of a geologic deposit cannot be accurately represented by a single value. To obtain a representative sample, multiple increments must be taken. Three methods can be used: (1) systematic sampling, where increments are spaced evenly in time or position; (2) random sampling, where increments are spaced at random, but a prerequisite number are taken; or (3) stratified random sampling, where the unit is divided by time or amount into a number of equal strata, and one or more increments are taken at random from each (Thomas 1992). Table 6.6 details certain minimum numbers of increments.

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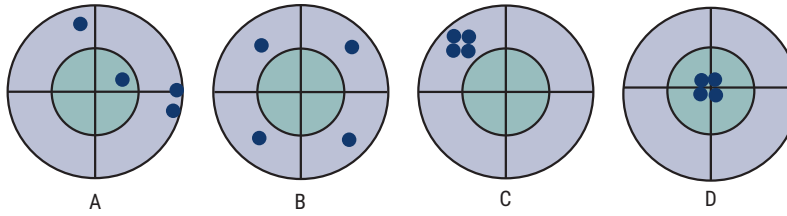

Source: Springett 1984

FIGURE 6.1 Accuracy and precision

IN SITU AND NON–IN SITU SAMPLING

In general, a better sample can be obtained by sampling the deposit or mineral commodity in its natural state (in situ). Channel samples and core drilling are examples of in situ sample techniques. Non–in situ sampling consists of sampling the material after it has been removed from its original deposit. An example of this would be a sample taken from a truck, boat, conveyor belt, or stockpile.

Grab Sampling

Randomly picking up a sample of the material can easily lead to a bias in the selection of the sample and inconclusive results. Grab sampling can, however, be used to obtain generalized information about the material sampled.

In Situ Sampling Methods

Channel, core, cuttings, and bulk sampling methods the in situ sample techniques described in the following paragraphs.

Channel Sampling

Channel sampling is recommended for obtaining a representative sample of a laminated deposit such as coal. Clean the area to be sampled to expose a fresh section. Normally, the material is sampled perpendicular to the bedding. Cut a channel of uniform width and depth into the seam and collect the material on a plastic sheet or metal plate placed on the floor. Figure 6.2 shows a composite method in which the entire cross section is collected as one sample, and Figure 6.3 illustrates a ply method in which each individual layer is sampled and recorded individually to make up the total cross section. A *ply* is a layer of coal seam differing in properties, like color and brightness, from boarding layers. This method allows additional information to be obtained—for example, the top of the seam might be high in sulfur and could be removed in a mining process before the remainder of the seam is recovered.

Core Sampling

Core sampling is the preferred method for obtaining a nonweathered sample of the deposit, along with roof and floor material that could make up seam dilution in mining operations. The sample can be recorded as a whole, or in plies (individual components). Bag, label, and record individual samples.

Cuttings Sampling

This method is considerably less accurate than core sampling. As drilling progresses downhole, samples of bit cuttings are collected over specified intervals. Although this method is better than no information at all, it gives only a generalized representation of the material sampled.

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TABLE 6.6 Minimum numbers of increments required for a coal shipment up to 1,000 metric tons

Sampling Situation	Common Sample for Total Moisture and General Analysis			Total Moisture Sample		General Analysis Sample		Size Analysis Sample
	Sized Coals: Dry Cleaned or Washed	Washed Smalls, 50 mm	Blended, Part Treated, Untreated, Run-of-Mine, and *Unknown* Coals	Sized Coals: Dry Cleaned or Washed; Unwashed Dry Coals	Washed Smalls (50 mm), Blended, Part Treated, Untreated, Run-of-Mine, and *Unknown* Coals	Sized Coals: Dry Cleaned, Washed, or Unwashed Dry Coals	Blended, Part Treated, Untreated, Run-of-Mine, and *Unknown* Coals	All Coals
Moving streams	20	35	35	20	35	20	35	40
Wagons and trucks; barges, grabs, or conveyors unloading ships	25	35	50	20	35	25	50	40
Holds of ships, stockpiles	35	35	65	20	35	35	65	40

Source: Osborne 1988

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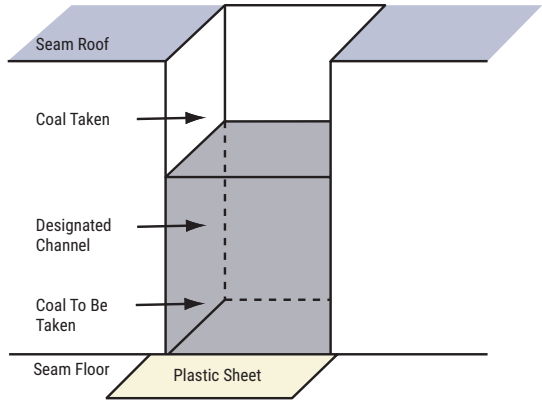

Source: Strauss and Atkinson 1987

FIGURE 6.2 Composite channel sample procedure

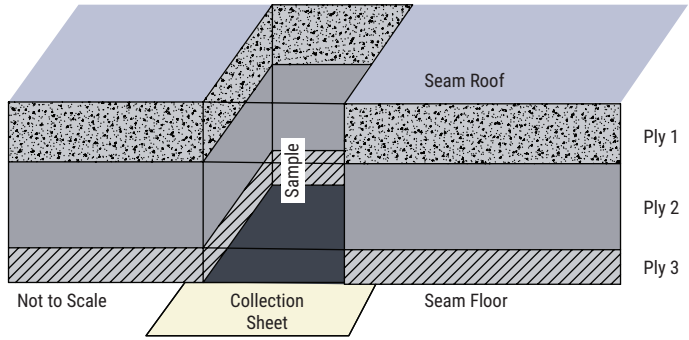

FIGURE 6.3 Ply channel sample procedure

Bulk Sampling

Larger volume samples taken from an outcrop or small pit or shaft are called *bulk samples*. They offer a larger representation of the material to be sampled than is available through a borehole. This method is generally used when size consistency, washability, or variability data are needed.

Non–In Situ Sampling Methods

Collecting samples after the mineral has been mined is required to determine the *as-mined* or *as-shipped* quality. These data will vary from the in situ quality because of mining dilution or processing after mining. The sample taken should be random and should consist of the entire cross section of material. Conduct the sampling in accordance with ASTM guidelines. The following procedures are a quick guide to obtaining accurate samples.

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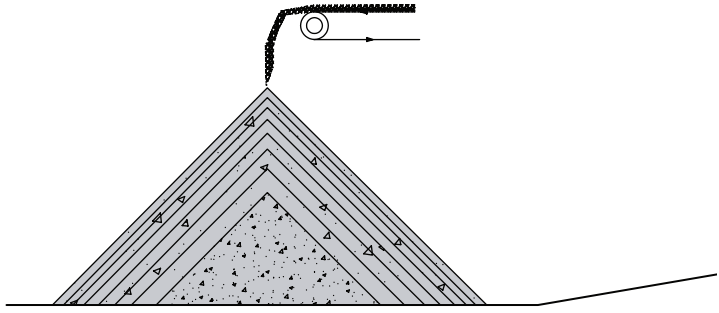

FIGURE 6.4 Stockpile cross section

Stockpile Sample Procedure

Mined minerals are commonly stored in a stockpile. Figure 6.4 illustrates the problem of obtaining a representative sample because of segregation and material crumbling within the pile. Avoid taking stockpile samples if possible.

But if sampling a stockpile is the only method possible, insert a sheet of thin metal or plywood (approximately 2 × 2 ft) corner first, and horizontally to a depth of at least 1 ft into the stockpile and one-third of the aboveground total height of the stockpile. The sheet should be inserted directly above the area to be sampled and at several locations around the stockpile. Obtain the sample from the stockpile directly below the sheet of metal or plywood. Discard material to a depth of approximately 6 in. by removing it with a shovel. Take samples until a sufficient amount of the material is obtained (Johnson and Forst 2000).

Conveyor Belt Sample Procedure

A representative sample can be obtained from a conveyor belt. Take a minimum of three increments and combine them into one field sample. Use a sweep-type sampler if available. Stop the belt and clear the material at least 6 in. ahead of and behind the sample. Carefully scoop all the material between the cleared areas into a sample bag, then collect the fines on the belt with a brush and add them to the sample bag (Johnson and Forst 2000).

Auger Sampling Procedure

Augers are generally used to obtain samples of material from transportation units such as trucks, rail cars, and barges. Take all increments at the full depth of the material, and be careful to exclude any segregation or stratification of the material. Collect a minimum of three secondary crushed increments per auger increment, evenly spaced throughout (Johnson and Forst 2000).

Sample Splitting

To analyze sampled material, it must first be reduced in size. Figure 6.5 is a sample reduction diagram that illustrates requirements for a gold operation.

The best method for reducing the size of a sample is to use a true mechanical splitting device, which is typically available in a laboratory. In the field, however, to reduce the amount of material to send to a laboratory while maintaining representation, use the following procedures.

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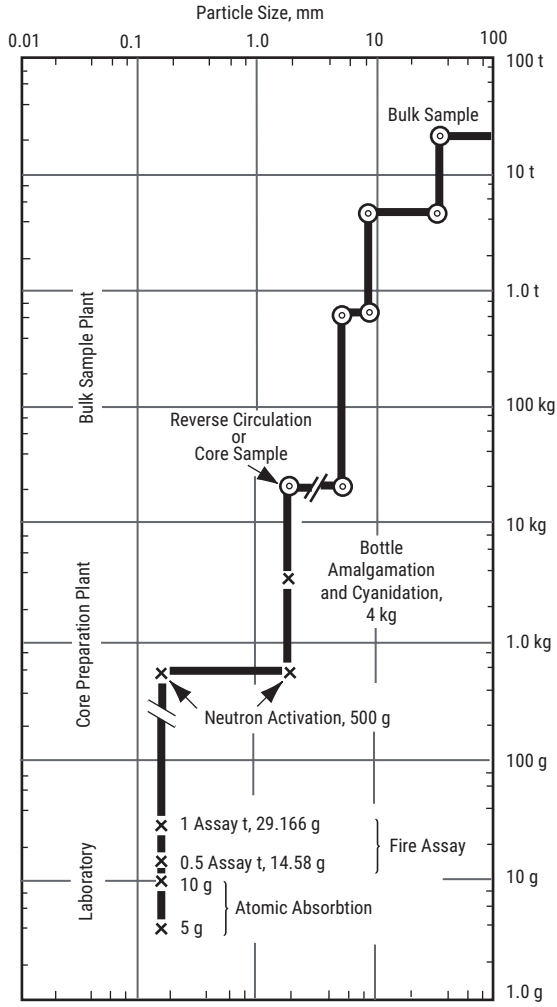

Source: Springett 1984

FIGURE 6.5 Sample reduction

Coning and Quartering

Coning and quartering, illustrated in Figure 6.6, is a very old method. The material to be sampled is mixed and carefully piled on top of a conical heap. Flatten the cone into a flat circular cake that maintains the cone's symmetry, and then divide this cake into four identical quarters. Take care to divide the cake with a 90-degree cross angle so that each section is strictly equivalent, then take the sample from one of the quarters or a combination of two quarters. Although this method is useful for splitting small samples, the alternate shovel method is an easier method for obtaining larger samples.

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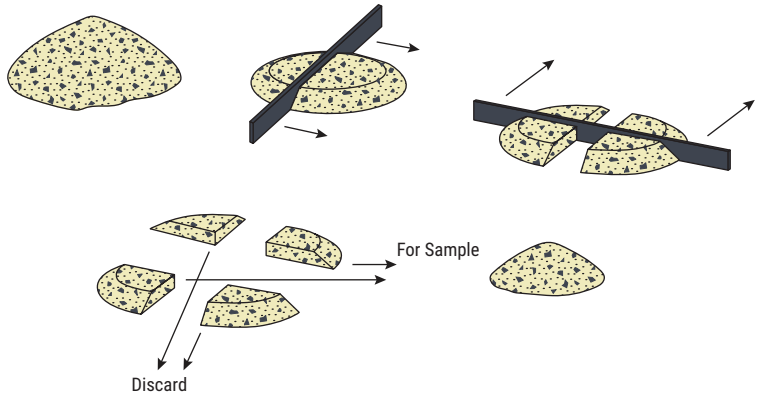

Source: Alakangas 2015

FIGURE 6.6 Coning and quartering

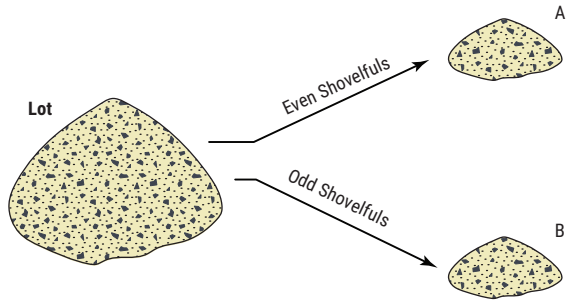

FIGURE 6.7 Alternate shoveling

Alternate Shoveling

Figure 6.7 illustrates the alternate shoveling method. Extract the material to be sampled one shovelful at a time and place it in two alternating distinct heaps. All shovelfuls should be approximately the same size, and each heap should consist of the same number of shovelfuls. One heap should contain only odd increments and the other only even increments. When the sample is selected at random, the sampling equity is preserved in the reduced sample.

Fractional Shoveling

Another method of sampling is shown in Figure 6.8. In this method, place the material to be sampled, one shovelful at a time, in several distinct heaps. All shovelfuls are approximately the same size, and each heap contains the same number of shovel increments. Take a sample at random from one or a combination of the individual heaps. The illustration shows this method with five individual heaps.

Sample Preparation

To maintain the integrity of a sample after it has been obtained in the field, yet to reduce it for analysis in a laboratory, use the procedures that follow. Table 6.7 outlines a procedure for a relatively homogeneous material with small ore particles; Table 6.8 describes a procedure for a nuggety-type ore such as gold.

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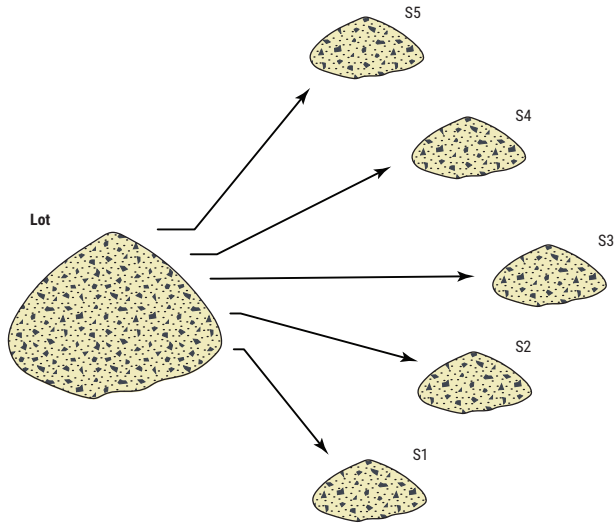

FIGURE 6.8 Fractional shoveling

TABLE 6.7 Representative sample preparation procedure

Dry Sample
Typical weight 1.0–4.5 kg (2–10 lb)
Crush
Typical product 2.4–2.0 mm (8–10 mesh)
Riffle Split
Retain 250 g (½ lb)
Pulverize
Typical product 150–100 µm (100–150 mesh)

Source: Gumble et al. 1992

TABLE 6.8 Sample preparation procedure for nuggety materials

Dry Sample
Typical weight 3.6–9.0 kg (8–20 lb)
Crush
Typical product 2.4–2.0 mm (8–10 mesh)
Riffle Split
Retain 1.8–4.5 kg (4–10 lb)
Plate Pulverize
Product 420 µm (40 mesh) or finer
Riffle Split
Retain 450–900 g (1–2 lb)
Pulverize in Ring Mill
Typical product 150–100 µm (100–150 mesh)

Source: Gumble et al. 1992

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Sampling Error

Table 6.9 contains a formula for calculating minimum sampling error.

TABLE 6.9 Pierre Gy's formula for calculating minimum sampling error

$$\sigma^2 FE = \left(\frac{1}{m_s} - \frac{1}{m_L}\right)\frac{1 - a_l}{a_l}[(1 - a_l)\lambda_c + (a_l + \lambda_g)]/fgd^3$$

where

σ^2FE* = (sampling error in %)^2
m_s = sample weight, g
m_L = total sample weight, g
a_l = mineral content, decimal %
λ_c = specific gravity of mineral, g/cm^3
λ_g = specific gravity of gangue, g/cm^3
l = liberation factor
f = particle shape factor
d = maximum particle size in sample, cm
g = granulometric factor

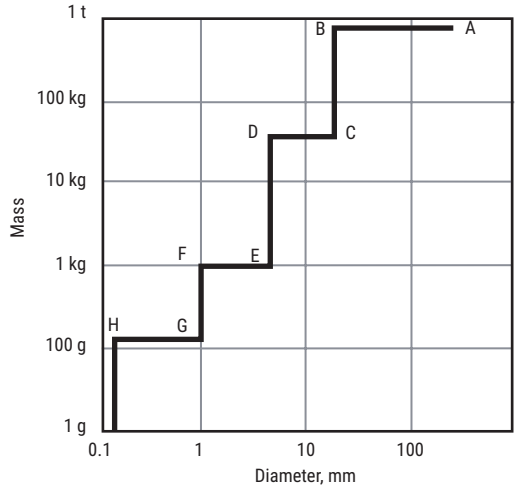
* Total sampling error = $\sqrt{\Sigma\sigma^2 FE}$.

Source: Schwarz et al. 1984

The usual method for reducing a core sample is to split the core and then crush half of it to ¼-in. After this ¼-in. material is blended, take a subsample of 450–900 g (1 or 2 lb), then grind this material to 100 mesh, blend again, and keep a few grams for analysis.

Figure 6.9 is a diagram illustrating sample preparation and reduction. Each point A–H represents a given quantity of material crushed to a given size. Sample preparation is a succession of reductions in weight and grain size. At each vertical line of the stairway, error is involved.

Figure 6.10 shows a sample preparation procedure for reducing a bulk sample of steam coal.



Source: David 1977

FIGURE 6.9 Sample preparation

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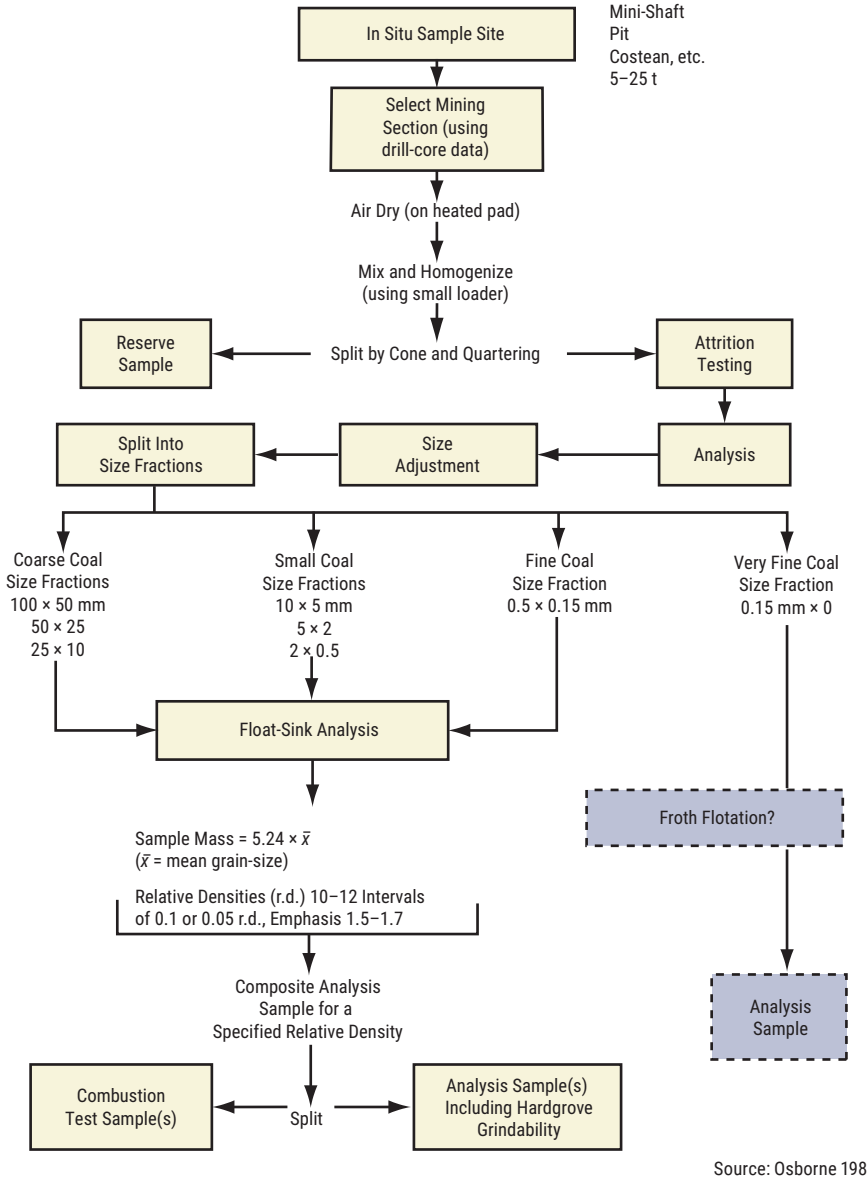
In Situ Sample Site
Mini-Shaft
Pit
Costean, etc.
5–25 t
Select Mining Section (using drill-core data)
Air Dry (on heated pad)
Mix and Homogenize (using small loader)
Reserve Sample
Split by Cone and Quartering
Attrition Testing
Split Into Size Fractions
Size Adjustment
Analysis
Coarse Coal Size Fractions 100 × 50 mm 50 × 25 25 × 10
Small Coal Size Fractions 10 × 5 mm 5 × 2 2 × 0.5
Fine Coal Size Fraction 0.5 × 0.15 mm
Very Fine Coal Size Fraction 0.15 mm × 0
Float-Sink Analysis
Froth Flotation?
Sample Mass = 5.24 × $\bar{x}$
($\bar{x}$ = mean grain-size)
Relative Densities (r.d.) 10–12 Intervals of 0.1 or 0.05 r.d., Emphasis 1.5–1.7
Composite Analysis Sample for a Specified Relative Density
Analysis Sample
Combustion Test Sample(s)
Split
Analysis Sample(s) Including Hardgrove Grindability

Source: Osborne 1988

FIGURE 6.10 Coal sample preparation

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TABLE 6.10 Commonly used analysis methods

Element	Digestion	Analysis by
Au	HCl, HNO_3 Fusion	Atomic absorption (AA) Fire assay
Ag	HCl, HNO_3 Fusion	AA Fire assay
Hg	HNO_3 Thermal	AA cold vapor Au film
As	$HClO_4$ + other various acids	AA hydride AA furnace
Sb	Various acids Fusion	AA hydride AA
Mo	$HClO_4$, H_2SO_4	AA Colorimetric
Cu	$HClO_4$ and various acids	AA Titrimetric
Pb	$HClO_4$ and various acids	AA Titrimetric
Zn	$HClO_4$ and various acids	AA Titrimetric
F	Fusion	Specific ion electrode
Tl	Various acids	AA Colorimetric
Te	Various acids	AA Colorimetric
W	Fusion	Colorimetric
Ba	Fusion	AA Inductively coupled plasma (ICP)
Sn	Fusion	AA
S	—	Leco induction furnace
Mn	Various acids	AA Colorimetric ICP
B	—	ICP

Source: Cardwell 1984

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ANALYSIS

The following list shows instruments and abbreviations now being used for analyzing geologic materials (Cardwell 1984).

- Atomic absorption (AA, atomic absorption spectroscopy [AAS])
  - Hydride generation
  - Graphite furnace
  - Cold vapor generation
- Colorimetry
- Fluorimetry
- Emission spectrometry (ES, E-Spec)
- Inductively coupled plasma (ICP, inductively coupled atomic plasma [ICAP], direct current inductively coupled plasma [DCICP])
- X-ray fluorescence (XRF)
- Neutron activation (NA, instrumental neutron activation analysis [INAA], prompt gamma-ray activation analysis [PGAA])
- Electron microprobe (EM)
- Ion chromatography (IC)

Table 6.10 gives guidelines for the instrument or method to be used, listed by element of interest.

ACKNOWLEDGMENTS

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CHAPTER 7

Mineral Products

R. Karl Zipf Jr., P.E.

INTRODUCTION

This chapter discusses the world's major mineral products from A to Z, describes their major uses both consumer and industrial, and lists standards pertinent to those mineral products. Most of these standards are developed by ASTM International (formerly American Society for Testing and Materials). Other standards originate from the American National Standards Institute (ANSI) and the American Water Works Association (AWWA). See the "References" section at the end of this chapter for how to access the complete list of ASTM and ANSI standards.

The purity standards set by ASTM or another organization for a particular commodity—metal or nonmetal—may dictate practices employed by producers of that commodity. Strict quality control during mining and processing is needed to ensure that the commodity purity and the allowable amounts of accessory minerals or trace elements are met. If ASTM or other recognized standards do not exist for a commodity, then producers and consumers must rely on their own standards suitable to their particular requirements.

ABRASIVES

Abrasives are synthetic or natural substances used to polish, abrade, scour, clean, or remove solid material by rubbing action or impact. Some major synthetic abrasives include boron carbide, boron nitride, chromium oxide, diamond, silicon carbide, titanium carbide, and tungsten carbide. Table 7.1 lists natural abrasives of varying hardness. Any of these minerals can be mined and processed for use as abrasives, but the most important are industrial diamond, corundum, emery, garnet, novaculite, silica sand, pumice, and diatomite (Hoffman 2006). Major ANSI standards governing minerals used as abrasives follow.

- ANSI B74.8-1987(R2001): *Ball Mill Test for Friability of Abrasive Grain*
- ANSI B74.4-1992(R2002): *Procedure for Bulk Density of Abrasive Grains*
- ANSI B74.18-1996: *Grading of Certain Abrasive Grain on Coated Abrasive Products*
- ANSI B74.12-2001: *Size of Abrasive Grain-Grinding Wheels, Polishing and General Industrial Uses*

TABLE 7.1 Natural abrasives of varying hardness

Superior Hardness, H > 7 on Mohs Scale	Intermediate Hardness, H Between 5.5 and 7		Inferior Hardness, H < 5.5
	Silica Abrasives	Other Rocks and Minerals	
Diamond, H = 10	Buhrstone	Argillaceous limestone	Apatite
Corundum, H = 9	Chalcedony	Basalt	Calcite
Emery, H = 7–9	Chert flint	Feldspar	Chalk
Garnet, H = 6.5–7.5	Novaculite	Granite	Clay
Staurolite, H = 7–7.5	Quartz	Mica schist	Diatomite
	Quartzite	Perlite	Dolomite
	Sandstone	Pumice and pumicite	Iron oxides
	Silica sand	Quartz conglomerate	Limestone
			Rottenstone
			Siliceous shale
			Silt
			Talc

Adapted from Hoffman 2006

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AGGREGATES

Aggregates include construction sand and gravel, crushed stone, and lightweight aggregates.

Construction Sand and Gravel

Sand is natural granular material derived from natural rock weathering and has a diameter of about 1.5 mm (1/16 in.). Gravel is rounded rock fragments derived from natural weathering and has a diameter larger than sand. Most sand and gravel are used in portland cement concrete, road base, fill, and asphalt concrete (Langer 2006a). ASTM and the American Association of State Highway and Transportation Officials (AASHTO) standards apply. A select few are shown in the following section, "Crushed Stone."

Crushed Stone

Crushed stone is produced by mining bedrock such as granite, limestone, or traprock, then crushing and sizing the product. Like sand and gravel, crushed stone is used in portland cement concrete, road base, fill, and asphalt concrete (Langer 2006b). ASTM and AASHTO standards apply. The standards cover

- Coarse size aggregate size,
- Aggregate base and subbase and soil aggregate,
- Aggregate for bituminous paving applications,
- Aggregate for portland cement concrete, and
- General practices.

Test procedures applicable to aggregates cover the following (Marek 1996):

- General testing
- Sampling and sample preparation
- Particle size analysis of aggregate
- Properties of fines in aggregates
- Tests to evaluate general quality of aggregate
- Deleterious materials in aggregate
- Testing aggregate in bituminous applications
- Aggregate base moisture–density–permeability relationships
- Strength parameters of aggregate base
- Specific gravity, absorption, and unit weight of aggregate
- Frictional properties of aggregate and pavements
- Measurements and indices of particle shape and texture

A select few standards and specifications follow:

- ASTM C778-17: *Standard Specification for Standard Sand*
- ASTM D1241-15: *Standard Specification for Materials for Soil-Aggregate Subbase, Base, and Surface Courses*
- ASTM D448-12(2017): *Standard Classification for Sizes of Aggregate for Road and Bridge Construction*
- ASTM C33/C33M-18: *Standard Specification for Concrete Aggregates*
- ASTM D692/D692M-15: *Standard Specification for Coarse Aggregate for Bituminous Paving Mixtures*
- ASTM C144-17: *Standard Specification for Aggregate for Masonry Mortar*
- ASTM C131/C131M-14: *Standard Test Method for Resistance to Degradation of Small-Size Coarse Aggregate by Abrasion and Impact in the Los Angeles Machine*
- ASTM C535-16: *Standard Test Method for Resistance to Degradation of Large-Size Coarse Aggregate by Abrasion and Impact in the Los Angeles Machine*

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Lightweight Aggregates

Lightweight aggregates are natural and synthetic materials used as bulk fillers in lightweight concrete, concrete masonry units, and insulating fill. Natural lightweight aggregates are rocks such as pumice and volcanic cinder. Synthetic lightweight aggregates include pyro-processed shale, clay, or slate and foamed and granulated slag. Ultra-lightweight aggregates are typically pyro-processed vermiculite or perlite (Bush et al. 2006). ASTM and AASHTO standards apply. A select few follow.

- ASTM C330/C330M-17a: *Standard Specification for Lightweight Aggregates for Structural Concrete*
- ASTM C331/C331M-17: *Standard Specification for Lightweight Aggregates for Concrete Masonry Units*
- ASTM C332/C332M-17: *Standard Specification for Lightweight Aggregates for Insulating Concrete*

ALUMINUM METAL AND BAUXITE

Bauxite is a sedimentary rock with a relatively high aluminum content formed by weathering of certain silicate rocks. It is the primary ore mineral for aluminum metal production and is also used in cement, abrasives, chemical, and refractory production. Bauxite is classified according to its application—metallurgical, cement, abrasive, chemical, or refractory—listed in order of importance (Hill and Sehnke 2006; Table 7.2).

TABLE 7.2 Chemical analysis of typical raw bauxite ore by grade

	Major Oxides			
Grade	**Al_2O_3, wt %**	**SiO_2, wt %**	**Fe_2O_3, wt %**	**TiO_2, wt %**
Metallurgical	50–55	0–15	5–30	0–6
Cement	45–55	<6	20–30	3
Abrasive	>55	<5	<6	>2.5
Chemical	>55–60	<5–18	<2	0–6
Refractory	>59–61	<1.5–5.5	<2	<2.5

Adapted from Hill and Sehnke 2006

Numerous ASTM specifications exist for the composition and strength of aluminum and aluminum alloys. A select few follow.

- ASTM B209-14: *Standard Specification for Aluminum and Aluminum-Alloy Sheet and Plate*
- ASTM B985-12(2016): *Standard Practice for Sampling Aluminum Ingots, Billets, Castings and Finished and Semi-Finished Wrought Aluminum Products for Compositional Analysis*
- ASTM E716-16: *Standard Practices for Sampling and Sample Preparation of Aluminum and Aluminum Alloys for Determination of Chemical Composition by Spark Atomic Emission Spectrometry*
- ASTM E1251-17a: *Standard Test Method for Analysis of Aluminum and Aluminum Alloys by Spark Atomic Emission Spectrometry*

ANTIMONY METAL AND COMPOUNDS

Antimony is produced from stibnite (Sb_2S_3) as a by-product during base metal mining and refining. Antimony metal is alloyed with lead for use in batteries and ammunition. The compound antimony trioxide (Sb_2O_3) is used as a flame retardant and in pigments (Anderson 2019). The ASTM standard for antimony metal purity is ASTM B237-01(2014): *Standard Specification for Refined Antimony.*

ARSENIC METAL AND COMPOUNDS

Arsenic is generally produced as a by-product in the smelting of complex copper, lead, zinc, gold, and silver ores. The primary use of arsenic compounds is in wood preservatives. Minor amounts of arsenic metal are used in nonferrous alloys. Gallium arsenide is an important semiconductor. ANSI or ASTM standards for arsenic compounds or arsenic metal do not appear to exist (Twidwell 2019).

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ASBESTOS

Asbestos is a generic name for six fibrous minerals used in commercial products (Virta 2006). Asbestos fibers are graded using the Quebec Standard test developed by the Asbestos Textile Institute and the Quebec Asbestos Mining Association (1975). ASTM has active standards for asbestos mineral products such as asbestos-cement siding and pipe and asbestos yarn that may affect asbestos producers. A select few follow.

- ASTM C296/C296M-00(2015): *Standard Specification for Asbestos-Cement Pressure Pipe*
- ASTM D1918/D1918M-95(2017): *Standard Test Method for Asbestos Content of Asbestos Textiles*

BARIUM MINERALS

Barite is a high-density mineral used primarily by the petroleum industry in drilling fluid. For use in the petroleum industry, the American Petroleum Institute (API) standard requires a minimum specific gravity of 4.2, a minimum barium sulfate content of 92%, particle size such that 97% passes a number 200 mesh sieve, and a soluble alkaline content less than 250 ppm. Barite used for drilling fluids in offshore applications must also meet U.S. Environmental Protection Agency limits on mercury and cadmium content. Minor uses for barite include glass manufacturing, heavy aggregate for concrete, pigments, and pharmaceuticals. Barium sulfate used in pigments must have a purity exceeding 95% and in the pharmaceutical industry, a purity exceeding 97.5% (Mills 2006). The ASTM standard for barium sulfate pigments is ASTM D602-81(2012): *Standard Specification for Barium Sulfate Pigments*.

BERYLLIUM METAL AND MINERALS

Beryllium is refined from the mineral bertrandite. Copper-beryllium alloy, beryllium metal, and other nickel- and aluminum-base beryllium alloys are used for electrical components and in the aerospace industry. Another important use is specialty glass such as radiation windows (Sabey 2006; Vidal 2019). ASTM specifications exist for several products containing beryllium, for example, ASTM B768-11(2016): *Standard Specification for Copper-Cobalt-Beryllium Alloy and Copper-Nickel-Beryllium Alloy Strip and Sheet*.

BISMUTH METAL AND COMPOUNDS

Bismuth is produced as a by-product during base metal refining. The major uses of bismuth are in chemicals, pharmaceuticals, and cosmetics. Bismuth metal is alloyed with lead, tin, cadmium, or indium to make low-melting-point alloys. No standards exist for purity bismuth metal.

Pharmaceutical-grade compounds are 99.99% pure and free from arsenic (Ciullo 1994). ASTM specifications exist for major products containing bismuth as a metal in alloys or as a compound that may affect producers of bismuth.

- ASTM B774-00(2014): *Standard Specification for Low Melting Point Alloys*
- ASTM B974/B974M-16. *Standard Specification for Free-Cutting Bismuth Brass Rod, Bar* and Wire
- ASTM B967/B967M-18: *Standard Specification for Copper-Zinc-Tin-Bismuth Alloy Rod, Bar and Wire*

BORON AND BORATE MINERALS

Borates are used in fiberglass and glass fibers; glass and ceramics; soaps and detergents; and agricultural chemicals; and to a small extent, as an alloy metal in steel (Carpenter and Kistler 2006). ASTM has active standards for certain boron and borate mineral products. A select few follow.

- ASTM A323-05(2015): *Standard Specification for Ferroboron*
- ASTM C992-16: *Standard Specification for Boron-Based Neutron Absorbing Material Systems for Use in Nuclear Fuel Storage Racks in a Pool Environment*
- ASTM C750-18: *Standard Specification for Nuclear-Grade Boron Carbide Powder*

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TABLE 7.3 Specifications for elemental bromine

Parameter	American Chemical Society Specification	Product Specification
Bromine content, %	99.5	99.9
Specific gravity, 20/15°C	None	3.1
Water, ppm	None	30
Chlorine, ppm	500	100
Organic halogen compounds, ppm	80	80
Nonvolatile matter, ppm	50	50
Iodine, ppm	10	10
Sulfur (as S), ppm	10	10
Heavy metals (as Pb), ppm	2	2
Nickel, ppm	2	2

Adapted from Frim and Ukeles 2006

BROMINE

Bromine has industrial uses in flame retardants, drilling fluids, agricultural chemicals, and water treatment. Table 7.3 summarizes the American Chemical Society specifications and typical product specifications for elemental bromine (Frim and Ukeles 2006). ANSI or ASTM standards for bromine-related products do not appear to exist.

CADMIUM

Cadmium is a by-product of zinc production and is used as a metal for plating and as a compound in pigments, plastics, and batteries (Edwards and Aume 2019). ASTM has standards for the numerous uses of cadmium or cadmium compounds. Two examples follow:

- ASTM B440-12(2017): *Standard Specification for Cadmium*
- ASTM B907-16: *Standard Specification for Zinc, Tin and Cadmium Base Alloys Used* as *Solders*

CEMENT

Portland cement is produced by heating a mixture of lime (CaO), silica, alumina (bauxite), and iron oxide in a kiln to form clinker made up of tricalcium silicate (Ca_3SiO_5), dicalcium silicate (Ca_2SiO_4), tricalcium aluminate ($Ca_3Al_2O_6$), and tetracalcium-aluminoferrite ($Ca_4Al_2Fe_2O_{10}$). This clinker is then ground to a fine powder to make type I portland cement, which is the most common variety. Portland cement, along with sand and crushed stone, make portland cement concrete a widely used construction material (Macfadyen 2006).

Numerous ASTM standards apply to the manufacture and use of portland cement, covering topics such as additives, aggregates, compositional analysis, lightweight aggregates and concrete, ready-mixed concrete, and test methods. A select few follow.

- ASTM C91/C91M-18: *Standard Specification for Masonry Cement*
- ASTM C125-18b: *Standard Terminology Relating to Concrete and Concrete Aggregates*
- ASTM C150/C150M-18: *Standard Specification for Portland Cement*
- ASTM C618/C618M-17a: *Standard Specification for Coal Fly Ash and Raw or Calcined Natural Pozzolan for Use in Concrete*

CESIUM COMPOUNDS

Cesium compounds are refined from the mineral pollucite and are used in oil-drilling fluids, catalysts, electronics, and experimental power generation. ANSI or ASTM standards for cesium compounds do not appear to exist (Thomas and Kay 2006).

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CHROMIUM METAL AND COMPOUNDS

Chromium metal is alloyed with steel to make corrosion-resistant stainless steel, which is used in a wide variety of industrial and consumer products. In addition to its use as an alloy ingredient in steel, chromite is used to make chromium-containing chemicals such as pigments and wood preservatives (Papp and Lipin 2006).

ASTM has standards applicable to chromite ores, chromium metal, and chromium-based pigments. A select few follow.

- ASTM E342-11(2016): *Standard Test Method for Determination of Chromium Oxide Content in Chrome Ores by Permanganate Titrimetry*
- ASTM A481-05(2015): *Standard Specification for Chromium Metal*
- ASTM A101-04 (2014)e1: *Standard Specification for Ferrochromium*
- ASTM A482/A482M-11 (2016): *Standard Specification for Ferrochrome-Silicon*
- ASTM D263-05(2011): *Standard Specification for Chrome Oxide Green Pigment*
- ASTM D211-67 (2012): *Standard Specification for Chrome Yellow and Chrome Orange Pigments*

CLAYS

Clays include ball and refractory clays, as well as bentonite, fuller's earth, kaolin, palygorskite-sepiolite, and other common clays and shales.

Ball Clay

Ball clays are common kaolinitic sedimentary clays used for ceramics such as floor tile, wall tile, and sanitary ware (McCuistion and Wilson 2006). Numerous ASTM standards exist for the raw materials, such as ball clay, which go into the manufacture of ceramics. A select few follow.

- ASTM C322-09(2018): *Standard Practice for Sampling Ceramic Whiteware Clays*
- ASTM C323-56(2016): *Standard Test Methods for Chemical Analysis of Ceramic Whiteware Clays*
- ASTM C837-09(2014): *Standard Test Method for Methylene Blue Index of Clay*

Bentonite

Bentonite clays are used for absorbents, drilling fluids, foundry sand, binder in iron ore pelletizing, slurry wall construction, and clay liners for containment facilities (Eisenhour and Reisch 2006).

Standards for absorbent materials no longer exist. API has specifications for bentonite used in drilling fluids (API STD 13a). The Steel Founders' Society of America (SFSA) has specifications for bentonite used in foundry sand (SFSA 13T-86). For determining the suitability of bentonite for iron ore pelletizing, producers measure the water absorption ability of bentonite using a plate water absorption test, but no ANSI or ASTM standards appear to exist. Standards for the use of bentonite in clay liners for containment facilities also do not appear to exist.

- ASTM D4380-12: *Standard Test Method for Density of Bentonitic Slurries*
- ASTM D5890-18: *Standard Test Method for Swell Index of Clay Mineral Component of Geosynthetic Clay Liners*
- ASTM D5891/D5891M-02(2016)e1: *Standard Test Method for Fluid Loss of Clay Component of Geosynthetic Clay Liners*

Common Clays and Shales

Common clays and shales are used for making bricks, vitrified pipe, and roofing tile (Keith and Murray 2006). Numerous ASTM specifications may apply to the products made from common clays and shales.

- ASTM C896-17a: *Standard Terminology Relating to Clay Products*
- ASTM C62-17: *Standard Specification for Building Brick (Solid Masonry Units Made from Clay or Shale)*

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- ASTM C4-04(2018): *Standard Specification for Clay Drain Tile and Perforated Clay Drain Tile*
- ASTM C1167-11(2017): *Standard Specifications for Clay Roof Tiles*

Fuller's Earth

Fuller's earth is a general term for certain clays with strong absorptive, binding, gelling, thickening, or decolorizing ability. Major uses are absorbents and binders (Pickering and Heivilin 2006). ANSI or ASTM standards for fuller's earth mineral products do not appear to exist.

Kaolin

Kaolin is primarily used as pigment in paper and paint, as a filler for rubber and plastic, and in ceramics and refractories. Each kaolin product has a certificate of analysis that reports the color, particle size, viscosity of a clay–water slurry, and abrasiveness. In North America, the color or whiteness is gauged in General Electric brightness method (GEB or GE) units using TAPPI Test Method T-534 om-15. Kaolin pigments with GE units ranging from 84 to 88 are considered standard brightness, while kaolin pigments with GE units greater than 89 are considered high brightness grades. Viscosity is measured using TAPPI/ANSI Test Method T-648 om-14 developed by the Technical Association of Pulp and Paper Industry. In Europe, brightness is gauged using ISO 2470 (Pruett and Pickering 2006). ANSI or ASTM standards for kaolin mineral products do not appear to exist. Other industrial organizations have standards for kaolin clay used in paper products.

Palygorskite and Sepiolite

Palygorskite and sepiolite clays are highly absorptive with an elongated shape that prevents particle flocculation. The major use is in drilling fluids when salt water or formation brines are present. However, many other uses also exist for these clays, for example as absorbent, anticaking agents and fillers (Murray and Zhou 2006). API STD 13a applies.

Refractory Clays

Refractory clays are used to make firebricks and mortar for furnaces, kilns, smelters, reaction vessels, and gasifiers. Specifications for refractory clay vary according to the different applications. Heat resistance is the most important property along with chemical inertness, thermal shock resistance, and impact strength. Evaluating clay deposits for suitability as refractory clays is done using the pyrometric cone equivalence test described in ASTM Test Method C24-56 (Harvey et al. 2006). Select ASTM standards follow.

- ASTM Test Method C24-56: *Standard Classification of Refractory Materials*
- ASTM C27-98(2018): *Standard Classification of Fireclay and High-Alumina Refractory Brick*
- ASTM F1323-90(2013): *Standard Specification for Brick, Insulating, High Temperature, Fire Clay*
- ASTM C1088-18: *Standard Specification for Thin Veneer Brick Units Made from Clay or Shale*
- ASTM F1097-17: *Standard Specification for Mortar, Refractory (High-Temperature, Air Setting*)

COAL

Coal has two primary uses: thermal (steam) coal for electric power generation and metallurgical (coking) coal for iron and steelmaking. ASTM has developed standard methods for the sampling and analysis of coal. A proximate analysis of coal provides the moisture content; volatile matter content; ash content; calorific value; and, by calculation, the fixed carbon content. The proximate analysis is the basis for classifying the coal by rank. An ultimate analysis of coal determines the carbon; hydrogen; sulfur, nitrogen; ash contents; and, by calculation, the oxygen content. The proximate and ultimate analyses are the basis for evaluating the market value of the coal and its suitability as fuel or coking coal. Additional analyses provide data on the major, minor, and trace elements in the coal ash (Post et al. 2011; Nadkarni 2008). Select ASTM standards pertinent to coal follow (Table 7.4).

- ASTM D2013M-18: *Standard Practice for Preparing Coal Samples for Analysis*
- ASTM D3172-13: *Standard Practice for Proximate Analysis of Coal and Coke*

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TABLE 7.4 Coal classification by rank*

Class	Group	Fixed Carbon Limits, % (dry mineral-matter-free basis)		Volatile Matter Limits, % (dry mineral-matter-free basis)		Calorific Value Limits, BTU/lb (moist mineral-matter-free basis)†		Agglomerating Character
		Greater Than or Equal to	Less Than	Greater Than	Less Than or Equal to	Greater Than or Equal to	Less Than	
Anthracite	Meta-anthracite	98	–	–	2	–	–	Non-agglomerating
	Anthracite	92	98	2	8	–	–	Non-agglomerating
	Semi-anthracite‡	86	92	8	14	–	–	Non-agglomerating
Bituminous	Low volatile bituminous	78	86	14	22	–	–	Commonly agglomerating§
	Medium volatile bituminous	69	78	22	31	–	–	Commonly agglomerating§
	High volatile A bituminous	–	69	31	–	14,000	–	Commonly agglomerating§
	High volatile B bituminous	–	–	–	–	13,000	14,000	Commonly agglomerating§
	High volatile C bituminous	–	–	–	–	11,500	13,000	Commonly agglomerating
						10,500	11,500	Agglomerating
Sub-bituminous	Sub-bituminous A	–	–	–	–	10,500	11,500	Non-agglomerating
	Sub-bituminous B	–	–	–	–	9,500	10,500	Non-agglomerating
	Sub-bituminous C	–	–	–	–	8,300	9,500	Non-agglomerating
Lignitic	Lignite A	–	–	–	–	6,300	8,300	Non-agglomerating
	Lignite B	–	–	–	–	–	6,300	Non-agglomerating

* This classification does not apply to certain coals.

† Refers to coal containing its natural inherent moisture but not including visible water on the surface of the coal.

‡ If agglomerating, classify in the low volatile group of the bituminous class.

§ It is recognized that there may be non-agglomerating varieties in these groups of the bituminous class, and that there are notable exceptions in the high volatile C bituminous group.

Source: ASTM International 2018

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- ASTM D7582-15: *Standard Test Methods for Proximate Analysis of Coal and Coke by Macro Thermogravimetric Analysis*
- ASTM D3173M-17a: *Standard Test Method for Moisture in the Analysis Sample of Coal and Coke*
- ASTM D3174-12: *Standard Test Method for Ash in the Analysis Sample of Coal and Coke*
- ASTM D3175-17: *Standard Test Method for Volatile Matter in the Analysis Sample of Coal and Coke*
- ASTM D3176-15: *Standard Practice for Ultimate Analysis of Coal and Coke*
- ASTM D5373-16: *Standard Test Methods for Determination of Carbon, Hydrogen and Nitrogen in Analysis Samples of Coal and Coke*
- ASTM D388-18a: *Standard Classification of Coals by Rank*
- ASTM D2492-02(2012): *Standard Test Method for Forms of Sulfur in Coal*
- ASTM D3682-13: *Standard Test Method for Major and Minor Elements in Combustion Residues from Coal Utilization Processes*
- ASTM D4326-13: *Standard Test Method for Major and Minor Elements in Coal and Coke Ash by X-Ray Fluorescence*
- ASTM D6722-11: *Standard Test Method for Total Mercury in Coal and Coal Combustion Residues by Direct Combustion Analysis*
- ASTM D6357-11: *Standard Test Methods for Determination of Trace Elements in Coal, Coke, and Combustion Residues from Coal Utilization Processes by Inductively Coupled Plasma Atomic Emission Spectrometry, Inductively Coupled Plasma Mass Spectrometry, and Graphite Furnace Atomic Absorption Spectrometry*

COBALT METAL AND COMPOUNDS

Cobalt is produced as a by-product from nickel and copper mining and is used in numerous specialty alloys, rechargeable batteries, electronics, and pigments (Edwards and Aume 2019). ASTM has numerous standards for certain products containing cobalt. A select few follow.

- ASTM B768-11(2016): *Standard Specification for Copper-Cobalt-Beryllium Alloy and Copper-Nickel-Beryllium Alloy Strip and Sheet*
- ASTM B818-03(2018): *Standard Specification for Cobalt-Chromium-Nickel-Molybdenum-Tungsten Alloy (UNS R31233) Plate, Sheet, and Strip*
- ASTM F1537-11: *Standard Specification for Wrought Cobalt-28Chromium-6Molybdenum Alloys for Surgical Implants*

COLUMBIUM (SEE NIOBIUM)

COPPER

The largest use of copper is in electrical equipment such as motors and generators, wiring, and power distribution. Copper is also used in construction and in alloys (Johnston et al. 2019). ASTM has numerous copper standards for the purity of copper metal and alloys and various products made from copper. A select few follow.

- ASTM B224-16: *Standard Classification of Coppers*
- ASTM E255-07(2014): *Standard Practice for Sampling Copper and Copper Alloys for the Determination of Chemical Composition*
- ASTM B846-11a: *Standard Terminology for Copper and Copper Alloys*
- ASTM B5-16: *Standard Specification for High Conductivity Tough-Pitch Copper Refinery Shapes*
- ASTM B49-17: *Standard Specification for Copper Rod for Electrical Purposes*
- ASTM B115-10(2016): *Standard Specification for Electrolytic Copper Cathode*
- ASTM B216-16: *Standard Specification for Tough-Pitch Fire-Refined Copper Refinery Shapes*

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DIATOMITE

Diatomite is a soft, siliceous sedimentary rock composed of the skeletal remains of microscopic one-celled algae and is used primarily as filter media, fillers, polishes, and cleansers. Each application has unique requirements and specifications. Trace element analysis may be required for diatomite used in food, beverage, or cosmetics products. When used as a filler, ASTM Test Method D686-93 may apply (Breese and Bodycomb 2006).

FELDSPARS

Feldspars are aluminosilicate minerals with varying amounts of calcium, potassium, and sodium, which are primarily used in glass and ceramics manufacturing (Potter 2006). Numerous ASTM standards exist for the raw materials, such as feldspar, which go into the manufacture of glass and ceramics. A select few follow.

- ASTM C371-09(2018): *Standard Test Method for Wire-Cloth Sieve Analysis of Nonplastic Ceramic Powders*
- ASTM C925-09(2014): *Standard Guide for Precision Electroformed Wet Sieve Analysis of Nonplastic Ceramic Powders*

FLUORSPAR

Fluorspar or fluorite is used to make hydrofluoric acid, ceramics, and flux in steel and aluminum making. Acid-grade fluorspar contains more than 97% calcium fluoride. Ceramic-grade fluorspar is sold in two grades. Ceramic grade 1 contains 90%–95% calcium fluoride, and ceramic grade 2 contains 85%–90%. Metallurgical grade fluorspar contains at least 80% calcium fluoride (Fulton and Miller 2006).

ASTM has a standard test method for analysis of fluorspar, and the ISO has at least 24 standards for testing fluorspar.

- ASTM E463-14a: *Standard Test Method for Determination of Silica in Fluorspar by Silico-Molybdate Visible Spectrophotometry*
- ASTM E815-17b: *Standard Test Method for Determination of Calcium Fluoride in Fluorspar by EDTA Complexometric Titrimetry*

GALLIUM METAL AND COMPOUNDS

Gallium is recovered from bauxite during aluminum production and used primarily in the semiconductor industry as gallium arsenide. Small amounts of gallium metal are used in low-melting-point alloys. ASTM standards for gallium do not appear to exist.

GARNET

Garnet is primarily used as an abrasive in sandpaper and grinding wheels and also as abrasive blast media for cleaning and water-jet cutting. It is also used in water filtration. Several organizations have developed standards applicable to garnet, depending on the application (Rapple 2006).

- ANSI Standard B74.4: *Procedure for Bulk Density of Abrasive Grains*
- AWWA B100-01
- MIL-A-22262
- SSPC ABM-1

GERMANIUM

Germanium is recovered as a by-product from base metal refining and is used in fiber optics, infrared optics, catalysts, electronics, and photovoltaics. ASTM specifications for germanium do not appear to exist.

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GILSONITE

Gilsonite is a solid hydrocarbon used in printing inks, asphalt pavement, roofing, and drilling fluids. Gilsonite is classified by fusing temperature into five ranges, and each range has different applications. ANSI or ASTM standards for Gilsonite mineral products do not appear to exist (Tripp and White 2006).

GLAUCONITE

Glauconite, informally known as *greensand*, is used in minor amounts for water treatment and soil conditioning. ANSI or ASTM standards for glauconite mineral products do not appear to exist (Dooley 2006).

GOLD

The primary uses for gold are monetary purposes, jewelry, and electronics. The purity of gold is expressed in *fineness,* where 1,000 fine gold is 100% pure gold (Marsden 2019). ASTM has specifications for gold metal and gold used in certain applications. A select few follow.

- ASTM B562-95(2017): *Standard Specification for Refined Gold*
- ASTM B541-01(2018): *Standard Specification for Gold Electrical Contact Alloy*
- ASTM B477-97(2017): *Standard Specification for Gold-Silver-Nickel Electrical Contact Alloy*
- ASTM B596-89(2017): *Standard Specification for Gold-Copper Alloy Electrical Contact Material*

GRAPHITE

Natural graphite is primarily used in refractories with smaller uses in batteries, bearings, and pencils. The two main categories of natural graphite are crystalline and amorphous. Crystalline graphite is divided into two classes—flake (coarse flake) and fines (fine flake)—and must contain at least 85% carbon and be within the −8 to +60 mesh size range. Amorphous graphite is graded on carbon content, which ranges from 50% to 94%, depending on source (Taylor 2006). ASTM standards exist for graphite used in nuclear reactor components. A select few follow. Standards for other graphite mineral products do not appear to exist.

- ASTM C781-18: *Standard Practice for Testing Graphite Materials for Gas-Cooled Nuclear Reactor Components*
- ASTM D7301-11(2015): *Standard Specification for Nuclear Graphite Suitable for Components Subjected to Low Neutron Irradiation Dose*

GYPSUM AND ANHYDRITE

Gypsum ($CaSO_4 \times 2H_2O$) and anhydrite ($CaSO_4$) are produced from evaporite deposits or flue gas desulfurization and are commonly used in wallboard, plaster products, portland cement manufacturing, agriculture, fillers, food, and pharmaceuticals (Sharpe and Cork 2006).

For construction products, ASTM has numerous specifications, and a few are in the following list. Other organizations also have standards for gypsum products depending on the application, such as the U.S. Food and Drug Administration and the U.S. Food Chemicals Codex.

- ASTM C22/C22M-00(2015): *Standard Specification for Gypsum*
- ASTM C471M-17ae1: *Standard Test Methods for Chemical Analysis of Gypsum and Gypsum Products*
- ASTM C472-99(2014): *Standard Test Methods for Physical Testing of Gypsum, Gypsum Plasters and Gypsum Concrete*
- ASTM C1396/C1396M-17: *Standard Specification for Gypsum Board*

HELIUM

Helium is produced as a by-product in natural gas production and is primarily used for cryogenics, pressurizing and purging, controlled atmospheres, welding, and leak detection. Several ASTM standards are pertinent to helium production and use. A select few follow.

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- ASTM D1945-14: *Standard Test Method for Analysis of Natural Gas by Gas Chromatography*
- ASTM E499/E499M-11(2017): *Standard Practice for Leaks Using the Mass Spectrometer Leak Detector in the Detector Probe Mode*

INDIUM

Indium is produced as a by-product from zinc refinement and is primarily used in the semiconductor industry, in low-melting-point alloys, and as a transparent coating on glass. ASTM has standards for indium metal and indium-containing products in several applications.

- ASTM B1002-16: *Standard Specification for Refined Indium*
- ASTM B774-00(2014): *Standard Specification for Low Melting Point Alloys*
- ASTM C752-18: *Standard Specification for Nuclear-Grade Silver-Indium-Cadmium Alloy*

IODINE

Iodine-containing compounds are primarily used in biocides and germicides, animal feeds, and pharmaceuticals. Iodine is generally sold in the form of crude iodine with a minimum purity of 99.5%; as resublimed iodine with a minimum purity of 99.9%; or as a variety of iodine salts such as potassium iodide, sodium iodide, and calcium iodide. Standards for iodine products do not appear to exist (Krukowski and Johnson 2006).

IRON AND STEEL

The primary source mineral for iron and steel is iron ore as either hematite (Fe_2O_3) or magnetite (Fe_3O_4). Ores containing more than 60% iron as hematite or magnetite are called *natural ore* or *direct-shipping ore*. Lower grade iron ores are upgraded to 61%–64% contained iron (Ripke et al. 2019). ASTM has standards for determining the iron content and other elements in iron ore.

- ASTM E877-13: *Standard Practice for Sampling and Sample Preparation of Iron Ores and Related Materials for Determination of Chemical Composition and Physical Properties*
- ASTM E246-10(2015): *Standard Test Methods for Determination of Iron in Iron Ores and Related Materials by Dichromate Titrimetry*
- ASTM E279-18: *Standard Test Method for Determination of Abrasion Resistance of Iron Ore Pellets, Lump, and Sinter by the Tumbler Test*
- ASTM E314-16: *Standard Test Methods for Determination of Manganese in Iron Ores by Pyrophosphate Potentiometry and Periodate Spectrophotometry Techniques*
- ASTM E1070-17a: *Standard Test Method for Determination of Phosphorous in Iron Ores by Phospho-Molybdenum-Blue Spectrophotometry*

KYANITE, ANDALUSITE, SILLIMANITE, AND MULLITE

The minerals kyanite, andalusite, sillimanite, and mullite, which are different crystal forms of anhydrous aluminum silicate (Al_2SiO_5), are used in the manufacture of heat-resistant refractory ceramics. ANSI or ASTM standards for this family of mineral products do not appear to exist (Sweet et al. 2006).

LEAD

Lead metal is produced from lead sulfide ores, and its primary use is in lead–acid batteries. Other uses are as radiation shielding, glass, and in bearing alloys (Siegmund 2019). ASTM standards pertinent to lead production follow.

- ASTM B29-14: *Standard Specification for Refined Lead*
- ASTM B749-14: *Standard Specification for Lead and Lead Alloy Strip, Sheet, and Plate Products*

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LIME

Chemical lime (CaO), produced by heating limestone, is used in steelmaking, nonferrous metallurgy, flue gas desulfurization, paper production, agriculture, and construction (Krukowski 2006a). ASTM and other organizations have developed specifications for lime depending on its end use. Select specifications from ASTM follow.

- ASTM C25-99: *Standard Test Methods for Chemical Analysis of Limestone, Quicklime, and Hydrated Lime*
- ASTM C110-05: *Standard Test Methods for Physical Analysis of Limestone, Quicklime, and Hydrated Lime*
- ASTM C400-98: *Standard Test Methods for Quicklime and Hydrated Lime for Neutralization of Waste Acid*
- ASTM C602-95a(2001): *Standard Specification for Agricultural Liming Materials*
- ASTM C911-99: *Standard Specification for Quicklime, Hydrated Lime, and Limestone for Chemical Uses*

LIMESTONE AND DOLOMITE

Limestone and dolomite are important raw material rocks for aggregate, cement manufacture, lime manufacture, glass-making, and agricultural uses. Specifications for limestone and dolomite depend on their end use. ASTM and AASHTO standards apply to limestone and dolomite used as aggregate in portland or asphalt cement concrete. See the list of relevant standards in the preceding section, "Crushed Stone" (Freas et al. 2006).

LITHIUM METAL AND COMPOUNDS

Lithium metal is electrolytically produced from brines and is alloyed with aluminum or magnesium for use in the aerospace industry. ASTM standards for lithium metal and lithium alloys do not appear to exist. Lithium ores and concentrates from pegmatites are used in the glass and ceramics industries. Various lithium-containing compounds are used in batteries and other high-technology applications. ASTM standards do not appear to exist for these products (Kunasz 2006).

MAGNESIUM METAL AND COMPOUNDS

Magnesium metal is produced electrolytically from brines and is used to make lightweight alloys for the aerospace, automotive, and other industries. Magnesite ($MgCO_3$) is the primary ore mineral used to produce dead-burned magnesia (MgO), which is used to make refractories in the iron and steel industry. Other magnesium compounds produced are magnesium hydroxide [$Mg(OH)_2$], magnesium chloride ($MgCl_2$), and magnesium sulfate ($MgSO_4$) for use in a wide variety of industrial processes. Specifications for these compounds are set by the consumer depending on the application (Kramer 2006). Select ASTM standards pertinent to magnesium metal, magnesium alloys, and magnesium minerals follow.

- ASTM B92/B92M-17: *Standard Specification for Unalloyed Magnesium Ingot and Stick for Remelting*
- ASTM B107/B107M-13: *Standard Specification for Magnesium-Alloy Extruded Bars, Rods, Profiles, Tubes, and Wire*
- ASTM E1652-15: *Standard Specification for Magnesium Oxide and Aluminum Oxide Powder and Crushable Insulators used in the Manufacture of Base Metal Thermocouples, Metal-Sheathed Platinum Resistance Thermometers, and Noble Metal Thermocouples*

MANGANESE METAL AND COMPOUNDS

The primary use of manganese is as an alloy metal in steel and also with copper and aluminum. In addition to metallurgical uses, manganese dioxide is used in batteries. ASTM specifications exist for the minimum content of manganese dioxide, maximum trace metal content, and maximum impurities in manganese ores. ASTM has standards for ferromanganese, silico manganese, and ferromanganese-silicon for use by the iron and steel industry (Corathers and Machamer 2006). Select ASTM standards pertinent to manganese ores and metal follow.

- ASTM E465-11(2017): *Standard Test Methods for Determination of Manganese (IV) in Manganese Ores by Redox Titrimetry*

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- ASTM E248-12: *Standard Test Methods for Determination of Manganese in Manganese Ores by Pyrophosphate Complexed Permanganate Potentiometric Titrimetry*
- ASTM A601/A601M-10(2015): *Standard Specification for Electrolytic Manganese Metal*
- ASTM A99-03(2014): *Standard Specification for Ferromanganese*
- ASTM A483-A483M-10(2015): *Standard Specification for Silicomanganese*
- ASTM A701/A701M-10(2015): *Standard Specification for Ferromanganese-Silicon*
- ASTM B138/B138M-11(2017): *Standard Specification for Manganese Bronze Rod, Bar, and Shapes*

MERCURY METAL AND COMPOUNDS

Mercury is used in industrial chemical manufacturing, electrical components, electronics, and fluorescent lights. ASTM standards for mercury in various applications do not appear to exist (Burton 2019).

MICA

By far, the largest use of mica is flake mica, either dry or wet ground, for use in joint compound for wallboard. Small amounts of wet-ground mica are used in cosmetics and pigments, and small amounts of sheet mica and related products are used in electrical insulators and capacitors (Schlanz and Tanner 2006). ASTM has standards for mica in various applications. A select few follow.

- ASTM D351-18: *Standard Classification of Natural Muscovite Block Mica and Thins Based on* Visual Quality
- ASTM D607-82(2012): *Standard Specification for Wet-Ground Mica Pigments*
- ASTM D716-86(2012): *Standard Test Methods for Evaluating Mica Pigment*
- ASTM D748-18: *Standard Specification for Natural Block Mica Films Suitable for Use in Fixed Mica-Dielectric Capacitors*
- ASTM D2131-97(2016): *Standard Classification for Natural Muscovite Mica Splittings*
- ASTM D1039-16: *Standard Test Methods for Glass-Bonded Mica Used as Electrical Insulation*

MOLYBDENUM METAL AND COMPOUNDS

Molybdenum is used primarily as an alloy metal in steel and superalloys. Smaller amounts of molybdenum compounds are used in lubricants, ceramics, and pigments. Molybdenum is a refractory metal (along with niobium, tantalum, tungsten, and rhenium) that is extraordinarily resistant to heat and wear (Amelunxen et al. 2019). Select ASTM specifications for molybdenum and molybdenum alloys follow.

- ASTM B386-03: *Standard Specification for Molybdenum and Molybdenum Alloy Plate, Sheet, Strip, and Foil*
- ASTM A132-04(2014): *Standard Specification for Ferromolybdenum*
- ASTM A146-04(2014): *Standard Specification for Molybdenum Oxide Products*

NEPHELINE SYENITE

Nepheline syenite is used in glass, ceramics, roofing granules, and as asphalt and concrete aggregate. ANSI or ASTM standards for nepheline syenite mineral products do not appear to exist (McLemore 2006).

NICKEL METAL AND COMPOUNDS

Nickel is used primarily as an alloy metal in stainless steel. It is also used in other steel alloys, nonferrous alloys, and in electroplating. Minor amounts of nickel compounds are used in catalysts and batteries (Bamber and Barnes 2019). Select ASTM specifications for nickel and nickel alloys follow.

- ASTM B39-79(2018): *Standard Specification for Nickel*
- ASTM B162-99(2014): *Standard Specification for Nickel Plate, Sheet, and Strip*
- ASTM A636-08(2018): *Standard Specification for Nickel Oxide Sinter*

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NIOBIUM

Niobium, formerly known as columbium, is used in high-grade structural steel, superalloys, and superconducting magnets. Niobium is a refractory metal that is extraordinarily resistant to heat and wear (Goode 2019). Select ASTM specifications for niobium and niobium alloys follow.

- ASTM A550-16: *Standard Specification for Ferrocolumbium (Ferroniobium)*
- ASTM B391-18: *Standard Specification for Niobium and Niobium Alloy Ingots*
- ASTM B652/B652M-10(2018): *Standard Specification for Niobium-Hafnium Alloy Ingots*
- ASTM B884-11: *Standard Specification for Niobium Alloy Billets, Bar, and Rod for Superconducting Applications*

NITROGEN AND NITRATES

Most nitrates used in fertilizer are synthetically produced from natural gas; however, a small quantity of natural nitrates is also produced. Fertilizers are labeled using the NPK system, where the first number is the percentage of elemental nitrogen in the product. ASTM or other organizations that set standards for fertilizers do not appear to exist (Harben and Theune 2006).

OLIVINE

The primary use for olivine is in metallurgy as a slag conditioner in blast- and electric-arc furnaces. Other uses for olivine are in refractories, abrasives, and as a high-density construction aggregate. Typically, olivine contains 45%–51% MgO; 40%–43% silica (SiO_2); and Fe, Al, Ca, and other oxides. The loss on ignition is critical because it indicates the presence of other undesirable compounds. ANSI or ASTM standards for olivine mineral products do not appear to exist (Harben and Smith 2006).

PEAT

Peat is an accumulation of anaerobically preserved, wetlands plants. Peat is used as a soil supplement in gardening, water filtration, and sometimes as a fuel (Hoffman and Austin 1994). ASTM has standards for peat.

- ASTM D4427-18: *Standard Classification of Peat Samples by Laboratory Testing*
- ASTM D2974-14: *Standard Test Methods for Moisture, Ash, and Other Organic Matter of Peat and Other Organic Soils*
- ASTM D2976-15: *Standard Test Method for pH of Peat Materials*

PERLITE

Perlite is a natural volcanic glass containing chemically bound water such that when crushed, ground, and heated rapidly, it expands (pops) into an anhydrous glass foam or microspheres. The lightweight perlite spheres have wide use in joint compound for wallboard, insulation, low-density aggregate, filter aids, horticulture, and low-density fillers (Barker and Santini 2006). The Perlite Institute (1984), the U.S. Food Chemicals Codex, and ASTM have standards for perlite in various applications.

- ASTM C549-18: *Standard Specification for Perlite Loose Fill Insulation*
- ASTM C728-17a: *Standard Specification for Perlite Thermal Insulation Board*
- ASTM C610-17: *Standard Specification for Molded Expanded Perlite Block and Pipe Thermal Insulation*

PHOSPHATE ROCK

Phosphate rock is processed into phosphate fertilizer for use in agriculture. Fertilizers are labeled using the NPK system, where the second number is the percentage of elemental phosphorus in the product. ASTM or other organizations that set standards for fertilizers do not appear to exist (Zhang et al. 2006).

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PLATINUM GROUP METALS

The platinum group metals include ruthenium, rhodium, palladium, osmium, iridium, and platinum. Major uses for these metals are in catalytic converters, jewelry, and electronics. ASTM standards exist for electroplating applications of these metals and other applications (Crundwell 2019).

- ASTM B867-95(2018): *Standard Specification for Electrodeposited Coatings of Palladium-Nickel for Engineering Use*
- ASTM B684/B684M-16: *Standard Specification for Platinum-Iridium Electrical Contact Materials*

POTASH

Potash is a generic term for several potassium salts including potassium chloride, potassium magnesium sulfate, potassium sulfate, and potassium nitrate. It is used primarily as fertilizer in agriculture, and minor amounts are used in the chemical industry. Muriate of potash (MOP or potassium chloride) is the primary product, and it has a minimum requirement of 60% potassium oxide (K_2O) content. Fertilizers are labeled using the NPK system, where the third number is the percentage of elemental potassium in the product. ASTM or other organizations that set standards for fertilizers do not appear to exist (Prud'homme and Krukowski 2006).

PUMICE, PUMICITE, AND VOLCANIC CINDER

These volcanic rocks have a cellular structure and are used in lightweight aggregates, insulators, absorbents, and abrasives. When used as lightweight aggregate, the following ASTM standard applies. ANSI or ASTM standards for other applications do not appear to exist (Presley 2006).

- ASTM C331/C331M-17: *Standard Specification for Lightweight Aggregates for Concrete Masonry Units*

PYROPHYLLITE

Pyrophyllite is a hydrous aluminum silicate similar to talc and used in refractories, pottery and tile, cement, building materials, glass fiber, agricultural chemicals, and paper. ANSI or ASTM standards for pyrophyllite mineral products do not appear to exist (Hida and Kitagawa 2006).

RARE EARTH ELEMENTS

The rare earth elements, yttrium plus lanthanum through lutetium in the periodic table, are used in glass polishing and ceramics, as catalysts in petroleum refining, as metallurgical additives and alloys, in catalytic converters, as permanent magnets, and in phosphors. ANSI or ASTM standards for rare earth element mineral products do not appear to exist (Castor and Hedrick 2006; Goode 2019).

RHENIUM

Rhenium is extracted from the flue gas of molybdenum production. It is used in high-temperature superalloys for jet engines and in catalysts for petroleum refining. Rhenium is a refractory metal that is extraordinarily resistant to heat and wear (Amelunxen et al. 2019). ASTM standards exist for a few alloys containing rhenium.

- ASTM F3273-17: *Standard Specification for Wrought Molybdenum-47.5 Rhenium Alloy for Surgical Implants (UNS R03700)*
- ASTM E696-07(2018): *Standard Specification for Tungsten-Rhenium Alloy Thermocouple Wire*
- ASTM F73-96(2017): *Standard Specification for Tungsten-Rhenium Alloy Wire for Electron Devices and Lamps*

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RUBIDIUM

Rubidium is chemically similar to potassium and is refined from the mineral lepidolite. It has limited and specialized applications in atomic clocks for high-precision timing. Other potential uses for rubidium are in power generation using the magnetohydrodynamic principle. No ASTM or other standards appear to exist for rubidium (Welham 2019).

SALT

Salt—sodium chloride—is used for chemical manufacturing, ice control, industrial processing, food processing, water treatment, and other applications. A wide variety of ASTM and other standards may apply to salt, depending on the application. Table 7.5 summarizes these specifications (Feldman 2006).

SELENIUM

Selenium is chemically similar to sulfur and is produced from selenides in many sulfide ores. The primary use of selenium is in glass production where it gives a red color. Other applications are in electronic components, batteries, and chemicals. ASTM standards for selenium products do not appear to exist (Crundwell 2019).

TABLE 7.5 Specifications applying to the salt industry

Type Specification	Title	Designation	Jurisdiction
Analysis	*Standard Methods for Chemical Analysis of Sodium Chloride*	ASTM E534(2018)	ASTM International
Food	*Sodium Chloride*	Food Chemicals Codex (FCC)	U.S. Pharmacopeial Convention
	Official Methods of Analysis of AOAC International: Salt	AOAC 925.55-1925 AOAC 925.56-1925 AOAC 925.57-1925	AOAC International
Highway	*Standard Specification for Sodium Chloride*	ASTM D632	ASTM International
	Sodium Chloride	AASHTO M143-03	American Association of State Highway Transportation Officials
	Handbook of Test Methods for Evaluating Chemical Deicers	SHRP-H-332	Strategic Highway Research Program
Medical	*Official Monograph on Sodium Chloride*	National Formulary (2005)	U.S. Pharmacopeial Convention
Reagent	*Sodium Chloride*	*Reagent Chemicals: Specifications and Procedures*, 10th ed.	American Chemical Society
Soil stabilization	*Standard Test Methods for Water-Soluble Chlorides Present as Admixtures in Graded Aggregate Road Mixes*	ASTM D1411	ASTM International
Table salt	*Salt, Table, Iodized*	A-A-20041B	U.S. Department of Agriculture
Water	*Federal Specification Commercial Item, Description, Sodium Chloride, Technical (Water Conditioning Grade)*	A-A-694D	General Services Administration
	AWWA Standard for Sodium Chloride	ANSI/AWWA-B200-03	American National Standards Institute/American Water Works Association

Adapted from Feldman 2006

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SILICA

Silicas include industrial sand and sandstone, pure quartz products, and specialty materials.

Industrial Sand and Sandstone

Industrial sand is used in glass and ceramics manufacturing, as foundry sand, as filler in industrial applications, in abrasives, and as proppant in the petroleum industry. Industrial sand may be 98.5%–99.5% SiO_2 depending on the application (Herron 2006). The ASTM standard is ASTM C146-94a(2014): *Standard Test Methods for Chemical Analysis of Glass Sand*.

High Pure and Ultra-High Pure Quartz

Pure quartz products are used in high-end, high-technology applications such as electronics, optics, specialty glass, and fiber optics. The purity of high-grade silica is greater than 99.9%, and that of the ultra-high grade is greater than 99.99%. Table 7.6 gives the elemental limits for these silica grades (Schlanz 2006). ANSI or ASTM standards for high-purity quartz mineral products do not appear to exist.

Specialty Silica Materials

Specialty silica materials are produced from rocks such as quartzite, sandstone, and novaculite that are predominantly silica (SiO_2), and are used to make filtration material, silicon metal, and ferrosilicon alloys (Krukowski 2006b). ANSI or ASTM standards for specialty silica mineral products do not appear to exist.

TABLE 7.6 Elemental limits—initial qualifications

Quartz Product	Fe	Al	K	Na	Ca	Li	Mg	Ti
High pure, maximum ppm	20	250	80	100	50	2	30	5
Ultra-high pure, maximum ppm	1.0	15.0	2.0	2.0	2.0	0.5	0.5	1.0

Adapted from Schlanz 2006

SILICON

Silicon metal of 96%–99% purity is made from quartzite or sand in an electric-arc furnace. Silicon metal is primarily used as ferrosilicon and in electrical steel for transformers. An important aluminum alloy called *silumin* contains up to about 35% silicon and is used for automotive castings. A small portion of silicon production is used in electronics such as photovoltaics, semiconductors, and integrated circuits (Lynch and Young 2019). Numerous ASTM standards apply to silicon in its many metallurgical uses.

- ASTM A922-05(2015): *Standard Specification for Silicon Metal*
- ASTM A100-07(2018): *Standard Specification for Ferrosilicon*
- ASTM A482/A482M-11(2016): *Standard Specification for Ferrochrome-Silicon*
- ASTM A483-A483M-10(2015): *Standard Specification for Silicomanganese*
- ASTM A495-06(2015): *Standard Specification for Calcium-Silicon Alloys*
- ASTM A701/A701M-10(2015): *Standard Specification for Ferromanganese-Silicon*

SILVER

Similar to gold and other precious metals, the purity of silver is expressed as *fineness* where 100% silver is 1,000 fine. Sterling silver is 925 fine or 92.5% silver and 75 parts (7.5%) copper. Silver is used in coinage, jewelry and tableware, medicine, and electronics (Marsden 2019). Select ASTM standards for silver and silver-containing products follow.

- ASTM B413-97a(2017): *Standard Specification for Refined Silver*
- ASTM B742-90(2012): *Standard Specification for Fine Silver Electrical Contact Fabricated Material*
- ASTM B617-98(2016): *Standard Specification for Coin Silver Electrical Contact Alloy*

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SODA ASH

Soda ash is the common name for sodium carbonate that is used to manufacture glass, chemicals, soaps and detergents, pulp and paper, and many other products (Santini et al. 2006). Select ASTM standards applicable to soda ash follow.

- ASTM E359-17: *Standard Test Methods for Analysis of Soda Ash (Sodium Carbonate)*
- ASTM D501-03(2016): *Standard Test Methods of Sampling and Chemical Analysis of Alkaline Detergents*

SODIUM SULFATE

A variety of evaporite minerals contain a form of sodium sulfate that is primarily used in detergents and soaps and also pulp and paper, glass, and textile manufacturing (Austin and Humphrey 2006). ASTM D800-05(2014): *Standard Test Methods of Chemical Analysis of Industrial Metal Cleaning Compounds* is applicable to sodium sulfate.

STONE

Both decorative stone and dimension stone are covered in this section.

Decorative Stone

Decorative stone is primarily used in building construction and landscaping. Unlike dimension stone, decorative stone is not cut and finished to specific dimensions and shapes. Specifications for decorative stone are not well developed because aesthetics are of principle importance. Testing methods exist for abrasion resistance and flexural strength if applicable.

Dimension Stone

Dimension stone is rock that is commonly quarried as rectangular blocks, then cut and finished to specific dimensions and shapes. Dimension stone is used in building construction and landscaping (Austin et al. 2006; Mead and Austin 2006). ASTM has a variety of specifications and test methods applicable to dimension stone. A select few follow.

- ASTM C406/C406M-15: *Standard Specification for Roofing Slate*
- ASTM C503/C503M-15: *Standard Specification for Marble Dimension Stone*
- ASTM C568/C568M-15: *Standard Specification for Limestone Dimension Stone*
- ASTM C615/C615M-18e1: *Standard Specification for Granite Dimension Stone*
- ASTM C616/C616M-15: *Standard Specification for Quartz-Based Dimension Stone*
- ASTM C629/C629M-15: *Standard Specification for Slate Dimension Stone*
- ASTM C97/C97M-18: *Standard Test Methods for Absorption and Bulk Specific Gravity of Dimension Stone*
- ASTM C99/C99M-18: *Standard Test Methods for Modulus of Rupture of Dimension Stone*
- ASTM C170/C170M-17: *Standard Test Method for Compressive Strength Testing of Dimension Stone*
- ASTM C241/241M-15e1: *Standard Test Method for Abrasion Resistance of Stone Subjected to Foot Traffic*

STRONTIUM MINERALS

The strontium-containing minerals celestite (strontium sulfate) and strontianite (strontium carbonate) are mined to produce strontium carbonate, which is used in ceramics and glass manufacturing, specifically for cathode-ray tubes (CRTs). ASTM D1649-01(2012): *Standard Specification for Strontium Chromate Pigment* is applicable to a strontium compound (Ober 2006a).

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SULFUR

Ninety percent of sulfur is used in the form of sulfuric acid for phosphate fertilizer production, petroleum refining, agricultural chemical production, copper ore leaching, and other industrial processes. Elemental sulfur is used in agriculture and industry. ASTM E223-16: *Standard Test Methods for Analysis of Sulfuric Acid* enables classification of the various grades of sulfuric acid and determination of impurities that are important factors governing its many uses (Ober 2006b).

TALC

Talc is used in paper, paints, plastics, ceramics construction products, and also cosmetics and pharmaceuticals (McCarthy et al. 2006). ASTM D605-82(2012): *Standard Specification for Magnesium Silicate Pigment (Talc)* is applicable to talc and talc products.

TANTALUM

Tantalum is extracted from minerals such as tantalite, niobite, or coltan, which is a mixture of niobite and tantalite, and is used in capacitors and in various superalloys. Tantalum is a refractory metal that is extraordinarily resistant to heat and wear (Welham 2019). ASTM B364-18: *Standard Specification for Tantalum and Tantalum Alloy Ingots* applies to tantalum and tantalum-containing alloys.

TELLURIUM

Tellurium is recovered as a by-product during electrolytic copper refining and is used in steel and copper alloys, solar cells, semiconductors, and chemicals. ASTM standards for tellurium-containing products do not appear to exist (Crundwell 2019).

THALLIUM

Thallium is obtained as a by-product from base metal refinement and is used in electronics, pharmaceuticals, and glass manufacturing (Goode 2019). No ASTM or other standards appear to exist for thallium.

THORIUM

Thorium is produced as a by-product during the refinement of rare earths. Thorium dioxide has minor uses in the production of incandescent filaments with tungsten, certain heat-resistant ceramics, and glass. Thorium also has potential use for nuclear energy (Goode 2019). No ASTM or other standards appear to exist for thorium.

TIN

The primary uses of tin are in solders, as a protective coating on other metals, as an alloy with copper to make bronze, and in bearing metal. Other uses are in superconducting magnet alloys and chemicals. Select ASTM standards for tin and tin-containing products follow.

- ASTM B339-12: *Standard Specification for Pig Tin*
- ASTM A623-16: *Standard Specification for Tin Mill Products, General Requirements*
- ASTM A625/A625M-13(2018): *Standard Specification for Tin Mill Products, Black Plate, Single-Reduced*

TITANIUM METAL AND COMPOUNDS

The primary use (93%) of titanium is as titanium dioxide in paint pigments. Titanium metal and its alloys are used in the aerospace, marine, and automotive industries, as well as consumer products (Murphy and Frick 2006a; Battle 2019a, 2019b; Imam and Froes 2019). Select ASTM standards for titanium and titanium-containing products follow.

- ASTM B348-13: *Standard Specification for Titanium and Titanium-Alloy Bars and Billets*
- ASTM A324-08(2013): *Standard Specification for Ferrotitanium*

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- ASTM D476-15: *Standard Classification for Dry Pigmentary Titanium Dioxide Products*
- ASTM D1394-76(2014): *Standard Test Methods for Chemical Analysis of White Titanium Pigments*

TRIPOLI

Tripoli—microcrystalline silica—is used in the paint, plastics, and abrasives industries (Bratney 2006). ANSI or ASTM standards for tripoli mineral products do not appear to exist.

TUNGSTEN METAL AND COMPOUNDS

Tungsten is primarily used to make tungsten-carbide cutting tools. Tungsten metal is used in steel alloys, superalloys, armaments, electronics, and lightbulbs. Tungsten is a refractory metal that is extraordinarily resistant to heat and wear (Kelley 2019). Select ASTM standards for tungsten and tungsten-containing products follow.

- ASTM B777-15: *Standard Specification for Tungsten Base, High-Density Metal*
- ASTM B760-07(2013): *Standard Specification for Tungsten Plate, Sheet, and Foil*
- ASTM F288-96(2014): *Standard Specification for Tungsten Wire for Electron Devices and Lamps*
- ASTM B663/B663M-16: *Standard Specification for Silver-Tungsten Carbide Electrical Contact Material*

URANIUM

Uranium is produced from a variety of uranium-bearing minerals and is used in high-density penetrators, nuclear weapons, and military and civilian nuclear power reactors (Schnell 2019). Select ASTM standards for uranium and uranium-containing products follow.

- ASTM C967-13: *Standard Specification for Uranium-Ore Concentrate*
- ASTM C1022-17: *Standard Test Methods for Chemical and Atomic Absorption Analysis of Uranium-Ore Concentrate*
- ASTM C1075-17: *Standard Practices for Sampling Uranium-Ore Concentrate*
- ASTM C696-11: *Standard Test Methods for Chemical, Mass Spectrometric, and Spectrochemical Analysis of Nuclear-Grade Uranium Dioxide Powders and Pellets*
- ASTM C753-16a: *Standard Specification for Nuclear-Grade, Sinterable Uranium Dioxide Powder*

VANADIUM

Vanadium is primarily used as an alloying metal in steel. Vanadium is also alloyed with titanium and aluminum to make lightweight materials for the aerospace industry (Lam and Harris 2019). ASTM A102-04(2014): *Standard Specification for Ferrovanadium* applies because ferrovanadium is the primary use of vanadium.

VERMICULITE

Vermiculite is an altered biotite mineral that expands when rapidly heated. The resulting lightweight product is used in horticultural products, lightweight aggregates, fire-resistant wallboard, and insulation (Hindman 2006). Select ASTM standards pertinent to vermiculite follow.

- ASTM C516-08(2013)e1: *Standard Specification for Vermiculite Loose Fill Thermal Insulation*
- ASTM C196-00(2016): *Standard Specification for Expanded or Exfoliated Vermiculite Thermal Insulating Cement*

WOLLASTONITE

Wollastonite is calcium metasilicate ($CaSiO_3$) with a needlelike crystal habit that has become a nontoxic substitute for asbestos. It is used for fire-resistant products, ceramics, metallurgy, paints and coatings, plastics, friction-brake materials, and specialty rubbers (Robinson et al. 2006). ASTM D6109-13: *Standard*

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Test Methods for Flexural Properties of Unreinforced and Reinforced Plastic Lumber and Related Products is pertinent to certain wollastonite-containing products.

ZEOLITES

Zeolites are hydrated aluminosilicates with alkali or alkaline earth metals that can selectively adsorb molecules based on their size, shape, or electric charge. Most zeolite production is used for odor mitigation, aquaculture, agriculture, radioactive barriers, effluent treatment, and other ion-exchange applications (Eyde and Holmes 2006). Select ASTM standards pertinent to zeolites follow.

- ASTM D4365-13: *Standard Test Method for Determining Micropore Volume and Zeolite Area of a Catalyst*
- ASTM D3906-03(2013): *Standard Test Method for Determination of Relative X-ray Diffraction Intensities of Faujasite-Type Zeolite-Containing Materials*
- ASTM D3942-03(2013): *Standard Test Method for Determination of the Unit Cell Dimension of a Faujasite-Type Zeolite*

ZINC METAL AND COMPOUNDS

Zinc is used as a protective coating metal on steel and as an alloy with copper to make brass. Zinc compounds are used in a variety of industrial processes (Moats and Free 2019). Select ASTM standards for zinc and zinc-containing products follow.

- ASTM B6-13: *Standard Specification for Zinc*
- ASTM B949-13: *Standard Specification for General Requirements for Zinc and Zinc Alloy Products*
- ASTM B633-15: *Standard Specification for Electrodeposited Coatings of Zinc on Iron and Steel*
- ASTM B852-16: *Standard Specification for Continuous Galvanizing Grade (CGG) Zinc Alloys for Hot-Dip Galvanizing of Sheet Steel*

ZIRCONIUM AND HAFNIUM METALS AND COMPOUNDS

Zirconium- and the associated hafnium-containing minerals are produced as by-products from titanium mineral and tin mineral mining and processing. Zirconium metal and alloys are used in nuclear power plants and the aerospace industries. Hafnium metal is used in control rods in nuclear power plants. Zirconium and the associated hafnium minerals are used in high-temperature ceramics, refractories, foundry sand, television glass, and specialty alloys (Murphy and Frick 2006b). Select ASTM standards pertinent to zirconium-based products follow.

- ASTM B493/B493M-14: *Standard Specification for Zirconium and Zirconium Alloy Forgings*
- ASTM B495-10(2017): *Standard Specification for Zirconium and Zirconium Alloy Ingots*
- ASTM B349/B349M-16: *Standard Specification for Zirconium Sponge and Other Forms of Virgin Metal for Nuclear Application*

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CHAPTER

8

Economics and Costing

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INTEREST FORMULAS

Interest is the cost of borrowing money. The following formulas can be used to calculate interest, present worth, future worth, uniform amount per interest period, or interest period.

Simple Interest

$$I = niP \quad \text{(EQ 8.1)}$$

Discrete Interest End-of-Period Dollar Values

Single payment compound amount factor: $F/P_{i,n} = (1 + i)^n$ (EQ 8.2)

Single payment present worth factor: $P/F_{i,n} = 1/(1 + i)^n$ (EQ 8.3)

Uniform series compound amount factor: $F/A_{i,n} = [(1 + i)^n - 1]/i$ (EQ 8.4)

Sinking fund deposit factor: $A/F_{i,n} = i/[(1 + i)^n - 1]$ (EQ 8.5)

Capital recovery factor: $A/P_{i,n} = i(1 + i)^n/[(1 + i)^n - 1]$ (EQ 8.6)

Uniform series present worth factor: $P/A_{i,n} = [(1 + i)^n - 1]/i(1 + i)^n$ (EQ 8.7)

Arithmetic gradient series factor: $A/G_{i,n} = (1/i) - \{n/[(1 + i)^n - 1]\}$ (EQ 8.8)

Effective annual interest rate: $E = (1 + i)^m - 1$ (EQ 8.9)

Continuous Interest Lump Sum End-of-Period Dollar Values

$$F/P_{r,n} = e^{nr} \quad \text{(EQ 8.10)}$$

$$P/F_{r,n} = 1/e^{nr} \quad \text{(EQ 8.11)}$$

$$F/A_{r,n} = (e^{nr} - 1)/(e^r - 1) \quad \text{(EQ 8.12)}$$

$$A/F_{r,n} = (e^r - 1)/(e^{nr} - 1) \quad \text{(EQ 8.13)}$$

$$P/A_{r,n} = (e^{nr} - 1)/(e^r - 1)e^{nr} \quad \text{(EQ 8.14)}$$

$$A/P_{r,n} = (e^r - 1)e^{nr}/(e^{nr} - 1) \quad \text{(EQ 8.15)}$$

where

A = uniform amount per interest period
E = effective annual interest
e = base of natural log (ln) = 2.7182818…
F = future worth, value, or amount
G = uniform gradient amount per interest period
i = interest rate per period
m = number of compounding periods per year
n = number of compounding periods
r = nominal interest rate compounded continuously
P = present worth, value, or amount

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NOMINAL AND EFFECTIVE INTEREST RATES

For interest periods other than annual compounding, use the following equation:

$$i = \frac{r}{m} \quad \text{(EQ 8.16)}$$

where

i = effective interest rate per compounding period, %
r = nominal annual interest rate, %
m = compounding period per year (quarters, months, etc.)

DISCRETE COMPOUND INTEREST TABLES

Tables 8.1 through 8.4 give values for single payment compound amount factor ($F/P_{i,n}$), single payment present worth factor ($P/F_{i,n}$), uniform series compound amount factor ($F/A_{i,n}$), and uniform series present worth factor ($P/A_{i,n}$).

CAPITALIZED COSTS

$$P = A/i \quad \text{(EQ 8.17)}$$

where

P = principal or initial amount
A = payment per period (perpetual)
i = interest rate per period

DEPRECIATION

Depreciation is a method that allows for the recovery of the cost or other basis of certain property over the time the property is used. It is an allowance for the wear and tear, deterioration, or obsolescence of the property. There are several methods that can be used to calculate depreciation. Straight line depreciation and the accelerated cost recovery system are only two of the methods.

Straight Line

$$D_j = (C - S_n)/n \quad \text{(EQ 8.18)}$$

where

D_j = depreciation in year j
C = cost
S_n = salvage value in year n
n = recovery period in years

Accelerated Cost Recovery System

The accelerated cost recovery system (ACRS) is another method to measure property depreciation. Table 8.5 shows modified ACRS depreciation rates. Other methods can be found in IRS publication 946, *How to Depreciate Property* (2017).

$$D_j = (\text{modified ACRS depreciation rate})(C) \quad \text{(EQ 8.19)}$$

where

D_j = depreciation in year j
C = cost

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TABLE 8.1 Single payment compound amount factor ($F/P_{i,n}$)

n	1%	2%	3%	4%	5%	6%	7%	8%
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.06000	1.07000	1.08000
2	1.02010	1.04040	1.06090	1.08160	1.10250	1.12360	1.14490	1.16640
3	1.03030	1.06121	1.09273	1.12486	1.15763	1.19102	1.22504	1.25971
4	1.04060	1.08243	1.12551	1.16986	1.21551	1.26248	1.31080	1.36049
5	1.05101	1.10408	1.15927	1.21665	1.27628	1.33823	1.40255	1.46933
6	1.06152	1.12616	1.19405	1.26532	1.34010	1.41852	1.50073	1.58687
7	1.07214	1.14869	1.22987	1.31593	1.40710	1.50363	1.60578	1.71382
8	1.08286	1.17166	1.26677	1.36857	1.47746	1.59385	1.71819	1.85093
9	1.09369	1.19509	1.30477	1.42331	1.55133	1.68948	1.83846	1.99900
10	1.10462	1.21899	1.34392	1.48024	1.62889	1.79085	1.96715	2.15892
11	1.11567	1.24337	1.38423	1.53945	1.71034	1.89830	2.10485	2.33164
12	1.12683	1.26824	1.42576	1.60103	1.79586	2.01220	2.25219	2.51817
13	1.13809	1.29361	1.46853	1.66507	1.88565	2.13293	2.40985	2.71962
14	1.14947	1.31948	1.51259	1.73168	1.97993	2.26090	2.57853	2.93719
15	1.16097	1.34587	1.55797	1.80094	2.07893	2.39656	2.75903	3.17217
16	1.17258	1.37279	1.60471	1.87298	2.18287	2.54035	2.95216	3.42594
17	1.18430	1.40024	1.65285	1.94790	2.29202	2.69277	3.15882	3.70002
18	1.19615	1.42825	1.70243	2.02582	2.40662	2.85434	3.37993	3.99602
19	1.20811	1.45681	1.75351	2.10685	2.52695	3.02560	3.61653	4.31570
20	1.22019	1.48595	1.80611	2.19112	2.65330	3.20714	3.86968	4.66096
21	1.23239	1.51567	1.86029	2.27877	2.78596	3.39956	4.14056	5.03383
22	1.24472	1.54598	1.91610	2.36992	2.92526	3.60354	4.43040	5.43654
23	1.25716	1.57690	1.97359	2.46472	3.07152	3.81975	4.74053	5.87146
24	1.26973	1.60844	2.03279	2.56330	3.22510	4.04893	5.07237	6.34118
t	1.28243	1.64061	2.09378	2.66584	3.38635	4.29187	5.42743	6.84848
n	**9%**	**10%**	**12%**	**15%**	**20%**	**25%**	**30%**	**40%**
1	1.090	1.100	1.120	1.150	1.200	1.250	1.300	1.400
2	1.188	1.210	1.254	1.323	1.440	1.563	1.690	1.960
3	1.295	1.331	1.405	1.521	1.728	1.953	2.197	2.744
4	1.412	1.464	1.574	1.749	2.074	2.441	2.856	3.842
5	1.539	1.611	1.762	2.011	2.488	3.052	3.713	5.378
6	1.677	1.772	1.974	2.313	2.986	3.815	4.827	7.530
7	1.828	1.949	2.211	2.660	3.583	4.768	6.275	10.541
8	1.993	2.144	2.476	3.059	4.300	5.960	8.157	14.758
9	2.172	2.358	2.773	3.518	5.160	7.451	10.604	20.661
10	2.367	2.594	3.106	4.046	6.192	9.313	13.786	28.925
11	2.580	2.853	3.479	4.652	7.430	11.642	17.922	40.496
12	2.813	3.138	3.896	5.350	8.916	14.552	23.298	56.694
13	3.066	3.452	4.363	6.153	10.699	18.190	30.288	79.371
14	3.342	3.797	4.887	7.076	12.839	22.737	39.374	111.120
15	3.642	4.177	5.474	8.137	15.407	28.422	51.186	155.568
16	3.970	4.595	6.130	9.358	18.488	35.527	66.542	217.795
17	4.328	5.054	6.866	10.761	22.186	44.409	86.504	304.913
18	4.717	5.560	7.690	12.375	26.623	55.511	112.455	426.879
19	5.142	6.116	8.613	14.232	31.948	69.389	146.192	597.630
20	5.604	6.727	9.646	16.367	38.338	86.736	190.050	836.683
21	6.109	7.400	10.804	18.822	46.005	108.420	247.065	1,171.356
22	6.659	8.140	12.100	21.645	55.206	135.525	321.184	1,639.898
23	7.258	8.954	13.552	24.891	66.247	169.407	417.539	2,295.857
24	7.911	9.850	15.179	28.625	79.497	211.758	542.801	3,214.200
25	8.623	10.835	17.000	32.919	95.396	264.698	705.641	4,499.880

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TABLE 8.2 Single payment present worth factor ($P/F_{i,n}$)

n	1%	2%	3%	4%	5%	6%	7%	8%
1	0.99010	0.98039	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593
2	0.98030	0.96117	0.94260	0.92456	0.90703	0.89000	0.87344	0.85734
3	0.97059	0.94232	0.91514	0.88900	0.86384	0.83962	0.81630	0.79383
4	0.96098	0.92385	0.88849	0.85480	0.82270	0.79209	0.76290	0.73503
5	0.95147	0.90573	0.86261	0.82193	0.78353	0.74726	0.71299	0.68058
6	0.94205	0.88797	0.83748	0.79031	0.74622	0.70496	0.66634	0.63017
7	0.93272	0.87056	0.81309	0.75992	0.71068	0.66506	0.62275	0.58349
8	0.92348	0.85349	0.78941	0.73069	0.67684	0.62741	0.58201	0.54027
9	0.91434	0.83676	0.76642	0.70259	0.64461	0.59190	0.54393	0.50025
10	0.90529	0.82035	0.74409	0.67556	0.61391	0.55839	0.50835	0.46319
11	0.89632	0.80426	0.72242	0.64958	0.58468	0.52679	0.47509	0.42888
12	0.88745	0.78849	0.70138	0.62460	0.55684	0.49697	0.44401	0.39711
13	0.87866	0.77303	0.68095	0.60057	0.53032	0.46884	0.41496	0.36770
14	0.86996	0.75788	0.66112	0.57748	0.50507	0.44230	0.38782	0.34046
15	0.86135	0.74301	0.64186	0.55526	0.48102	0.41727	0.36245	0.31524
16	0.85282	0.72845	0.62317	0.53391	0.45811	0.39365	0.33873	0.29189
17	0.84438	0.71416	0.60502	0.51337	0.43630	0.37136	0.31657	0.27027
18	0.83602	0.70016	0.58739	0.49363	0.41552	0.35034	0.29586	0.25025
19	0.82774	0.68643	0.57029	0.47464	0.39573	0.33051	0.27651	0.23171
20	0.81954	0.67297	0.55368	0.45639	0.37689	0.31180	0.25842	0.21455
21	0.81143	0.65978	0.53755	0.43883	0.35894	0.29416	0.24151	0.19866
22	0.80340	0.64684	0.52189	0.42196	0.34185	0.27751	0.22571	0.18394
23	0.79544	0.63416	0.50669	0.40573	0.32557	0.26180	0.21095	0.17032
24	0.78757	0.62172	0.49193	0.39012	0.31007	0.24698	0.19715	0.15770
25	0.77977	0.60953	0.47761	0.37512	0.29530	0.23300	0.18425	0.14602
n	9%	10%	12%	15%	20%	25%	30%	40%
1	0.91743	0.90909	0.89286	0.86957	0.83333	0.80000	0.76923	0.71429
2	0.84168	0.82645	0.79719	0.75614	0.69444	0.64000	0.59172	0.51020
3	0.77218	0.75131	0.71178	0.65752	0.57870	0.51200	0.45517	0.36443
4	0.70843	0.68301	0.63552	0.57175	0.48225	0.40960	0.35013	0.26031
5	0.64993	0.62092	0.56743	0.49718	0.40188	0.32768	0.26933	0.18593
6	0.59627	0.56447	0.50663	0.43233	0.33490	0.26214	0.20718	0.13281
7	0.54703	0.51316	0.45235	0.37594	0.27908	0.20972	0.15937	0.09486
8	0.50187	0.46651	0.40388	0.32690	0.23257	0.16777	0.12259	0.06776
9	0.46043	0.42410	0.36061	0.28426	0.19381	0.13422	0.09430	0.04840
10	0.42241	0.38554	0.32197	0.24718	0.16151	0.10737	0.07254	0.03457
11	0.38753	0.35049	0.28748	0.21494	0.13459	0.08590	0.05580	0.02469
12	0.35553	0.31863	0.25668	0.18691	0.11216	0.06872	0.04292	0.01764
13	0.32618	0.28966	0.22917	0.16253	0.09346	0.05498	0.03302	0.01260
14	0.29925	0.26333	0.20462	0.14133	0.07789	0.04398	0.02540	0.00900
15	0.27454	0.23939	0.18270	0.12289	0.06491	0.03518	0.01954	0.00643
16	0.25187	0.21763	0.16312	0.10686	0.05409	0.02815	0.01503	0.00459
17	0.23107	0.19784	0.14564	0.09293	0.04507	0.02252	0.01156	0.00328
18	0.21199	0.17986	0.13004	0.08081	0.03756	0.01801	0.00889	0.00234
19	0.19449	0.16351	0.11611	0.07027	0.03130	0.01441	0.00684	0.00167
20	0.17843	0.14864	0.10367	0.06110	0.02608	0.01153	0.00526	0.00120
21	0.16370	0.13513	0.09256	0.05313	0.02174	0.00922	0.00405	0.00085
22	0.15018	0.12285	0.08264	0.04620	0.01811	0.00738	0.00311	0.00061
23	0.13778	0.11168	0.07379	0.04017	0.01509	0.00590	0.00239	0.00044
24	0.12640	0.10153	0.06588	0.03493	0.01258	0.00472	0.00184	0.00031
25	0.11597	0.09230	0.05882	0.03038	0.01048	0.00378	0.00142	0.00022

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TABLE 8.3 Uniform series compound amount factor ($F/A_{i,n}$)

n	1%	2%	3%	4%	5%	6%	7%	8%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.01000	2.02000	2.03000	2.04000	2.05000	2.06000	2.07000	2.08000
3	3.03010	3.06040	3.09090	3.12160	3.15250	3.18360	3.21490	3.24640
4	4.06040	4.12161	4.18363	4.24646	4.31013	4.37462	4.43994	4.50611
5	5.10101	5.20404	5.30914	5.41632	5.52563	5.63709	5.75074	5.86660
6	6.15202	6.30812	6.46841	6.63298	6.80191	6.97532	7.15329	7.33593
7	7.21354	7.43428	7.66246	7.89829	8.14201	8.39384	8.65402	8.92280
8	8.28567	8.58297	8.89234	9.21423	9.54911	9.89747	10.25980	10.63663
9	9.36853	9.75463	10.15911	10.58280	11.02656	11.49132	11.97799	12.48756
10	10.46221	10.94972	11.46388	12.00611	12.57789	13.18079	13.81645	14.48656
11	11.56683	12.16872	12.80780	13.48635	14.20679	14.97164	15.78360	16.64549
12	12.68250	13.41209	14.19203	15.02581	15.91713	16.86994	17.88845	18.97713
13	13.80933	14.68033	15.61779	16.62684	17.71298	18.88214	20.14064	21.49530
14	14.94742	15.97394	17.08632	18.29191	19.59863	21.01507	22.55049	24.21492
15	16.09690	17.29342	18.59891	20.02359	21.57856	23.27597	25.12902	27.15211
16	17.25786	18.63929	20.15688	21.82453	23.65749	25.67253	27.88805	30.32428
17	18.43044	20.01207	21.76159	23.69751	25.84037	28.21288	30.84022	33.75023
18	19.61475	21.41231	23.41444	25.64541	28.13238	30.90565	33.99903	37.45024
19	20.81090	22.84056	25.11687	27.67123	30.53900	33.75999	37.37896	41.44626
20	22.01900	24.29737	26.87037	29.77808	33.06595	36.78559	40.99549	45.76196
21	23.23919	25.78332	28.67649	31.96920	35.71925	39.99273	44.86518	50.42292
22	24.47159	27.29898	30.53678	34.24797	38.50521	43.39229	49.00574	55.45676
23	25.71630	28.84496	32.45288	36.61789	41.43048	46.99583	53.43614	60.89330
24	26.97346	30.42186	34.42647	39.08260	44.50200	50.81558	58.17667	66.76476
25	28.24320	32.03030	36.45926	41.64591	47.72710	54.86451	63.24904	73.10594
n	9%	10%	12%	15%	20%	25%	30%	40%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0900	2.1000	2.1200	2.1500	2.2000	2.2500	2.3000	2.4000
3	3.2781	3.3100	3.3744	3.4725	3.6400	3.8125	3.9900	4.3600
4	4.5731	4.6410	4.7793	4.9934	5.3680	5.7656	6.1870	7.1040
5	5.9847	6.1051	6.3528	6.7424	7.4416	8.2070	9.0431	10.9456
6	7.5233	7.7156	8.1152	8.7537	9.9299	11.2588	12.7560	16.3238
7	9.2004	9.4872	10.0890	11.0668	12.9159	15.0735	17.5828	23.8534
8	11.0285	11.4359	12.2997	13.7268	16.4991	19.8419	23.8577	34.3947
9	13.0210	13.5795	14.7757	16.7858	20.7989	25.8023	32.0150	49.1526
10	15.1929	15.9374	17.5487	20.3037	25.9587	33.2529	42.6195	69.8137
11	17.5603	18.5312	20.6546	24.3493	32.1504	42.5661	56.4053	98.7391
12	20.1407	21.3843	24.1331	29.0017	39.5805	54.2077	74.3270	139.2348
13	22.9534	24.5227	28.0291	34.3519	48.4966	68.7596	97.6250	195.9287
14	26.0192	27.9750	32.3926	40.5047	59.1959	86.9495	127.9125	275.3002
15	29.3609	31.7725	37.2797	47.5804	72.0351	109.6868	167.2863	386.4202
16	33.0034	35.9497	42.7533	55.7175	87.4421	138.1085	218.4722	541.9883
17	36.9737	40.5447	48.8837	65.0751	105.9306	173.6357	285.0139	759.7837
18	41.3013	45.5992	55.7497	75.8364	128.1167	218.0446	371.5180	1,064.6971
19	46.0185	51.1591	63.4397	88.2118	154.7400	273.5558	483.9734	1,491.5760
20	51.1601	57.2750	72.0524	102.4436	186.6880	342.9447	630.1655	2,089.2064
21	56.7645	64.0025	81.6987	118.8101	225.0256	429.6809	820.2151	2,925.8889
22	62.8733	71.4027	92.5026	137.6316	271.0307	538.1011	1,067.2796	4,097.2445
23	69.5319	79.5430	104.6029	159.2764	326.2369	673.6264	1,388.4635	5,737.1423
24	76.7898	88.4973	118.1552	184.1678	392.4842	843.0329	1,806.0026	8,032.9993
25	84.7009	98.3471	133.3339	212.7930	471.9811	1,054.7912	2,348.8033	11,247.1990

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TABLE 8.4 Uniform series present worth factor ($P/A_{i,n}$)

n	1%	2%	3%	4%	5%	6%	7%	8%
1	0.99010	0.98039	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593
2	1.97040	1.94156	1.91347	1.88609	1.85941	1.83339	1.80802	1.78326
3	2.94099	2.88388	2.82861	2.77509	2.72325	2.67301	2.62432	2.57710
4	3.90197	3.80773	3.71710	3.62990	3.54595	3.46511	3.38721	3.31213
5	4.85343	4.71346	4.57971	4.45182	4.32948	4.21236	4.10020	3.99271
6	5.79548	5.60143	5.41719	5.24214	5.07569	4.91732	4.76654	4.62288
7	6.72819	6.47199	6.23028	6.00205	5.78637	5.58238	5.38929	5.20637
8	7.65168	7.32548	7.01969	6.73274	6.46321	6.20979	5.97130	5.74664
9	8.56602	8.16224	7.78611	7.43533	7.10782	6.80169	6.51523	6.24689
10	9.47130	8.98259	8.53020	8.11090	7.72173	7.36009	7.02358	6.71008
11	10.36763	9.78685	9.25262	8.76048	8.30641	7.88687	7.49867	7.13896
12	11.25508	10.57534	9.95400	9.38507	8.86325	8.38384	7.94269	7.53608
13	12.13374	11.34837	10.63496	9.98565	9.39357	8.85268	8.35765	7.90378
14	13.00370	12.10625	11.29607	10.56312	9.89864	9.29498	8.74547	8.24424
15	13.86505	12.84926	11.93794	11.11839	10.37966	9.71225	9.10791	8.55948
16	14.71787	13.57771	12.56110	11.65230	10.83777	10.10590	9.44665	8.85137
17	15.56225	14.29187	13.16612	12.16567	11.27407	10.47726	9.76322	9.12164
18	16.39827	14.99203	13.75351	12.65930	11.68959	10.82760	10.05909	9.37189
19	17.22601	15.67846	14.32380	13.13394	12.08532	11.15812	10.33560	9.60360
20	18.04555	16.35143	14.87747	13.59033	12.46221	11.46992	10.59401	9.81815
21	18.85698	17.01121	15.41502	14.02916	12.82115	11.76408	10.83553	10.01680
22	19.66038	17.65805	15.93692	14.45112	13.16300	12.04158	11.06124	10.20074
23	20.45582	18.29220	16.44361	14.85684	13.48857	12.30338	11.27219	10.37106
24	21.24339	18.91393	16.93554	15.24696	13.79864	12.55036	11.46933	10.52876
25	22.02316	19.52346	17.41315	15.62208	14.09394	12.78336	11.65358	10.67478
n	**9%**	**10%**	**12%**	**15%**	**20%**	**25%**	**30%**	**40%**
1	0.91743	0.90909	0.89286	0.86957	0.83333	0.80000	0.76923	0.71429
2	1.75911	1.73554	1.69005	1.62571	1.52778	1.44000	1.36095	1.22449
3	2.53129	2.48685	2.40183	2.28323	2.10648	1.95200	1.81611	1.58892
4	3.23972	3.16987	3.03735	2.85498	2.58873	2.36160	2.16624	1.84923
5	3.88965	3.79079	3.60478	3.35216	2.99061	2.68928	2.43557	2.03516
6	4.48592	4.35526	4.11141	3.78448	3.32551	2.95142	2.64275	2.16797
7	5.03295	4.86842	4.56376	4.16042	3.60459	3.16114	2.80211	2.26284
8	5.53482	5.33493	4.96764	4.48732	3.83716	3.32891	2.92470	2.33060
9	5.99525	5.75902	5.32825	4.77158	4.03097	3.46313	3.01900	2.37900
10	6.41766	6.14457	5.65022	5.01877	4.19247	3.57050	3.09154	2.41357
11	6.80519	6.49506	5.93770	5.23371	4.32706	3.65640	3.14734	2.43826
12	7.16073	6.81369	6.19437	5.42062	4.43922	3.72512	3.19026	2.45590
13	7.48690	7.10336	6.42355	5.58315	4.53268	3.78010	3.22328	2.46850
14	7.78615	7.36669	6.62817	5.72448	4.61057	3.82408	3.24867	2.47750
15	8.06069	7.60608	6.81086	5.84737	4.67547	3.85926	3.26821	2.48393
16	8.31256	7.82371	6.97399	5.95423	4.72956	3.88741	3.28324	2.48852
17	8.54363	8.02155	7.11963	6.04716	4.77463	3.90993	3.29480	2.49180
18	8.75563	8.20141	7.24967	6.12797	4.81219	3.92794	3.30369	2.49414
19	8.95011	8.36492	7.36578	6.19823	4.84350	3.94235	3.31053	2.49582
20	9.12855	8.51356	7.46944	6.25933	4.86958	3.95388	3.31579	2.49701
21	9.29224	8.64869	7.56200	6.31246	4.89132	3.96311	3.31984	2.49787
22	9.44243	8.77154	7.64465	6.35866	4.90943	3.97049	3.32296	2.49848
23	9.58021	8.88322	7.71843	6.39884	4.92453	3.97639	3.32535	2.49891
24	9.70661	8.98474	7.78432	6.43377	4.93710	3.98111	3.32719	2.49922
25	9.82258	9.07704	7.84314	6.46415	4.94759	3.98489	3.32861	2.49944

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TABLE 8.5 Modified ACRS depreciation rates

Year	Recovery Period					
	3-Year, %	5-Year, %	7-Year, %	10-Year, %	15-Year, %	20-Year, %
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.462
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

Source: IRS 2017

Book Value

Book value is the value of the asset at the end of its life. Book value does not necessarily equal market value.

$$BV = \text{initial cost} - \Sigma D_j \quad \textbf{(EQ 8.20)}$$

where

BV = book value
D_j = depreciation in year j

DEPLETION ALLOWANCE

Depletion is the using up of natural resources by mining, drilling, quarrying stone, or cutting timber. The depletion deduction allows an owner or operator to account for the reduction of a product's reserves. Depletion allowance can be calculated via cost (Whitney and Sibbald 1992) or percentage. Chapter 9, "Depletion," in IRS publication 535, *Business Expenses* (2018) contains all information on depletion.

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Cost Depletion

$$CD_n = (CB - \Sigma D_i)\,[U_n/(U_n + U_r)] \quad \textbf{(EQ 8.21)}$$

where

CD_n = cost depletion allowance in year n
CB = original cost basis
ΣD_i = accumulated depletion in preceding years (both cost and percentage)
U_n = units of product sold during year n
U_r = units of product remaining at year end (units in reserves)

Percentage Depletion

The depletion percentages for minerals are shown in Table 8.6. This information is taken directly from U.S. Code Title 26 (which is Internal Revenue Code Subtitle A Income Taxes, Chapter 1 Normal taxes and surtaxes, Subchapter I Natural resources, Part I Deductions, Section 613 Percentage depletion, b Percentage depletion rates).

TABLE 8.6 Depletion percentages for minerals

(1) 22%

(A) Sulfur and uranium

(B) If from deposits in the United States—anorthosite, clay, laterite, nepheline syenite (to the extent that alumina and aluminum compounds are extracted therefrom), asbestos, bauxite, celestite, chromite, corundum, fluorspar, graphite, ilmenite, kyanite, mica, olivine, quartz crystals (radio grade), rutile, block steatite talc, zircon, and ores of the following metals: antimony, beryllium, bismuth, cadmium, cobalt, columbium, lead, lithium, manganese, mercury, molybdenum, nickel, platinum and platinum group metals, tantalum, thorium, tin, titanium, tungsten, vanadium, and zinc

(2) 15%—if from deposits in the United States

(A) Gold, silver, copper, and iron ore

(B) Oil shale [except shale described in paragraph (5)]

(3) 14%

(A) Metal mines [if paragraph (1)(B) or (2)(B) do not apply], rock asphalt, and vermiculite

(B) If paragraph (1)(B), (5), or (6)(B) do not apply—ball clay, bentonite, china clay, sagger clay, and clay used or sold for use for purposes dependent on its refractory properties

(4) 10%

Asbestos [if paragraph (1)(B) does not apply], brucite, coal, lignite, perlite, sodium chloride, and wollastonite

(5) 7½%

Clay and shale used or sold for use in the manufacture of sewer pipe or brick, and clay, shale, and slate used or sold for use as sintered or burned lightweight aggregates

(6) 5%

(A) Gravel, peat, pumice, sand, scoria, shale [except shale described in paragraph (2)(B) or (5)], and stone [except stone described in paragraph (7)]

(B) Clay use, or sold for use, in the manufacture of drainage and roofing tile, flower pots, and kindred products

(C) If from brine wells—bromine, calcium chloride, and magnesium chloride

(7) 14%

All other minerals, including, but not limited to, aplite, barite, borax, calcium carbonates, diatomaceous earth, dolomite, feldspar, fullers earth, garnet, Gilsonite, granite, limestone, magnesite, magnesium carbonates, marble, mollusk shells (including clam shells and oyster shells), phosphate rock, potash, quartzite, slate, soapstone, stone (used or sold for use by the mine owner or operator as dimension stone or ornamental stone), thenardite, tripoli, trona, and [if paragraph (1)B does not apply] bauxite, flake graphite, fluorspar, lepidolite, mica, spodumene, and talc (including pyrophyllite), except that, unless sold on bid in direct competition with a bona fide bid to sell a mineral listed in paragraph (3), the percentage shall be 5% for any such other mineral [other than slate to which paragraph (5) applies] when used, or sold for use, by the mine owner or operator as rip rap, ballast, road material, rubble, concrete aggregates, or for similar purposes. For purposes of this paragraph, the term "all other minerals" does not include:

(A) soil, sod, dirt, turf, water, or mosses;

(B) minerals from seawater, the air, or similar inexhaustible sources; or

(C) oil and gas wells.

For the purposes of this subsection, minerals (other than sodium chloride) extracted from brines pumped from a saline perennial lake within the United States shall not be considered minerals from an inexhaustible source.

Adapted from IRS 2011

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EFFECTIVE TAX RATE

$$\text{Effective tax rate} = s + f(1 - s) \quad \textbf{(EQ 8.22)}$$

where

s = marginal state tax rate, decimal
f = marginal federal tax rate, decimal

BENEFIT COST RATIO

The benefit cost ratio (BCR) is a ratio comparing present worth net positive cash flows to present worth net negative cash flows. BCR is also known as the *profitability index* and can be calculated using the following equation. If the BCR is greater than 1, then the project economics are considered satisfactory. If the BCR is equal to 1, then the project economics are at the breakeven point. And if the BCR is less than 1, the project economics are unsatisfactory.

$$\text{BCR} = \frac{\text{present worth net positive cash flow at } i^*}{\left|\text{present worth net negative cash flow at } i^*\right|} \quad \textbf{(EQ 8.23)}$$

where i^* is the discount rate.

PRESENT VALUE RATIO

The present value ratio (PVR) is a variation of the BCR and can be determined using the following equations. If the PVR is greater than 0, then the project economics are considered satisfactory. If the PVR is 0, then the project economics are at the breakeven point. And if the PVR is less than 0, the project economics are unsatisfactory.

$$\text{PVR} = \frac{\text{net present value at } i^*}{\left|\text{present worth net negative cash flow at } i^*\right|} \quad \textbf{(EQ 8.24)}$$

$$\text{PVR} = \text{BCR} - 1 \quad \textbf{(EQ 8.25)}$$

INFLATION

Inflation is a measure of the *increase* in cost of goods and services from year to year. Figure 8.1 is a diagram explanation of the relationship between escalated present worth calculations and constant dollar present worth calculations.

$$i = (1 + f)(1 + i') - 1 \quad \textbf{(EQ 8.26)}$$

where

i = escalated dollar rate of return
i' = constant dollar rate of return
f = inflation rate

CONSUMER PRICE INDEX

The consumer price index (CPI) is a measure of the average change in prices for goods and services purchased by the consumer. The CPI data are available for utility, automotive fuel, and food items; and the data are available for different U.S. geographical regions. Table 8.7 shows the CPI for all items in the United States from 1960 through 2017. CPI can be used to calculate inflation/deflation.

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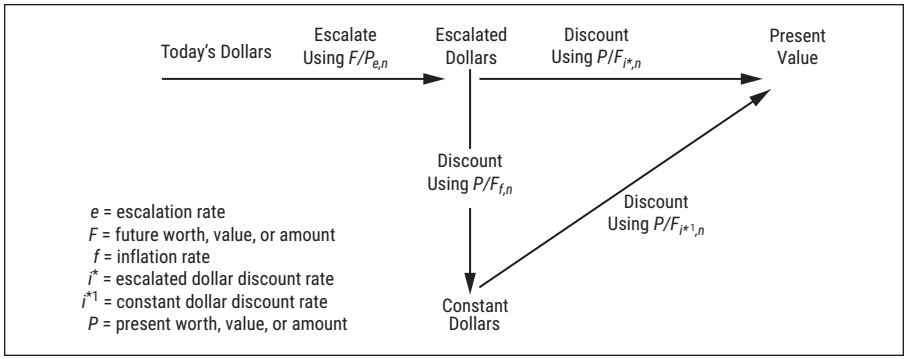

Source: Stermole et al. 2019

FIGURE 8.1 Equivalent escalated dollar and constant dollar present value calculations

TABLE 8.7 Consumer price index

Consumer Price Index (major expenditures—all items: 1982–1984 = 100*)											
Year	Index	Year	Index	Year	Index	Year	Index	Year	Index	Year	Index
1960	29.6	1970	38.8	1980	82.4	1990	130.7	2000	172.2	2010	218.1
1961	29.9	1971	40.5	1981	90.9	1991	136.2	2001	177.1	2011	224.9
1962	30.2	1972	41.8	1982	96.5	1992	140.3	2002	179.9	2012	229.6
1963	30.6	1973	44.4	1983	99.6	1993	144.5	2003	184.0	2013	233.0
1964	31.0	1974	49.3	1984	103.9	1994	148.2	2004	188.9	2014	236.7
1965	31.5	1975	53.8	1985	107.6	1995	152.4	2005	195.3	2015	237.0
1966	32.4	1976	56.9	1986	109.6	1996	156.9	2006	201.6	2016	240.0
1967	33.4	1977	60.6	1987	113.6	1997	160.5	2007	207.3	2017	245.1
1968	34.8	1978	65.2	1988	118.3	1998	163.0	2008	215.3	2018	251.1
1969	36.7	1979	72.6	1989	124.0	1999	166.6	2009	214.5		

* 1982–1984 = 100 because the United States currently uses these years as the reference base period with a factor of 100.

Source: Council of Economic Advisers 2001; U.S. Bureau of Labor Statistics 2018

CAPITAL STRUCTURE

Capital structure of an entity is defined as the way the entity utilizes capital to provide optimal value to the entity, owners, shareholders, and so forth. Financial statements are used to report the status of the entity and assist managers in determining the entity's optimum capital structure.

Financial statements include income statements, balance sheets, and statements of cash flow. Statements of cash flow provide information on cash flows from operations, cash flows from investing activities, and cash flows from financing activities of the entity. The balance sheet provides information on assets and liabilities of the entity (Table 8.8).

The income statement provides information on the profitability of the entity (Table 8.9).

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TABLE 8.8 Components of capital structure (balance sheet)

Assets =	Liabilities +	Equity
Current assets	Current liabilities	Common shares
Cash	Accounts payable	Preferred shares
Marketable securities	Notes payable	Share premium above par
Accounts receivables	Current portion of long-term debt	Retained earnings
Inventory	Taxes	
	Lease payments	
Long-term assets	Long-term liabilities	Subordinated debt?
Mine/plant	Bank loans	
Buildings	Bonds	
Equipment	Debentures	
Land	Long-term notes	
Mineral rights	Leases	
Ore reserves/resources	Subordinated debt	

Source: Tinsley 1992

TABLE 8.9 Components for developing cash flows (income statement)

Calculation	Component
	Revenue
Less	Royalties
Equal	Gross income from mining
Less	Operating costs
Equal	Net operating income
Less	Depreciation and amortization allowance
Equal	Net income after depreciation and amortization
Less	Depletion allowance
Equal	Net taxable income
Less	State income tax
Equal	Net federal taxable income
Less	Federal income tax
Equal	Net profit after taxes
Add	Depreciation and amortization allowances
Add	Depletion allowance
Equal	Operating cash flow
Less	Capital expenditures
Less	Working capital
Equal	Net annual cash flow

Source: Gentry and O'Neil 1984

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NET PRESENT VALUE

Table 8.10 presents typical factors to consider when developing cash-flow analysis for net present value (NPV) analysis.

TABLE 8.10 Factors for consideration in cash-flow analysis

Preproduction Period	
Exploration expenses	Land and mineral rights
Water rights	Environmental costs
Mine and plant capital requirements	Development costs
Sunk costs	Financial structure
Working capital	Administration
Production Period	
Price	Capital investment—replacement and expansions
Processing costs	Royalty
Recovery	Mining cost
Postconcentrate cost	Development cost
Reserves and percent removable	Exploration cost
Grade	General and administration
Investment tax credit	Insurance
State taxes	Production rate in tons per year
Federal taxes	Financial year production begins
Depletion rate	Percent production not sent to processing plant
Depreciation	Operating days per year
Postproduction Period	
Salvage value	Contractual and reclamation expenditures

Source: Laing 1977

NPV can be determined using the following equation.

$$\text{NPV} = -|CF_0| + \sum_{j=1}^{n} CF_j \left(\frac{P}{F}, i\%, j \right) \quad \textbf{(EQ 8.27)}$$

where

NPV = net present value in dollar amount
CF_0 = initial cash flow (assumed to be negative as it is an outlay) in dollar amount
CF_j = cash flow for period j in dollar amount
P/F = single payment present worth factor
i = interest, %
j = period (= 1 to n)

RATE OF RETURN

Rate of return (ROR) is the series of cash flows at interest (i) where the NPV is 0. ROR can be found by trial and error or substituting different i values; computer programs can determine this value as well.

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PAYBACK PERIOD

The payback period is the time required for an initial investment to be recovered. Payback period calculations ignore the time-value of money and should be used judiciously.

$$|CF_0| = \sum_{j=1}^{PBP} CF_j \quad \text{(EQ 8.28)}$$

where

PBP = payback period (j periods)
CF_0 = initial cash investment in dollar amount
CF_j = cash inflows for period j
j = number of periods, generally years (could also be months, quarters, etc.)

If yearly cash inflows are equal, then

$$\text{PBP} = \frac{|CF_0|}{\text{YCF}} \quad \text{(EQ 8.29)}$$

where

PBP = payback period, years
CF_0 = initial cash investment in dollar amount
YCF = yearly cash inflow for equal inflows or average yearly cash flow in dollar amount

COSTING

Costs are an important component of capital structure. To maintain profitability, costs must be controlled. Cost estimation and cost reporting are significant elements.

Cost Estimation Using the Six-Tenths Rule

One method to estimate costs of similar items of different capacities is to use the six-tenths rule, which is indicated by the following equation (Katell and Wellman 1968):

$$C_a = C_b\,(A/B)^{0.6} \quad \text{(EQ 8.30)}$$

where

C_a = cost of item of size or capacity A
C_b = cost of similar item of size or capacity B
A = size or capacity of item A
B = size or capacity of item B

The exponential factor varies from 0.1 to greater than 1. For mining projects, typical values range from 0.5 to 0.9 (Gentry and O'Neil 1984).

Cost Reports

Table 8.11 is an example of a monthly cost report for a mine and mill. Tables 8.12 and 8.13 are examples of monthly income statements.

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TABLE 8.11 Monthly cost report for a mine and a mill

	Actual, $	Budget, $	Variance	
			Amount	Percent
Mine Department				
Production				
Ore tons mined	1,120,000	1,250,000	−130,000	90
Waste tons mined	2,017,000	2,005,000	12,000	101
Ore grade, % Cu	0.80	0.78	0.02	103
Cost, $				
Operating labor	920,000	876,000	(44,000)	105
Mechanical labor	756,000	779,000	23,000	97
Salary labor	105,000	104,000	(1,000)	101
Fuel	947,000	976,000	29,000	97
Operating supplies	876,000	894,000	18,000	98
Mechanical supplies	1,002,000	808,000	(194,000)	124
Miscellaneous and administrative	14,000	28,000	14,000	50
Allocated overhead	420,000	393,000	(27,000)	107
Total	5,040,000	4,858,000	(182,000)	104
Mill Department				
Production				
Recovery	87.8%	88.3%	−0.5%	99
Product grade	96.4%	96.3%	0.1%	100
Copper produced, lb	15,733,760	17,218,500	−1,484,740	91
Cost, $				
Operating labor	187,000	189,000	2,000	99
Mechanical labor	205,000	212,000	7,000	97
Salary labor	52,100	49,600	(2,500)	105
Power	530,000	565,000	35,000	94
Operating supplies	164,000	176,000	12,000	93
Mechanical supplies	107,500	105,000	(2,500)	102
Miscellaneous and administrative	15,000	21,000	6,000	71
Allocated overhead	116,000	113,000	(3,000)	103
Total	1,376,600	1,430,600	54,000	96

Source: Cavender 1999

TABLE 8.12 Example of a monthly income statement: full absorption costing basis

Full Absorption Costing Basis, $000	Actual, $	Budget, $	Variance	
			Amount, $	Percent
Sales at $1/lb Cu	15,734	17,219	1,485	91
Less: Cost of goods sold				
Mining cost	5,040	4,858	(182)	104
Milling cost	1,377	1,431	54	96
Gross margin	9,317	10,930	1,613	85
Less: Selling, general, and administrative costs	8,143	7,725	(418)	105
Operating profit	1,174	3,205	2,031	37

Source: Cavender 1999

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TABLE 8.13 Example of monthly income statement: variable costing basis

Variable Costing Basis, $000	Actual, $	Budget, $	Variance	
			Amount, $	Percent
Sales (at $1/lb Cu)	15,734	17,219	1,485	91
Less: Variable costs				
Mining cost	4,217	4,073	(144)	104
Milling cost	1,097	1,138	41	96
Variable selling, general, and administrative costs	6,514	6,180	(334)	105
Contribution margin	3,905	5,827	1,922	67
Less: Fixed costs				
Mining cost	823	785	(38)	105
Milling cost	280	292	13	96
Fixed selling, general, and administrative costs	1,629	1,545	(84)	105
Operating profit	1,174	3,205	2,031	37

Source: Cavender 1999

ROYALTIES

Royalties are payments made to the owner or lessor of the property to allow extraction and sale of minerals by a mining entity from the property. Royalty payments are generally based on a percentage of revenue, and may be in addition to other fees imposed to allow extraction of minerals from a property. Royalties may be paid to federal, state, and/or private landowners. Federal leases generally require a royalty payment in addition to rent payments. Typical royalty rates on federal lands are as follows (30 U.S.C. ch. 3A):

- Coal (surface), 12.5%
- Coal (underground), 8%
- Phosphates, 5%
- Sodium (includes chlorides, sulfates, carbonates, borates, silicates, or nitrates of sodium), 2%
- Sulfur, 5%
- Potash (includes chlorides, sulfates, carbonates, borates, silicates, or nitrates of potassium), 2%

These royalty rates are applicable to the gross value of the product at the point of shipment to the nearest market.

Net Smelter Return

Net smelter return (NSR) is the revenue from the sales of a metal/nonmetal product less refining and transportation costs. NSR can be used to calculate royalty payments to a landowner or lessor.

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CHAPTER

9

Project Evaluation

Keith E. Dyas, P.E.

FEASIBILITY STUDIES AND DUE DILIGENCE

A *project evaluation* includes all the activities conducted to develop information for determining a project's economic viability. The results are included in a *feasibility study*, which integrates the data into a project plan and analyzes the economics. A feasibility study examines all phases of an investment proposal in as much detail as necessary to justify dropping it or continuing expenses through the next stage. A project evaluation progresses through several levels of feasibility as additional data are accumulated. Generally, feasibility levels advance from conceptual (prefeasibility) to preliminary to intermediate to final. After each level, investigators decide, based on economics, environmental considerations, and market timing, whether to advance to the next level and ultimately whether to build. Different companies use different terminology to define levels of feasibility.

Investigators must be careful to include all required elements and to conduct their studies at the appropriate level of investigation. If the investigation is preliminary, additional time and expense should not be spent to secure detail that belongs in the intermediate or final level.

Table 9.1 is a checklist of minimum reporting requirements for feasibility studies. The process has three steps: conceptual, prefeasibility, and feasibility.

TABLE 9.1 Minimum report content for engineering studies

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Introduction			
Location, topography, and climate			
Site location map	Basic map	Preliminary map showing claims and boundaries	Detailed map showing all claims and boundaries
Topography map	Basic map showing site topography	Preliminary map showing site topographic features	Detailed topographic map; aerial surveys verified with ground controls and surveys
Property ownership	Review of property lease	Review of property lease; claims list provided; mineral rights known	Property lease and rights secured and controlled; claims list and map provided; mineral rights secured
Current status and history			
Historical chronology	Basic presentation	Full presentation	Detailed presentation
Past production (if any)	Basic presentation	Full presentation	Detailed presentation
Exploration and Geology			
Geologic description			
Review	Preliminary review	Preliminary site-specific analysis	Detailed site-specific analysis
Data posting	Review of available existing maps	Detailed geologic mapping with cross-sections, lithology and mineralogy, and domains	Deposit well defined with three-dimensional mapping, geologic maps, long sections, level plans, lithology and mineralogy, and domains

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Exploration and Geology (continued)			
Geologic assessment	Preliminary	Basic assessment and review	Detailed assessment of structures/rock contacts, alteration, mineralization, deposit trends
Mineralogical sampling and analysis	Limited sampling; preliminary assessment	Preliminary mineralogical sampling and analysis; preliminary mineralogical study	Detailed mineralogical sampling and mapping; detailed mineralogical study
Drilling, sampling, and assaying			
Drill hole parameters	Wide-spaced drilling as appropriate	Initial infills of wide-spaced drilling; preliminary grid patterns	Close-spaced drilling on a detailed grid pattern to support calculated reserve categories
Underground drilling	Review of existing data	Drilling if accessible	Detailed drilling if accessible
Samples	Preliminary; some outcrop samples	Geophysical and geotechnical sampling; test pits	All sampling programs complete
Drilling and assay data	Preliminary check of existing drill hole data	Check of drill holes (coordinates, elevations, angles, etc.), check assays, angled hole vs. vertical hole comparison; assay flow diagram, dependable database	Check of drilling holes (coordinates, elevations angles, etc.); check assays, angled hole vs. vertical hole, etc.; check assays, angled hole vs. vertical hole comparison, twin hole drilling; assay flow diagram; validated database
Quality assurance/quality control (QA/QC) protocol and data	Defined QA/QC protocol that verifies sample and assay results; protocol should include blanks, standard reference material, coarse and pulp duplicates, field duplicates, and third-party check assays; verification samples should constitute minimum of 10% of the sample stream	Defined QA/QC protocol that verifies sample and assay results; protocol should include blanks, standard reference material, coarse and pulp duplicates, field duplicates, and third-party check assays; verification samples should constitute minimum of 10% of the sample stream	Defined QA/QC protocol that verifies sample and assay results; protocol should include blanks, standard reference material, coarse and pulp duplicates, field duplicates, and third-party check assays; verification samples should constitute a minimum of 10% of the sample stream
Condemnation drilling	None	None	Areas under waste dumps, tailings, and plant drilled
Resources/Reserves (internationally recognized standards*)			
Resources	Measured, indicated, and inferred	Measured, indicated, and inferred	Measured, indicated, and inferred
Geologic controls	Assumed	Established from geologic data and/or variograms	Well established from geologic data
Tonnage factors	Preliminary assessment if available	Preliminary analysis and determinations	Detailed analysis and determinations
Statistical analysis	Preliminary analysis and determinations	Preliminary analysis and determinations	Detailed analysis and determinations

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Resources/Reserves (internationally recognized standards*) (continued)			
Geostatistical analysis	Preliminary analysis and determinations	Preliminary analysis and determinations	Detailed analysis and determinations
Pit optimization	Pit limit optimization software	Pit analysis and determinations	Pit limit optimization software
Reserves	Only resources estimated	Proven over probable	Proven and probable
Calculation parameters	Usually no reserves estimated	Known or estimated	Detailed analysis and determinations including dilution and losses
Pit optimization	Pit limit optimization software	Pit analysis and determinations	Pit limit optimization software
Cutoff grade (COG) equations	Manually calculate to estimate minable inventory or use strategic mine planning software	Manually calculate to estimate minable inventory (underground project) or use strategic mine planning software (underground and open pit)	Manually calculate to estimate minable inventory (underground project) or use strategic mine planning software (underground and open pit)
Mining			
Mining method	Assumed between open pit and underground	Specific method identified, generic equipment	Method and mine plan finalized including optimization of specialized mining equipment and equipment types
Open pit mine plan			
Pit slopes	Assumed	Preliminary 3-D model based on preliminary estimates by rock type and basic geotechnical data	3-D model defined by geotechnical data from structural mapping and oriented core holes
Pit design	Simple Lerchs–Grossman (LG) cone outline of final pit	Preliminary pit design from optimized analysis; preliminary haul road incorporated, trade-off studies completed	Detailed, optimized pit designs with phases and access for equipment operation
Waste dumps	Simple outline of final dumps	Preliminary design for total waste tonnage; incremental and final outline of dumps, trade-off studies completed	Dump sites identified from geotechnical data; final waste tonnages determined with incremental phases, yearly, and final dump outlined
Underground mine plan			
	Assumed mining system; general outline of mine plan and development	Preliminary mining system identified from geologic and geotechnical data; preliminary outline of mine plan and development plan and development including mine access	Specific mining system identified from geologic and geotechnical data; detailed outline of mine plan and development including mine access

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Mining (continued)			
All mining operations			
Production schedule	Basic schedule based on assumed mine life	Annual schedule of life of mine of ore and waste tonnages and grade, plant grade, recovery and production; some sequence optimization, e.g., optimize grade or quality profile	Detailed schedules with monthly time increments from ramp-up to steady state, then annual thereafter, for life of mine; should match budget timeline schedule; multiple iterations to optimize mining sequence; report ore/product quality and waste tonnages and grades plus plant recovery and production
Capital cost estimate	Order-of-magnitude, factored or from similar operations	Preliminary equipment list; budget or historical price quotes; some factoring	Detailed equipment list; firm price quotes for all major equipment items; all capital items identified
Operating cost estimate	Order-of-magnitude, factored or from similar operations	Quantified estimates for labor, power, and consumables; budget, or historical price quotes for unit prices; some factoring	Detailed engineering estimate by project area based on quotes and studies
Processing			
Ore sampling and test work	Laboratory bench-scale data and ore characterization data used in combination with plant benchmarking to develop preliminary recovery and throughput	Range of flow sheets with recovery and throughput developed from bench-scale testing and often verified with pilot scale work, especially for new processes	Sampling of core for variability testing to identify ranges of throughput and recovery backed up with locked-cycle and/or pilot testing
Processing Engineering and Design			
Production rate and product(s)	First estimate of production rate and product(s)	Preliminary mining and processing rates and plant product(s)	Fixed mining and processing rates and plant product(s)
Design basis	Preliminary using factored estimates	General design basis; preliminary engineering drawings; trade-off studies completed	Complete design basis; basic engineering drawings essentially complete
Design concept	Outline of design criteria and specifications incorporating area/regional climatic conditions	Design criteria established for construction site incorporating known site climatic conditions	Design specifications defined incorporating known site climatic conditions
Tailings containment	Identify possible sites and capacities	Identify several options and determine the best site for sites	Finalize sites, location; develop geotechnical data for site and for tailings; generate general arrangement drawings and preliminary specifications
Process description	General	Narrative: 1%–2% of detail engineering complete	Detailed: 5%–15% of detail engineering complete

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Processing Engineering and Design (continued)			
Layout	Approximate geographic locations and site map; no general arrangement drawings	Optimization of facility locations on site map showing topography; simple general arrangement drawings of major equipment items	Exact geographic locations on site map with topography; detailed general arrangement drawings; detailed layout of all facilities
Flow sheets	Assumed flow sheet from known processes; simple block diagram	Establishment of probable flow sheet from preliminary test work data; major process flow diagrams; initial determinations of material and heat balances	Detailed flow sheet based on comprehensive beneficiation test program, detailed equipment list; diagrams for all process flows; material and heat balances finalized
Civil work	Rough topographic maps; no soil conditions considered or quantities estimated	Rough topographic maps; soil conditions report for foundation determinations; basic preliminary quantities	Detailed topographic maps with soil conditions identified for foundation design, loadings, and quantities
Equipment specifications	Major equipment items listed	Preliminary listing of major equipment items with initial sizings and specifications	Complete listing of major equipment items with detailed sizings and specifications
Architectural	None	Sketches	Exterior elevations only
Piping/HVAC	None	Preliminary piping and instrumentation diagram (P&ID)	Major P&ID
Electrical distribution	None	Basic one-line diagram	All design one-line diagram
Motors	None	General description	Detailed list of major items with horsepower
Instrumentation	None	General description	Detailed list of components
Infrastructure			
Facilities	General overview with types of support facilities described	All required support facilities identified, sizes and quantities estimated	All necessary support facilities identified, sized, and costed
Communications	Communications requirements identified	Communications systems study	Communications licensing and standards known
Power	Overview of power availability and regional unit power costs	Power sources and requirements identified; unit costs obtained from power source	Power requirements and unit costs derived from detailed engineering study; unit costs from quotes
Hydrology			
Water sources	Estimated using regional data	Preliminary hydrology study	Specific water source identified
Water usage	Factored plant volume and unit costs	Required plant water volumes and unit costs estimated	Requisite plant volumes and unit costs derived from detailed engineering/geotechnical studies
Dewatering	Dewatering parameters identified	Dewatering parameters estimated	Dewatering parameters confirmed and plan defined

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Hydrology (continued)			
Settling	Preliminary evaluation of project setting for potentially significant environmental and social constraints for site data	Preliminary evaluation of the project's impact on the environment; schedule of environmental and social other permitting requirements; evaluate project setting for potentially significant environmental and social permitting constraints from site data	Characterization of all the project's potential impacts; finalize schedule of environmental and/or other permitting requirements; evaluate project setting for potentially significant environmental and social permitting constraints
Data posting	Collect and review all available, existing data for environmental and social studies, assessments or audits; regulatory inspections, waste handling practices, management plans, and all applicable laws and regulations; social, training or safety programs identified	Collect and review available data from existing database for environmental studies, assessments or audits; regulatory inspections, waste handling practices; management plans; and all applicable laws and regulations; plans; initiate baseline data gathering; social, training, and health/safety programs identified	All requisite environmental data for project are identified; site sampling and analyses are complete; detailed review of the type, scope, and schedule for producing environmental and social government reports; comprehensive gathering and evaluation of baseline environmental and social conditions; social, training, and health/safety programs confirmed
Environmental impact statement/environmental assessment (EIS/EA)	None	Draft EIS/EA initiated	Draft EIS/EA submitted to regulatory authorities
Reporting and plans	Conceptual plans for mitigating any identified environmental issues	Preparation of environmental and social plans and monitoring programs; preliminary sediment and erosion control plan; conceptual closure plan; evaluation of acid rock drainage; geotechnical stability review of waste dumps and tailings dam; preliminary impact mitigation plan; preliminary spill and emergency response plan	Environmental characteristics used in project design; environmental plans and monitoring programs finalized; sediment and erosion control plan; finalize management plans for tailings and waste rock; management plan finalized for solid and hazardous wastes; finalize impact mitigation plan; geotechnical stability analysis of all major facilities; finalize closure plan; final analysis of acid rock drainage; finalize spill and emergency response plan
Monitoring	Not considered	Outline of a site environmental monitoring plan	Complete environmental monitoring plan
Permit requirements	General overview	Comprehensive overview and listing of required permits	Detailed evaluation of all pertinent authorizations and permitting requirements and schedule for obtaining operating license

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Project Development Schedule			
Development plan	Development period and mine life estimated	Development period and overall schedule estimated; mine life determined; development schedule set	Detailed development schedule; mine life known; development schedule finalized
Project master schedule	Estimated showing start and end of construction; Gantt bar chart of major work elements	Gantt bar chart with overall time frames; schedule outline for detailed engineering; QA/QC program outlined; preliminary construction schedule; preliminary project execution plan	Gantt bar chart with overall time frames and project flow planning; detailed project level schedule showing project deliverables and detailed engineering; construction plan (CP) schedule; major milestones identified; project control system outlined; QA/QC and safety program finalized; preliminary project procedures manual; project design basis finalized
Capital Cost Estimate			
Basis			
Civil, structural, architectural, piping/HVAC, electrical, instrumentation, construction labor, productivity, material, volumes/amounts, material/equipment, pricing infrastructure	Order-of-magnitude based historic data or factoring	Estimates from historical factors, percentages, and vendor quotes based on material volumes	Detailed from estimates; engineering 15%–25% complete; multiple vendor quotes
Contractors	Included in unit cost or as a percentage of total cost	Percentage of direct cost by cost area for contractor; historic for subcontractors	Written quotes from contractor and subcontractors
Engineering, procurement, and construction management (EPCM)	Percentage of estimated construction cost	Percentage of detailed construction cost	Calculated estimate from EPCM firm
Pricing	Free on board (FOB) mine site including all taxes and duties	FOB mine site including all taxes and duties	FOB mine site including all taxes and duties
Owner's	Historic estimate	Estimate from experience factored for similar project	Estimate prepared from detailed zero-based budget
Environmental compliance	Factored from historic experience	Estimate from experience factored for similar project	Estimate prepared from detail zero-based budget for design engineering and specific permit requirements
Escalation	Typically not considered	Based on company's current budget percentage	Based by cost area with risk

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TABLE 9.1 Minimum report content for engineering studies (continued)

Description	Preliminary Economic Assessment (scoping study)	Prefeasibility Study	Feasibility Study
Capital Cost Estimate (continued)			
Working capital	Factored from historic experience	Estimate from experience factored for similar project	Estimate prepared from detail zero-based budget
Accuracy	±50%	±25%	±15%
Contingency	25%	15%	10%
Operating Cost Estimate			
Basis	Order-of-magnitude estimate	Estimates for unit rates and quantified estimates with some factoring	Detailed from zero-based budget; minimal factoring
Operating quantities	General		
Unit cost	Historic unit costs factoring		
Accuracy	±35%	±25%	±15%
Economic Evaluation			
Financial analysis	Preliminary assessment of principal economic parameters	Assessment of the principal economic parameters	Full assessment of all principal economic parameters
Marketing and commodity price(s)	Industry knowledge and consensus pricing	Preliminary market analysis to confirm project placement, quality targets and production constraints; consensus pricing	Detailed marketing study and consensus pricing or price forecasts
Royalties and taxes	Preliminary assessment	Preliminary analysis and determinations	Detailed analysis with tax authority opinion
Smelting, refining, and freight	Historic data	Budgetary quotes	Firm quotes
Cash flow analysis	Simple cash flow	Preliminary cash flow	Formal, detailed cash flow
Economic criteria	Simple internal rate of return (IRR) and net present value (NPV) (pre- and after-tax)	Preliminary IRR and NPV (pre- and after-tax)	Fully defined IRR, NPV, return on investment (ROI), and payback period (pre- and after-tax)
Sensitivity analysis	Basic analysis to minimal amount of project variables	Preliminary to select key project variables	Numerous analyses to all key project variables
Risk Evaluation			
Risk assessment	General overview	Fatal flaw analysis	Risk workshop; formal Monte Carlo analysis and fatal flaw analysis
Project	Preliminary overview of geology, engineering, and environmental	Preliminary environmental, country, permitting, technology, and business; detailed geology and engineering	Detailed geology, engineering, environmental, legal, permitting, country, technology, business, and financial

* Internationally Recognized Standards include the following:

1. National Instrument NI-43-101, NI-43-101CP.
2. See the JORC Code (JORC 2012).
3. SEC (2019).
4. Resources and Reserves Committee (2017).
5. Environmental aspects should consider applicable local, national, and international guidelines including International Finance Corporation (IFC) and Equator Principle standards.

Source: RPMGlobal 2015

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A *conceptual study* is a preliminary evaluation of a project. Although the level of drilling and sampling must be sufficient to define a resource adequately, flow sheet development, cost estimation, and production scheduling are often based on limited test work and engineering design. This level of study is useful for defining subsequent engineering inputs and further required studies, but it is not appropriate for economic decision-making.

The *prefeasibility study* represents an intermediate step between the conceptual and final feasibility studies. Cost estimates are of the order of ±30% accuracy, but are not considered sufficiently accurate for final decision-making.

The *feasibility study*, considered the bankable document, is detailed and accurate enough to be used for positive go decisions and financing purposes. Cost estimates are ±20% accurate or better. Mine plans show materials movements and ore grades on an annual basis. Flow sheet development is based on extensive test work, material balances, and general arrangement drawings. Economic evaluation is based on cash flow calculations for the life of the defined reserve.

Principles of a Resource/Reserve Classification for Minerals

The following principles are based on U.S. Bureau of Mines and U.S. Geological Survey information (USBR and USGS 1980). Long-term public and commercial planning must be predicated on the probability of discovering new deposits, developing economic extraction processes for currently unworkable deposits, and knowing which resources are immediately available. Thus, resources must be continuously reassessed in the light of new geologic knowledge, progress in science and technology, and shifts in economic and political needs. Known resources should be classified from two standpoints: (1) purely geologic or physical/chemical characteristics, such as grade, quality, tonnage, thickness, and depth of the material in place; and (2) profitability analysis based on costs of extracting and marketing, the material in a given economy at a given time. The former constitutes important objective scientific information of the resource and a relatively unchanging foundation on which the latter more variable economic delineation can be based.

The revised classification system, generally designed for all mineral materials, is shown graphically in Figures 9.1 and 9.2; components and their usage are described in the text. The classification of mineral and energy resources is necessarily arbitrary because definitional criteria do not always coincide with natural boundaries. The system can be used to report the status of mineral and energy-fuel resources for the nation or for specific areas.

Cumulative Production	Identified Resources		
	Demonstrated		
	Measured	Indicated	Inferred
ECONOMIC	Reserves		Inferred Reserves
MARGINALLY ECONOMIC	Marginal Reserves		Inferred Marginal Reserves
SUBECONOMIC	Demonstrated Subeconomic Resources		Inferred Subeconomic Resources
Other Occurences	Includes Nonconventional and Low-Grade Materials		

Adapted from USBR and USGS 1980

FIGURE 9.1 Major elements of mineral-resource classification, excluding reserve base and inferred reserve base

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Cumulative Production	Identified Resources		
	Demonstrated		
	Measured	Indicated	Inferred
ECONOMIC	Reserves		Inferred
MARGINALLY ECONOMIC	Base		Reserve
SUBECONOMIC	Base		Base
Other Occurences	Includes Nonconventional and Low-Grade Materials		

Adapted from USBR and USGS 1980

FIGURE 9.2 Reserve base and inferred reserve base classification categories

Resource/Reserve Definitions

A dictionary definition of resource, "something in reserve or ready if needed," has been adapted for mineral and energy resources to comprise all materials, including those only surmised to exist, that have present or anticipated future value (USBR and USGS 1980).

- **Resource.** This is a concentration of naturally occurring solid, liquid, or gaseous material in or on the Earth's crust in such a form and amount that economic extraction of a commodity from the concentration is currently or potentially feasible.
- **Original resource.** The amount of a resource before production is the original resource.
- **Identified resources.** Resources whose location, grade, quality, and quantity are known or estimated from specific geologic evidence; they include economic, marginally economic, and subeconomic components. To reflect varying degrees of geologic certainty, these economic divisions can be subdivided into *measured*, *indicated*, and *inferred*.
  - **Demonstrated.** The sum of measured plus indicated equals demonstrated.
    - **Measured.** Quantity is computed from dimensions revealed in outcrops, trenches, workings, or drill holes; grade and/or quality are computed from the results of detailed sampling. The sites for inspection, sampling, and measurement are spaced so closely and the geologic character is so well defined that size, shape, depth, and mineral content of the resource are well established.
    - **Indicated.** Quantity and grade and (or) quality are computed from information similar to that used for measured resources, but the sites for inspection, sampling, and measurement are farther apart or are otherwise less adequately spaced. The degree of assurance, although lower than that for measured resources, is high enough to assume continuity between points of observation.
  - **Inferred.** Estimates are based on assumed continuity beyond measured and/or indicated resources, for which there is geologic evidence. *Inferred resources* may or may not be supported by samples or measurements.
  - **Reserve base.** That part of an identified resource that meets specified minimum physical and chemical criteria related to current mining and production practices, including those for grade, quality, thickness, and depth is the reserve base. It is the in-place demonstrated (measured plus indicated) resource from which reserves are estimated. It may encompass those parts of the resources that have a reasonable potential for becoming economically available within planning horizons beyond those that assume proven technology and current economics. The *reserve base* includes those resources that are currently economic (*reserves*), marginally economic (*marginal reserves*), and some of those that are currently subeconomic (*subeconomic resources*). The term *geologic reserve* has been generally applied by others to the *reserve base* category, but it may include the *inferred reserve base* category; it is not a part of this classification system.
  - **Inferred reserve base.** The in-place part of an identified resource from which inferred reserves are estimated is the inferred reserve base. Quantitative estimates are largely predicated on knowledge of the geologic character of a deposit and for which there may be no samples or measurements. The estimates are grounded on an assumed continuity beyond the reserve base, for which there is geologic evidence.

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- **Reserves.** The part of the reserve base that can be economically extracted or produced at the time of determination is the reserve. The term *reserves* need not signify that extraction facilities are in place and operative. Reserves include only recoverable materials; thus, terms such as *extractable reserves* and *recoverable reserves* are redundant and are not a part of the classification system.
- **Marginal reserves.** The part of the reserve base which, at the time of determination, borders on being economically producible is the marginal reserve. Its essential characteristic is economic uncertainty. Included are resources that would be producible, given postulated changes in economic or technologic factors.
- **Economic.** This term implies that profitable extraction or production under defined investment assumptions has been established, analytically demonstrated, or assumed with reasonable certainty.
- **Subeconomic resources.** The part of identified resources that does not meet the economic criteria of reserves and marginal reserves is the subeconomic resource.
- **Undiscovered resources.** Resources, the existence of which are only postulated, comprising deposits that are separate from identified resources meet this description. Undiscovered resources may be postulated in deposits of such grade and physical location as to render them economic, marginally economic, or subeconomic. To reflect varying degrees of geologic certainty, undiscovered resources can be divided into either hypothetical or speculative resources:
  - **Hypothetical resources.** These are undiscovered resources that are similar to known mineral bodies and that may be reasonably expected to exist in the same producing district or region under analogous geologic conditions. If exploration confirms their existence and reveals enough information about their quality, grade, and quantity, they are reclassified as identified resources.
  - **Speculative resources.** Undiscovered resources that may occur in either unknown types of deposits in favorable geologic settings where mineral discoveries have not been made, or in types of deposits as yet unrecognized for their economic potential are considered speculative. If exploration confirms their existence and reveals enough information about their quantity, grade, and quality, they are reclassified as identified resources.
- **Restricted resources/reserves.** That part of any resource/reserve category that is restricted from extraction by laws or regulations is restricted. For example, *restricted reserves* meet all the requirements of reserves except that they are restricted from extraction by laws or regulations.

DUE DILIGENCE PROCESS

Although due diligence is followed in other industries, RPMGlobal (2011) outlines the specifics for the mining industry in this section. The due diligence process for a new or expanding mine project normally involves the review of a bankable-level feasibility study that has been prepared by a mining company, a mineral engineering company, or alternatively, a mining company working in concert with a mineral engineering company. This work is carried out under the direction of a project manager assisted by a multidisciplinary review team consisting of professionals selected with a focus on matching specific backgrounds and experience to the particular project to be reviewed.

Completeness Review

The first step in a due diligence review is to ensure that all the major elements normally associated with mine project feasibility work are fully addressed. These elements are as follows:

- Geology and resource estimate
- Mining and reserve estimate
- Processing
- Environmental and social evaluation and permitting
- Infrastructure and administration
- Project schedule
- Economics

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Site Visit

The next step in the due diligence process usually consists of a site visit. In the case of a grassroots project, the site-visit team generally includes a geologist, a mining engineer, and an environmental specialist. A process engineer normally arranges a visit to the metallurgical laboratory where the test work has been carried out. In the case of an ongoing mining operation, the process engineer would normally be part of the site-visit team.

During the site visit, the technical team reviews local infrastructure, performs a general inspection of the property, examines exploratory drifts and available core, checks for adequacy and review sampling methods (geologic, metallurgical, geotechnical, etc.), discusses environmental management, and collects all relevant data. It is always desirable for the diligence-review team to meet with the sponsoring company's technical team during or shortly after the site visit.

Technical Review

After the site visit, a technical review is carried out to confirm that the level of engineering and accuracy of cost estimates are consistent with feasibility (i.e., bankable) requirements.

Geology and Resource Estimate

Sound geological interpretation and appropriate sampling are the foundations of a successful mining project. Geological and geostatistical methodology, assumptions, and resource calculations are reviewed, and requisite statistical checks are performed.

Mining and Reserve Estimate

Project economics are reviewed to determine that appropriate cutoff grades have been calculated in the preparation of the mine plan and reserve estimate. Opinions are developed as to the suitability of the proposed mining method, equipment selection, and capital- and operating-cost forecasts. Mine plans are reviewed to confirm that the production schedule is feasible and accurately reflected in the cash flow.

Process Plan Engineering and Construction

Metallurgical test work, mass balances, reagent requirements, plant flow sheet and process design, recovery estimates, plant construction and operations, as well as forecast capital and operating costs are reviewed.

Geotechnical and Hydrological Engineering

Geotechnical investigations and designs for mine and plant construction are reviewed. Mine-waste design including tailings-dam structures, heap-leach pads, impoundments, and waste rock disposal areas; and overall water-balance requirements are reviewed for conformance with appropriate design standards.

Environmental and Social Impact Assessment

Analysis of the potential environmental and social impacts of a mining project are an important consideration in the due diligence process. Typically, the identification of potential environmental and social impacts, and appropriate mitigation measures to offset these impacts, are articulated in an environmental and social impact assessment (ESIA), or in other documentation that captures this same intent. Often this documentation has been prepared to demonstrate compliance with environmental and social regulatory requirements that are contained in host country law. As such, it is important to verify the current permitting status of the project within the context of host country law, and to identify potential obstacles that may prevent the timely obtainment of all required permits to implement the project.

In addition to an evaluation of a project's permit status, the following items are typically addressed in the due diligence review:

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- An evaluation of the thoroughness and quality of available environmentally and socially related documentation, including appropriateness of methodology used and summary interpretations
- An evaluation of the comprehensiveness and suitability of baseline data
- Investigation of any legacy contamination from past mining practices
- An opinion of the adequacy of waste disposal methods
- An appraisal of the potential for the project to give rise to adverse community or regional reaction
- An appraisal of the appropriateness and reasonability of closure/reclamation planning and costs
- An appraisal of the appropriateness and reasonability of project environmental management system, management system, management monitoring plans, and environmental and community relations staffing
- A high-level opinion of the project's alignment with identified international standards

The nature of the environmental and social analysis for a given project largely depends on the financial institution requesting the industrial engineering assessment. For example, there are currently 94 financial institutions that are signatories to the Equator Principles (EPs), which comprise a common baseline and framework for the implementation of social and environmental policies, procedures, and standards related to project financing activities. The EPs, in turn, are largely based on the International Finance Corporation (IFC) Performance Standards and World Bank Group's sector-specific environmental, health, and safety guidelines.

Infrastructure and Administration

Mine and plant infrastructure are reviewed to determine if necessary services, such as communications, fire control, industrial and potable water, power, roads, safety and medical facilities, security, and sewage treatment, are adequate and that capital and operating costs reflect these services. Mine and plant administrative, supervisory, and technical personnel, and organization structure are reviewed. General and administrative costs are checked. Additional proposed facilities such as maintenance shops, warehouses, employee housing, common area maintenance (CAM) facilities, and explosive storage are assessed.

Logistics and Constructability

Many mining projects are being built in remote locations in developing countries. Whether the site is in the frozen tundra of Far East Russia or the jungles of Indonesia, the logistics and constructability of new projects require careful scrutiny to make certain that project scheduling and capital allotments are adequate.

Management and Organization

Review and comment on the level of management expertise available to the facility, and provide an opinion on possible shortcomings in the management/organization structure.

Material Contracts

Material contracts relating to the development/operation of the mine are reviewed.

Economic Analysis

The detailed cash flow analysis prepared by the owners or client is reviewed or, alternatively, an independent cash flow analysis is prepared. Should it be necessary, revisions in the cash flow and sensitivity studies are completed to represent alternative scenarios requested by the client.

Report Preparation

A professional technical due diligence report is prepared at the conclusion of the work. The report summarizes all the results and work done including data, pertinent assumptions, and methodologies used. Risks and concerns regarding the project are clearly identified and quantified where possible.

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Project Completion

Under a true project-finance scenario, lending institutions universally include project completion covenants as part of the loan agreements. Completion requirements under these covenants typically focus separately on (1) physical and/or mechanical completion followed by (2) the performance and/or production completion. Lending institutions frequently request assistance in defining these covenants with specific and measurable requirements. In addition, a client may request a review of proposed engineering and construction contracts at this time.

Construction Monitoring

The role of the industrial engineer (IE) during the construction period depends on the complexity of the project and the needs of the financing entity. Typically, an IE is retained to monitor construction progress to ensure that construction keeps pace with the capital outlays and that construction matches the feasibility study or accepted development plan. Any revisions to the approved plan are generally reviewed and approved by the IE before implementation. Construction-related reports are reviewed monthly and periodic site trips are a common requirement.

Physical or Mechanical Completion

Physical or mechanical completion occurs when all infrastructure is in place, the mine is developed to a point where it can produce ore on a continuous basis at the required tonnage and grade, and the process plant is built to design specification. Typically, the IE verifies that the major components are in place, operational, and consistent with the development plan approved by the financial institution.

Performance or Production Completion

A typical performance or production completion test is performed over a 90- to 180-day period depending on the covenants of the loan agreement. During the test period, the mine and process plant must operate at agreed-on minimum capacities, ore grades, and recoveries, with unit costs and consumptions rates not exceeding agreed-on levels. Often completion requirements include performance tests of major process circuits such as crushers, grinding mills, conveyors, and flotation cells as well as reconciliation of ore produced against the geologic model used in the feasibility study.

Monitoring of the production test period by the independent consultant requires close cooperation with the sponsor company. As with the physical completion test, once the production test is successfully completed, an approximate certification document is executed.

Production Monitoring

It is not uncommon for the IE to monitor project operations until completion of the payback of the loan. This work may include a review of monthly reports and annual or quarterly site visits. Table 9.2 is a due diligence checklist developed by RPMGlobal (2011).

Geostatistics

Geostatistics is a generic name for a family of techniques, which are used for mapping of surfaces from limited sample data and the estimation of values at unsampled locations. First developed almost 60 years ago by Georges Matheron and named in honor of Danie Krige, these methods are now widely used in the minerals industry and have disseminated out into many other fields where spatial data is studied (Clark, n.d.).

Geostatistical estimation is a two-stage process:

1. Study the gathered data to establish the predictability of values from place to place in the study area. This study results in a graph known as a *semivariogram*, which models the difference between a value at one location and the value at another location according to the distance and direction between them.

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2. Estimate values at those locations that have not been sampled. This process is known as *kriging*. The basic technique, called *ordinary kriging*, uses a weighted average of neighboring samples to estimate the unknown value at a given location. Weights are optimized using the semivariogram model, the location of the samples and all the relevant interrelationships between known and unknown values. The technique also provides a standard error, which may be used to quantify confidence levels.

TABLE 9.2 Due diligence checklist

GEOLOGY
- Regional geology
- Mining district history—property ownership
- Property geology
- Ore genesis
- Mineralization
- Weathering and post mineralization
- Summary of controlling geology
- Exploration and database compilation
- Exploration history
- Regional mapping
- Topography
- Drilling and sampling
- Drill hole location map
- Geotechnical logging
- Bulk density determinations
- Geological logging
- Hydrological modeling
- Sampling and sample prep
- Assaying
- Quality assurance/quality control procedures—check assays
- Twinned holes or samples
- Assays by size fractions, sampling protocol
- Surface trenching, pit samples or channel
- Excluded samples and reasons
- Chain of custody program
- Database description
- Checks of duplicates, repeats, etc.
- Check randomly selected samples
- Downhole bias contamination checks
- Database adjustments

RESOURCE ESTIMATION
- Geological model development
- Identification of controlling lithology
- Construction of rock/lithology model
- Date declustering
- Block model definition
- Modeling method
- Block statistics
- Basic statistics and cumulative frequency
- Population identification
- Correlation
- Compositing
- Composite statistics
- Variography
- Capping confidence limits
- Existing workings
- Model validation
- Constrained pit resources
- Resources statement

MINING

All Mines
- Existing project description
- Ore type descriptions
- Cutoff grade assumptions
- Grade control method
- Metallurgical impacts
- Stockpiling
- Stockpile aging
- Dilution and recovery
- Hydrology and water management

Surface Mining
- Whittle/Lerchs–Grossman (LG) pit optimization
- Pit design parameters
- Artificial pit restrictions
- Tonnage optimization curve
- Pit shell check
- Geotechnical analysis pit slopes
- Pit design basis from clones/LG
- Stope and road design
- Ultimate pit design
- Ultimate pit reserves
- Phase plans
- Phased pit resources
- Production schedule
- Schedule optimization
- Pit advance maps with dumps
- Pit water control and hydrology
- Project-specific conditions
- Dump design—geotechnical geochemical
- Dump design—geometry
- Dump reclamation considerations

Underground Mining
- Primary access (shaft/decline/adit)
- Primary development (main haulage)
- Secondary development (slope access)
- Mining method
- Stope design
- Ventilation problems (CO, radon, methane, etc.)
- Ventilation design
- Power distribution
- Water distribution
- Compressed air distribution
- Emergency evacuation
- Pumping
- Rock mechanics and ground support methods
- Backfill type and design
- Stope production capacities
- Stope extraction schedule

General Mining
- Equipment selection
- Operating parameters
- Special considerations
- Mechanical parameters
- Drilling
- Blasting
- Loading
- Hauling
- Support
- Operating hour summary
- Staffing
- Training
- Engineering and survey
- Management

MILLING
- Metallurgical background
- Metallurgical sample locations
- Metallurgical test work
- Crushing test work
- Grinding test work
- Metallurgical recovery
- Metallurgical trouble spots
- Flow sheet
- Capacity
- Operating parameters
- Special considerations
- Mill production schedule
- Mechanical parameters
- Staffing
- Training
- Management

INFRASTRUCTURE
- Access roads
- Power supply
- Water supply
- Housing
- Logistics, warehousing for supplies
- Port facilities
- Product transport
- Air service
- Medical care
- Communications
- Security
- Administrative staffing
- Accounting
- Personal
- Government relations
- Purchasing
- Information management
- Site layout and facilities

ENVIRONMENTAL
- Site background/baseline
- Climate
- Vegetation
- Seismology
- Special conditions
- Air quality
- Hydrology, surface
- Hydrology, subsurface
- Biota
- Acid generation potential
- Waste dump chemistry
- Endangered species
- Local population
- Regulatory agencies
- Permit requirements/status
- Site impacts
- Reclamation
- Structure foundation characterization
- Regulatory history

CAPITAL COSTS
- General cost summary
- Mining
- Processing
- Infrastructure
- Sustaining capital
- Working capital
- Owner's costs
- Engineering, procurement, and construction management (EPCM) costs

OPERATING COSTS
- Mining
- Processing
- Infrastructure
- Assessed charges
- Capitalized component breakout

MARKETING
- Market analysis
- Concentrate sales agreements
- Dore sales agreements
- Freight and insurance

MATERIAL CONTRACTS
- Construction contracts
- Power supply
- Off-take/refining
- Fuel and consumables

MANAGEMENT/ORGANIZATION
- Organization structure
- Labor relations
- Adequacy of training

PROJECT SCHEDULE
- Site background/baseline
- Task list
- Critical path chart

ECONOMIC ANALYSIS
- Project revenues
- Exclusions
- Taxes and customs
- Royalties
- Financing
- Discount rate
- Cash flow projection
- Sensitivity

Source: RPMGlobal 2011

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PREPARING REPORTS FOR THE U.S. SECURITIES AND EXCHANGE COMMISSION

The U.S. Securities and Exchange Commission (SEC) has specific rules for preparing property disclosure reports. Here is a summary of those reporting rules for mining industry registrants (SEC 2018):

- Rules are consistent with global standards set by the Committee for Mineral Reserves International Reporting Standards (CRIRSCO).
- Specified information must be disclosed in the company's Securities Act and Exchange Act filings concerning mineral resources that have been determined on one or more of its properties.
- Non-reserve estimates, such as mineral resources, are permitted to be disclosed only in limited circumstances.
- Documentation must be prepared by a mining expert, who is designated the "qualified person."
- A dated and signed technical report summary is required from the qualified person or persons, which identifies and summarizes the information reviewed and conclusions determined to be on each material property. A registrant must file the technical report summary as an exhibit to the relevant Commission filing when disclosing mineral reserves or mineral resources for the first time or when there is a material change in the mineral reserves or mineral resources from the last technical report summary filed for the property.
- The qualified person is required to use a price for each commodity that provides a reasonable basis for establishing estimates of mineral resources or mineral reserves, which may be a historical or forward-looking price, as long as the qualified person discloses and explains his or her reasons for using the selected price, including the material assumptions underlying the selection.
- The quantitative presumptions regarding when a registrant's mining operations are deemed material and when a change in previously reported estimates of mineral resources or mineral reserves are also considered material are both eliminated.
- The registrant is permitted to file a technical report summary to support its disclosure of exploration results.
- The disclosure of exploration targets is permitted in Commission filings if accompanied by certain specified cautionary and explanatory statements.
- The qualified person is permitted to include inferred resources in an economic analysis that the qualified person opts to include in an initial assessment as long as certain conditions are met.
- *Mineral reserve* is defined to include diluting materials and allowances for losses that may occur when the material is mined or extracted.
- The qualified person is permitted to conduct either a prefeasibility or final feasibility study to support a determination of mineral reserves even in high-risk situations.
- Historical estimates of mineral resources or reserves are permitted to be used in filings pertaining to mergers and acquisitions if the registrant is unable to update the estimate prior to the completion of the relevant transaction, provided that the registrant discloses the source and date of the estimate, and does not treat the estimate as a current estimate.

Governing Definitions of a Report on Exploration, Resources, and Reserves

- **Transparency.** The reader of a public report is provided with clear and unambiguous information.
- **Materiality.** The public report contains all relevant information to make reasoned and balanced decisions.
- **Competence.** The work is required to be performed by a suitably qualified and experienced mining professional.

Figure 9.3 shows the relationship between exploration information and mineral resources and reserves as required for SEC reporting purposes. The *modifying factors* are considerations used to convert measured and indicated mineral resources to proven and probable reserves including mining, processing, metallurgical, economic, marketing, legal, environmental, infrastructure, social, and governmental factors (SME Resources and Reserves Committee 2017).

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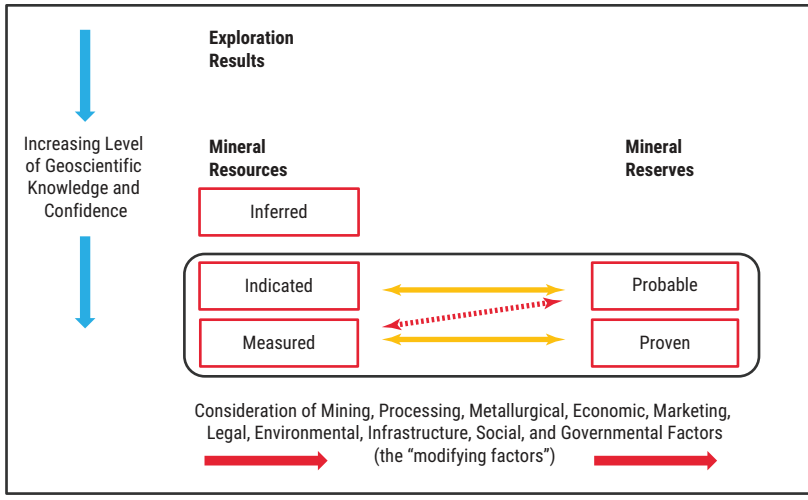

Source: SME Resources and Reserves Committee 2017

Figure 9.3 Exploration information and mineral resources and reserves framework

Standard Format for a Technical, Prefeasibility, or Feasibility Report

Following are the recommended sections to include in a technical, prefeasibility, or feasibility report. It is not necessary to include all of these sections in every project.

Section 1: Summary
Section 2: Introduction
Section 3: Reliance on Other Specialists
Section 4: Property Description and Location
Section 5: Accessibility, Climate, Local Resources, Infrastructure, and Physiography
Section 6: History
Section 7: Geological Setting and Mineralization
Section 8: Deposit Types
Section 9: Exploration
Section 10: Drilling
Section 11: Sample Preparation, Analyses, and Security
Section 12: Data Verification
Section 13: Mineral Processing and Metallurgical Testing
Section 14: Mineral Resource Estimates
Section 15: Mining Methods and Mine Plan
Section 16: Recovery and Processing Methods
Section 17: Project Infrastructure
Section 18: Market Studies and Contracts
Section 19: Environmental Studies, Permitting, and Social or Community Impact
Section 20: Capital and Operating Costs
Section 21: Economic Analysis
Section 22: Mineral Reserve Estimates
Section 23: Adjacent Properties
Section 24: Other Relevant Data and Information

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Section 25: Interpretation and Conclusions

Section 26: Recommendations

Section 27: References

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CHAPTER

10

Mining Methods

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The objective of this chapter is to present an overview of conventional mechanical mining methods—both surface and underground—that can be applied to the extraction of metal and nonmetal ores; coal; as well as sand, gravel, and crushed stone. Approximate production data for commodities mined in the United States is presented to illustrate the relative importance of various mining methods for different commodities (Table 10.1). Tables are used to compare the applicability and relative merits of various mining methods. Photographs and drawings illustrate the most common mining methods. A glossary of common mining terms and their definitions concludes the chapter.

Table 10.2 contains ore deposit genesis in order of induration or hardening of rocks over time. This table is useful for a start in mining method selection giving shape characteristics and hardness. Table 10.3 shows underground mining method selection related to deposit characteristics. Table 10.4 offers a matrix comparing mining methods with characteristics of using the method. Table 10.5 is a classification of surface mining methods and suggests suitable excavation methods given the deposit shape and characteristics. Table 10.6 is a classification of underground mining methods and suggests suitable excavation methods given the deposit shape and considering the rock strength and degree of support required.

TABLE 10.1 Commodity mining production data summary*

Material	Year	Surface Mining		Underground Mining		Total Mining
		MT†	Percent	MT	Percent	MT
Metal ores (Nelson 2011)	2007	1,330	98.6	19	1.4	1,350
Nonmetal ores (Nelson 2011)	2007	3,040	95.9	129	4.1	3,170
Coal (Nelson 2011)	2007	795	69.3	352	30.7	1,147
Construction sand and gravel (USGS 2016)	2015	—	—	—	—	931
Crushed stone (USGS 2016)	2015	—	—	—	—	1,390

* Approximate production data for various commodities mined in the United States by different mining methods.

† MT = million short tons.

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TABLE 10.2 Deposits related to geometry, genesis, and strength (in order of induration or hardening of rocks)

Deposits Type	Geometry	Genesis	Strength and Stiffness, Deposit/Country Rock	Examples
Alluvium (placers)	Tabular-flat	Surface-stream action deposition (fans, deltas, meanders, braids)	Poor/poor	Sand and gravel; precious metals and stones (tin)
Erosion surface (swamps)	Tabular-flat and thin (possible folding)	Swamps (possible dynamic metamorphism)	Poor/poor to good	Coal
Disseminated	Massive	Underground channels and multifaceted advance	Poor/poor	Hydrothermal ores (porphyry coppers and sulfides)
Vein (can be rehealed)	Tabular-inclined (pipes, chimney shoots)	Major underground channels (fissures), gouge, alteration (reheal)	Poor to good/good	Hydrothermal ores (porphyry coppers and sulfides)
Evaporites	Tabular-flat-thick	Interior drainage	Good/good	Salt, phosphates
Sedimentary (bedded)	Tabular-flat-thick	Shallow seas	Good/good	Limestone, sandstone
Metamorphic	Tabular-flat-thick	Dynamic and/or thermal	Good/good	Marble, slate
Igneous (magnetic)	Massive	Plutonic emplacement	Good/good	Granite, basalt, diabase

Source: Adler and Thompson 2011

TABLE 10.3 Underground mining method selection related to deposit characteristics

Deposit Depth	Rock Strength	Deposit Characteristics	Mining Method	Commodities Mined
Deep—requires underground mining	Strong to moderate competent rock—amenable to unsupported openings	Tabular, flat, thin, large size	Room and pillar	Coal, stone, some metals, some nonmetals
		Tabular, flat, thick, large size	Stope and pillar (vertical crater retreat)	Metals
		Tabular, steep, thin, any size	Shrinkage stoping	Metals
		Tabular, steep, thick, large size	Sublevel stoping	Metals
	Moderate to weak incompetent rock—openings require support	Variable shape, steep, thin, any size, rock stronger than backfill	Overhand cut-and-fill stoping	Metals
		Variable shape, steep, thin, any size, rock weaker than backfill	Underhand cut-and-fill stoping	Metals
		Tabular, steep, thin, small size	Stull stoping	Metals
		Any shape, any dip, thick, any size	Square-set stoping	Metals
	Moderate to weak rock—amenable to caving	Tabular, flat, thin, large size	Longwall	Coal, some metals, some nonmetals
		Tabular or massive, steep, thick, large size	Sublevel caving	Metals
		Massive, steep, thick, large size	Block caving	Metals

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TABLE 10.4 Factors to be considered when selecting a mining method

Method	Relative Cost	Flexibility/Selectivity	Percent Recovery/ Percent Dilution	Environment	Safety and Health	Output, t/h, and Productivity, t/employee	Miscellaneous
Placers and dredging	0.05	Low/high	High/low	High impact, and water pollution	Fair	Moderate	Need water; impact of weather
Open cast	0.10	Moderate/moderate	High/low	Blasting can lead to frequent claims and water pollution	Fair	High	Flat topography and impact of weather
Open pit	0.10	Moderate/moderate	High/low	Ground disturbance, waste piles, and some water problems	Slope stability (slides)	High	Impact of weather
Quarry	1.00	Low/high	High/high	Ground disturbance and waste piles	Slope stability	Very low	Skilled workers and impact of weather
Room and pillar (coal)	0.30	High/high	50–80/20	Subsidence and water pollution	Ground control and ventilation	High	Pillaring common
Stope and pillar	0.30	High/high	75/15	Good	Ground control and ventilation	High	Benching common
Sublevel stope	0.40	Low/low	75/15	Fill to avoid subsidence	Less, blast from long holes	Moderate	Fill common
Shrinkage stoping	0.50	Moderate/moderate	80/10 plucking during draw	Fill to avoid subsidence	Poor floor (collapse) and stored broken mineral*	Low	Tie up ⅔ of ore
Cut and fill	0.60	Moderate/high	100/0	Fill to avoid subsidence	Some	Low	Sort in stope
Timbered square set	1.00	Moderate/high	100/0	Fill to avoid subsidence	Smolder, and fall (of personnel)	Very low	Sort in stope
Longwall	0.20	Low/low	80/10	Subsidence and water pollution	Good	Very high	High capital ≤12° dip ≤2.4-m (8-ft) thick
Sublevel caving (top slicing)	0.50	Low/low	90/20	Severe subsidence disruption	Fair and stored broken mineral*	High	Cave width ≥9.2 m (30 ft)
Block caving	0.20	Low/low	90/20	Severe subsidence disruption	Air blasts and stored broken mineral*	High	Tie up mineral

*Can pack (cement), oxidize, and smolder.

Source: Adler and Thompson 2011

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TABLE 10.5 Classification of surface mining methods

Shape, Attitude (dip)	Deposit Characteristics	Stripping Ratio	Waste Handling	Excavation Method	Example Mining Method
Tabular—Flat	Near surface	Low	Onsite	Hydraulic, scoop, dig	Placers—hydraulic mining, dredging
					Solution—at depth
	Shallow	Moderate	Cast	Scoop, dig, blast	Open cast (strip)—area, contour, mountain top
				Auger or highwall miner (after open cast mining)	Auger or highwall mining
Tabular—Inclined	Moderate	Moderate (remove hanging wall)	Haul to waste dump	Blast	Open pit
	Deep	High (remove both hang wall and footwall)	Haul to waste dump	Blast	Open pit
				Saw, jet pierce (joints)	Quarry
Massive	Full range	Depends on depth	Haul to waste dump	—	Open pit

Source: Adler and Thompson 2011

TABLE 10.6 Classification of underground mining methods based on deposit geometry and support

Deposit Shape, Attitude (dip)	Degree of Support		
	Unsupported (open stopes)	Supported	Caved
Tabular Flat (mobile bulk handling)	Room and pillar; stope and pillar	Some degree of artificial support for room and pillar and stope and pillar	Longwall; retreat room and pillar
Inclined (mixed bulk handling)	Above with scrapers, large tunnel section (on strike)	Above with scrapers, large tunnel section with artificial support	Longwall
Steep (gravity bulk handling)	Coal hydraulicking; sublevel stoping; vertical crater retreat; shrinkage stoping	Shrinkage stoping; cut-and-fill stoping; timbered stoping; fill as needed; gravity fill as needed	Sublevel caving; top slicing
Massive	Immediately above mine in vertical slices; fill with gravity placement; to remove pillars, mine and then fill horizontal lifts*		Immediately above in horizontal lifts; block caving

* For ground control problems, especially those associated with coal, treat as if they were to be extracted by thick-seam and/or multiple-seam mining. As pressure increases (especially with depth), or as rock strength decreases, shift right for suitable method (toward supported and caved).

Source: Adler and Thompson 2011

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SURFACE MINING

Surface mining occurs either by open pit, or open cast. In open pit mining, all rock is extracted out of the excavation or pit. For open cast, the overburden is laterally displaced into an adjacent pit, and the excavation face moves in strips. Hence open cast is also called *strip mining* and occurs principally in tabular minerals such as coal.

Open Pit Method for Metals, Nonmetals, and Aggregates

Open pit mining is the most extensively used mining method from both a tonnage and value perspective for extracting metals such as copper; gold; iron ore; industrial minerals, such as phosphates; and crushed stone, sand, and gravel. A large-scale open pit mine for copper or gold ores may be up to a few kilometers wide and 1 km deep and encompass a surface area of a few square kilometers or more. Figure 10.1 shows a typical large-scale open pit mine for metal ores producing more than 10 million stpy (short tons per year) of ore and waste rock. Benches, haul roads, working benches (pushbacks), and the final pit wall are shown. Figure 10.2 shows a typical small-scale open pit mine for construction aggregate producing less than about 1 million stpy of crushed stone. This type of mine can measure several hundred meters wide and several hundred meters deep and cover an area less than 1 km^2.

The basic unit operations in open pit mining are drilling, blasting, loading, and hauling followed by crushing. Typical unit operations in an open pit mine include (1) drilling operations using a large blasthole drill and small track drill; (2) blasting operations using a bulk explosives truck to load blastholes with bulk explosives, typically ammonium nitrate and fuel oil (ANFO) blasting material; (3) excavating broken rock using a large cable shovel and small front-end loader; (4) hauling rock using large and small haulage trucks; and (5) crushing ore for subsequent milling.

Courtesy of Spencer Musick

Figure 10.1 Large-scale open pit mine for metal ore

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Courtesy of San Rafael Rock Quarry

Figure 10.2 Small-scale open pit mine for construction aggregate

The equipment size and the number of machines employed in these unit operations depend on the production requirements of the mine. These unit operations usually occur on a working bench or pushback (Figure 10.3) that may be several hundred meters wide depending on the size of the mining operation. By using a sequence of pushbacks as shown in Figure 10.4, the open pit can (1) produce a relatively constant amount of ore for the mill, (2) move a relatively constant amount of waste rock, and (3) maintain a relatively constant stripping ratio over the life of the mine. In addition, the use of pushbacks minimize the amount of costly pre-stripping early in the mine life and maintain a constant stripping ratio throughout the entire mine life.

The bench (Figure 10.5) is the basic geometric unit of any open pit mine. The bench height typically ranges from 10 to 20 m and is usually determined by the size of the drilling equipment employed at the mine, which in turn depends on the size of the mining operation. The bench face angle is usually about 75 degrees but varies depending on the rock character at the mine. The bench width from toe to crest is usually about 10 m wide, but again can vary depending on operational and safety considerations. Pit walls usually contain haul roads to access the lowest levels of the pit. The slope angle from the pit crest to the toe of the lowest bench in a pit wall that contains a working bench or haul road is called an *intermediate slope angle* (Figure 10.5). The final or overall slope angle is created when the pit reaches its economic limits. The overall or final slope angle is the slope angle from the pit crest to the toe of the lowest bench, and typically ranges from 40 to 60 degrees depending on the rock character.

Blasting in open pit mining (Figure 10.6) is usually done with ANFO and related blasting products. The typical blasthole pattern is defined by the bench height, burden, spacing, and powder column of the blastholes. The drill holes are normally drilled to a depth below the bench level to facilitate fragmentation and improve excavation.

Open Cast Method for Coal and Tabular Nonmetals

Open cast mining is the most widely applied method for extracting coal and other near-surface bedded mineral deposits. A large-scale open cast mine for coal using a dragline for overburden removal may be up to 100 m deep and encompass a surface area of several square kilometers or more. Figure 10.7 shows a typical large-scale open cast mine for coal producing more than 10 million stpy of coal and waste rock.

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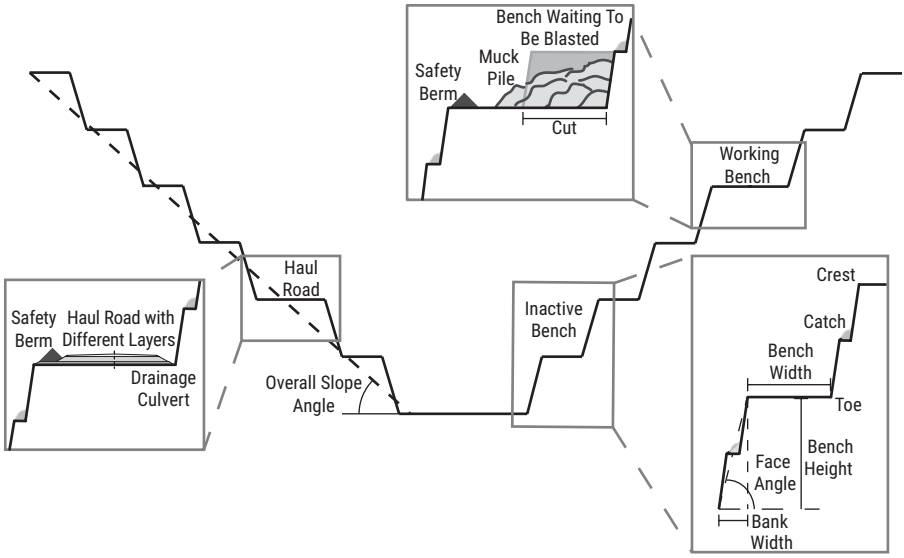
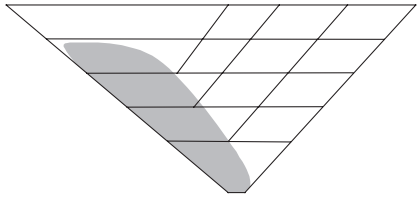


Source: Wetherelt and van der Wielen 2011

Figure 10.3 Typical open pit geometry



Source: Whittle 2011

Figure 10.4 Typical mining sequence to extract an ore body using four pushbacks

The features of an open cast mine are (1) topsoil removal, (2) pre-stripping overburden to provide level grade and meet digging/dumping dimensional requirements for the dragline, (3) overburden removal using a dragline, (4) coal removal using a cable shovel and trucks, (5) leveling overburden with a dozer, (6) topsoil replacement, and (7) revegetation of the mined area. Significant permanent waste dumps are not needed, and mine rehabilitation can be progressively performed at the same rate as mine revegetation.

A large-scale open cast mine for coal using a bucket wheel excavator for overburden removal (Figure 10.8; a conveyor belt transports the waste rock to the dump site) may be several hundred meters deep. The ideal condition for large-scale, open cast coal mining is relatively flat topography such that the depth of overburden is approximately constant. In mountainous topography with flat-lying coal seams, the contour method is practical. This method usually applies to medium-scale coal mining operations that may be up to 200 m deep and may encompass a surface area of a few square kilometers. Figure 10.9 shows a typical cross section of an open cast coal mine using the contour method with blasthole drilling and blasting. Features of contour mining are (1) overburden removal using a bulldozer, front-end loader, and trucks; (2) coal removal using a front-end loader and trucks; (3) overburden replacement in the mined-out area; (4) leveling overburden with a bulldozer; and (5) revegetation of mined area. One challenge of contour mining is to replace the overburden back onto the original bench with slopes that replicate the original contour, which is a requirement of many countries' permits.

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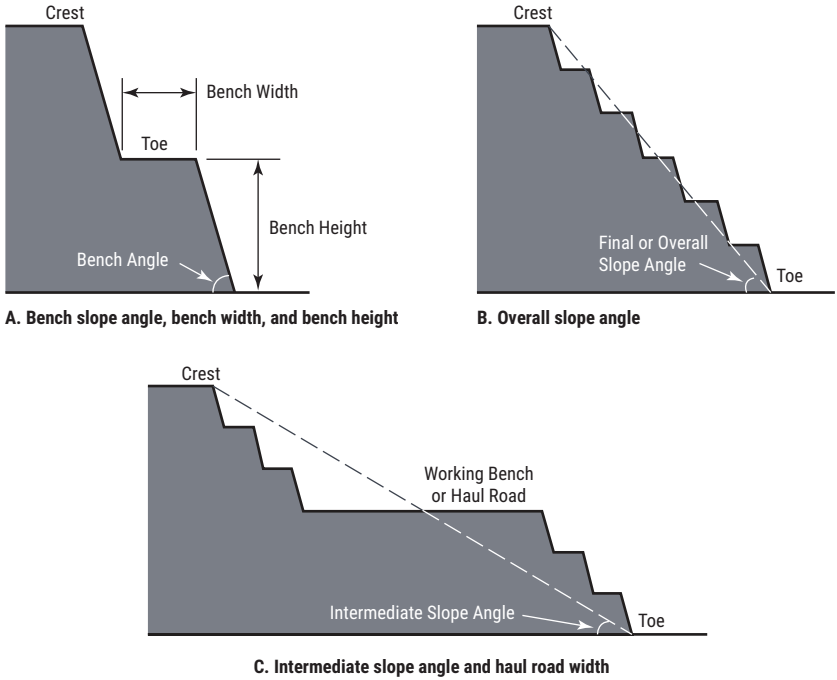


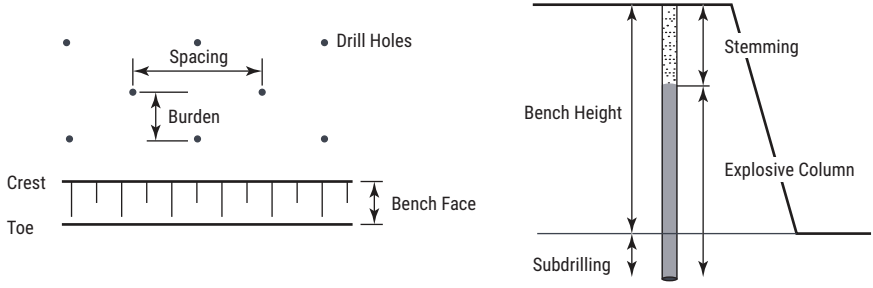
A. Bench slope angle, bench width, and bench height

B. Overall slope angle

C. Intermediate slope angle and haul road width

Courtesy of R.K. Zipf

Figure 10.5 Typical pit wall geometry in an open pit mine



A. Spacing and burden of blastholes

B. Bench height, explosive column, stemming, and subdrilling

Courtesy of R.K. Zipf

Figure 10.6 Typical blasting practice in an open pit mine

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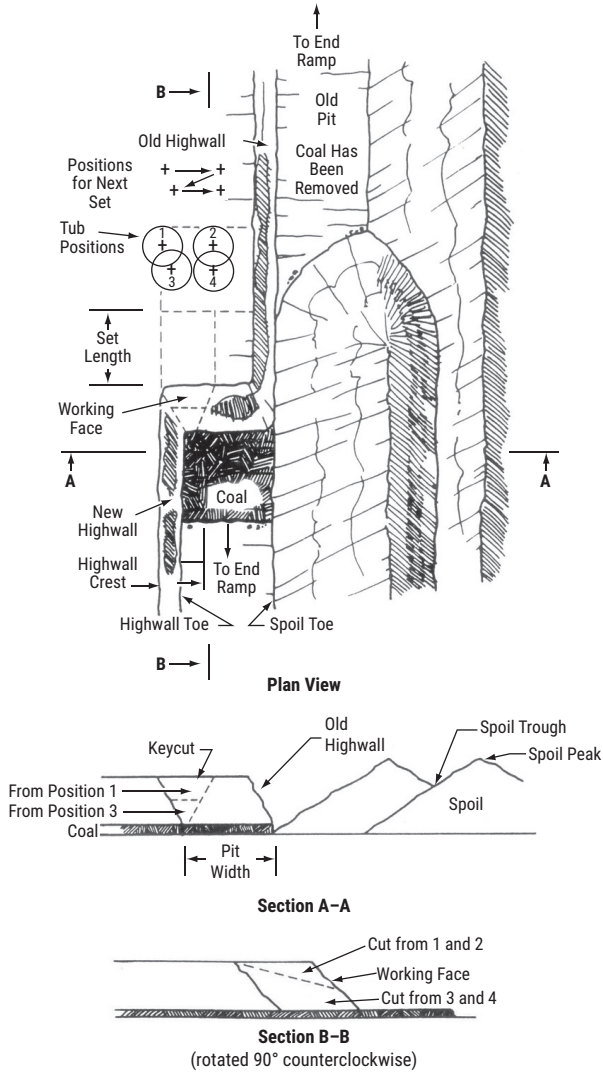

Source: Humphrey 1990

Figure 10.7 Digging positions in a dragline pit

The basic unit operations in open cast mining (Figures 10.7 and 10.9) are pre-stripping, drilling, blasting, overburden removal, coal removal, and reclamation. The equipment size and the number of machines employed in these unit operations depend on the production requirements of the mine. The dragline (Figure 10.7) is the most common machine employed for overburden removal. Coal is removed by an excavator or shovel and loaded into trucks for removal from the mine.

The range diagram (Figure 10.10) is the common tool for planning dragline operations in open cast mining. In the simplest case, the range diagram gives mining engineers the maximum depth of overburden (H_o) that can be removed by a dragline with a specified reach (R_d) without any rehandle of overburden material.

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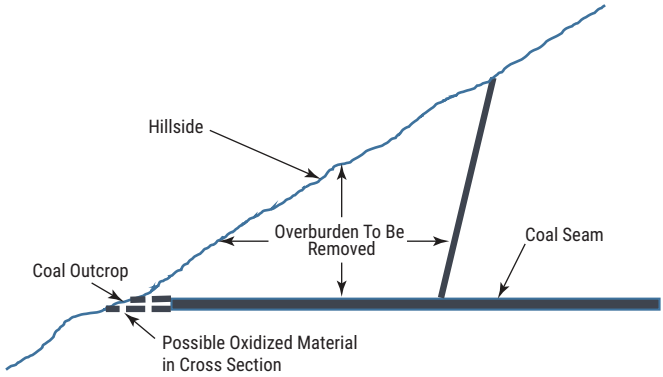
Courtesy of Raimond Spekking

Figure 10.8 Overburden removal in a large-scale open cast mine for coal using the bucket wheel excavator with scale shown by the bulldozer at the lower left



Source: Schissler 2004

Figure 10.9 Contour mining

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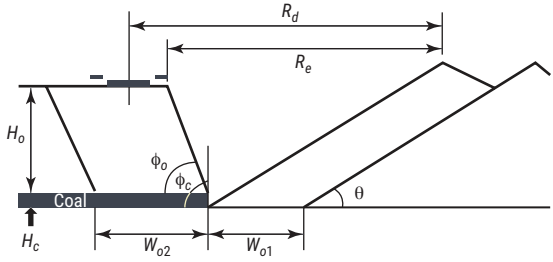

Adapted from Kennedy 1990

Figure 10.10 Side cast range diagram and equations for dragline operation

$$R_e = \frac{H_o}{\tan \phi_o} + \frac{H_c}{\tan \phi_c} + \frac{W_{o2} H_o F_s}{W_{o1} \tan \theta} + \frac{W_{o1}}{4} \quad \text{(EQ 10.1)}$$

$$H_o = \frac{\left(R_e - \frac{W_{o1}}{4} - \frac{H_c}{\tan \phi_c}\right)}{\left(\frac{1}{\tan \phi_o} + \frac{W_{o2} F_s}{W_{o1} \tan \theta}\right)} \quad \text{(EQ 10.2)}$$

where

R_e = dragline effective radius required to place a spoil pile so that the pile remains in the previous pit
R_d = dragline dump radius from the center of the dragline tub
H_o = overburden height
$\phi \leq 90$ degrees
H_c = coal thickness
W_{o1} and W_{o2} = not necessarily equal; pit width numerically subscripted from old to new
F_s = swell factor

Blasting in open cast mining uses a pattern like that shown in Figure 10.6, but without any subdrilling below the coal seam. In many open cast coal mines, blast casting is used to propel much of the overburden material to the spoil side of the mine. Blast casting consumes more explosives, but it decreases the amount of spoil material moved by the dragline.

UNDERGROUND MINING

In underground mining, various methods, such as block caving, are used to extract metal ores, whereas room-and-pillar methods are used for metal and nonmetal ores as well as coal. Longwall mining is used for tabular ores such as coal, potash, and trona.

Block Caving Method for Metal Ores

The block caving method is a broadly deployed, low-cost method to extract massive deposits of copper and associated metals. When the surface open pit mining method of a deposit reaches its economic limit, the underground block cave mining method is deployed to extract deeper ore reserves. A large-scale block cave mine may be up to 1 km wide. The undercut and production levels may be at a depth of 500 m or more, and the thickness of the caved ore zone may range from 100 to 200 m. The rock mass for successful block caving is usually classified as weak.

The basic operations at a block cave mine are (1) development of haulage and loading drifts usually by drill, blast, and muck methods on a level below the ore to be extracted; (2) development of undercut drifts also by drill, blast, and muck methods; (3) long-hole drilling and blasting to initiate the caving process; (4) ore production using load-haul-dump (LHD) vehicles; and (5) haulage out of the mine using conveyors

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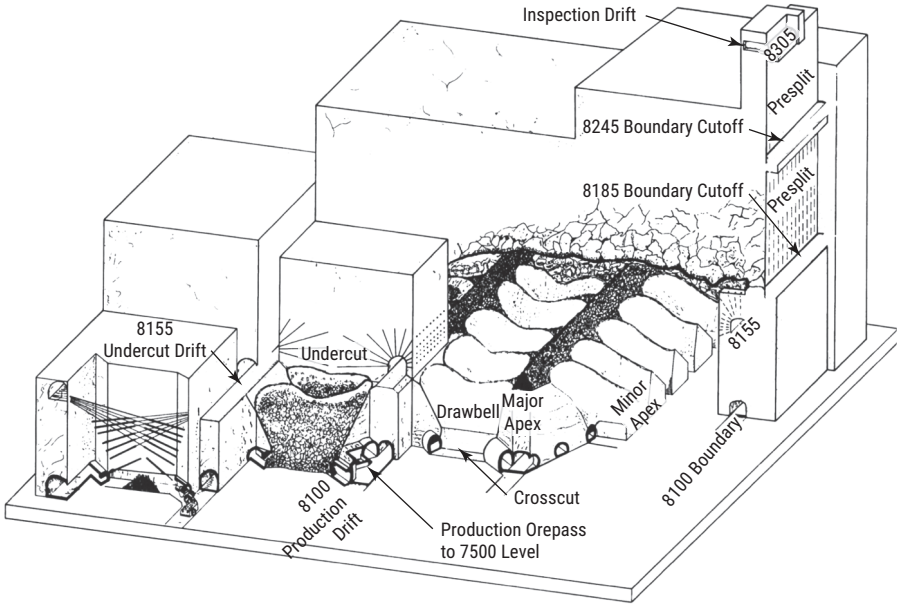
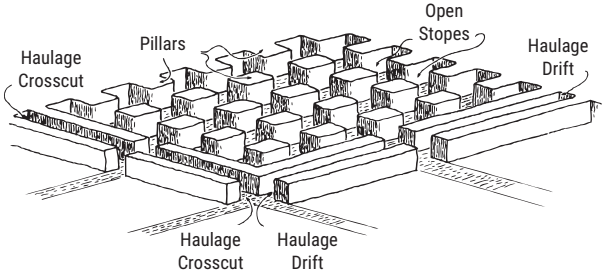


Source: Doepkin 1998

Figure 10.11 Large-scale block caving mine for metal ores



Source: Stebbins 2011

Figure 10.12 Development requirements for room-and-pillar mining

and skips. Figure 10.11 shows a typical large-scale block caving mine for metal ores producing more than 1 million stpy of ore and waste rock. Features shown are alternating undercut and production drifts, long holes from the undercut drift, the initial undercut before caving, and the caving zone. LHDs are the typical production vehicle in mechanized block caving mines.

Room-and-Pillar Method for Metal and Nonmetal Ores

The room-and-pillar method (Figure 10.12) is a widely used, low-cost method to extract flat-lying to inclined deposits in competent rock. The method is used for metals such as copper and lead; nonmetals such as limestone, trona, potash, and salt; as well as construction aggregates. A large-scale room-and-pillar mine may be hundreds of meters wide, kilometers long, and encompass an area of several square kilometers at a depth of up to 500 m, depending on the geology and value of the ore material. For room-and-pillar mines

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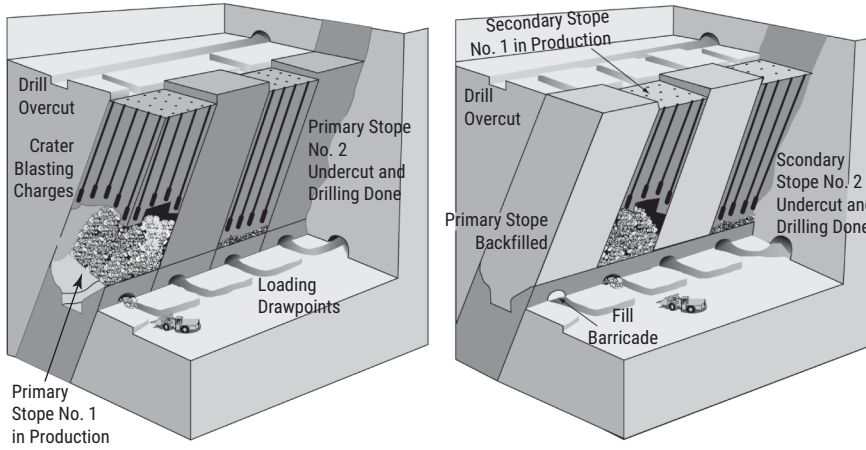
in metal, nonmetal, or construction aggregate deposits, the height of the mine can range from 3 to 6 m for a single-pass operation to 20 m or more for a multiple-pass operation with benches. The rooms may be 4–20 m wide depending on rock conditions, and the pillars may be 6–20 m wide or more depending on overburden depth and geologic conditions such as roof and floor lithology. The rock for successful room-and-pillar mining is usually classified as good or better to allow large roof spans with minimal support such as spot or pattern rock bolting. Roof support is determined by many factors including roof and floor lithology, seam or vein thickness, regional geology, room width, and federal regulations.

The basic operations at a room-and-pillar mine are (1) development of rooms on the upper level using drill, blast, and muck methods; (2) support of rooms using rock bolts only; and (3) mining benches below the development level using drill, blast, and muck methods. A typical medium-scale multiple-pass room-and-pillar operation used for mining thick metal and nonmetal ores and construction aggregate is capable of producing up to about 1 million stpy of ore and waste rock. Common features of such an operation are drilling horizontal blastholes into the face with a drill jumbo, loading broken rock with a front-end loader, hauling broken rock via truck, and drilling vertical blastholes with a track drill into the bench.

Other Modern Methods for Metal Ores

Other common, highly mechanized methods applied for extraction of high-value metal ores are as follows:

- **Stope-and-pillar or vertical crater retreat** (VCR) (Figure 10.13). This method is applied to thick, near-vertical veins in competent rock. The ore rock is typically strong and can allow the roof span on the blasthole development level with minimal support. The surrounding rock is usually moderate to strong to minimize dilution within the large stopes. Haulage workings are usually located in competent footwall rock.
- **Sublevel caving** (Figure 10.14). This method is applied to thick, near-vertical to inclined ore bodies. The ore rock and the hangingwall rock are typically weak. Haulage workings are usually located in competent footwall rock.
- **Sublevel stoping** (Figure 10.15). This method is also applied to thick, near-vertical veins; however, the ore rock is weak and only allows a small development opening for long-hole drilling and blasting. The surrounding rock is usually moderate to strong to minimize dilution within the large stopes. Haulage workings are usually located in competent footwall rock.



A. Primary stopes mined **B. Secondary stopes mined**

Source: Hamrin 2001

Figure 10.13 Stope-and-pillar mining with vertical crater retreat mining

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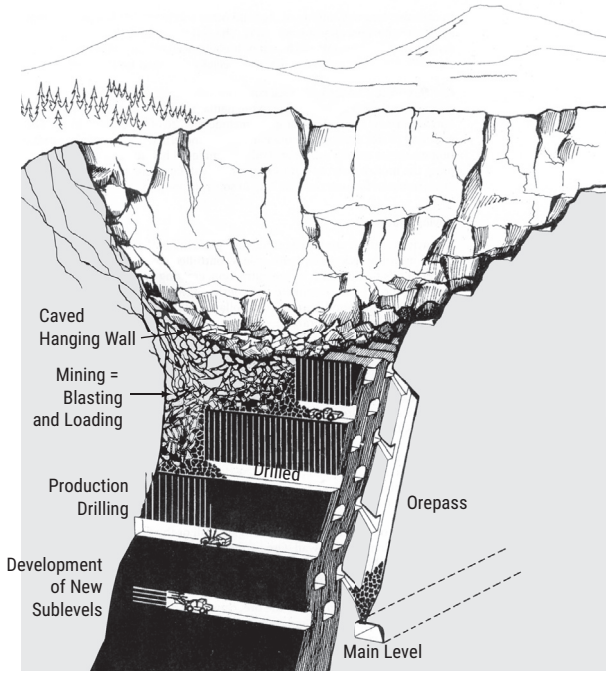
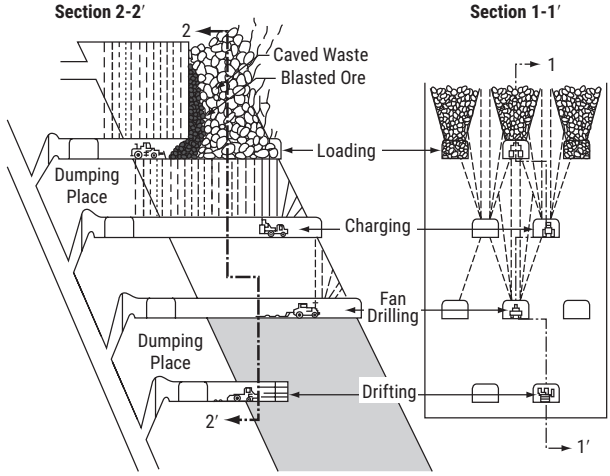


Courtesy of Atlas Copco

A. Ore body overview



B. Cross section

Source: Kvapil 1992

Figure 10.14 Sublevel caving

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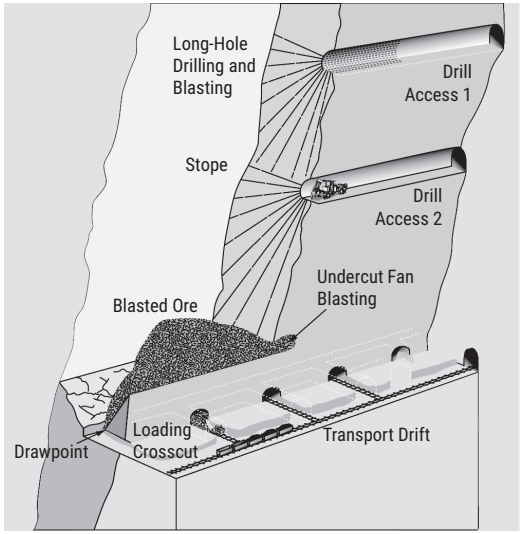
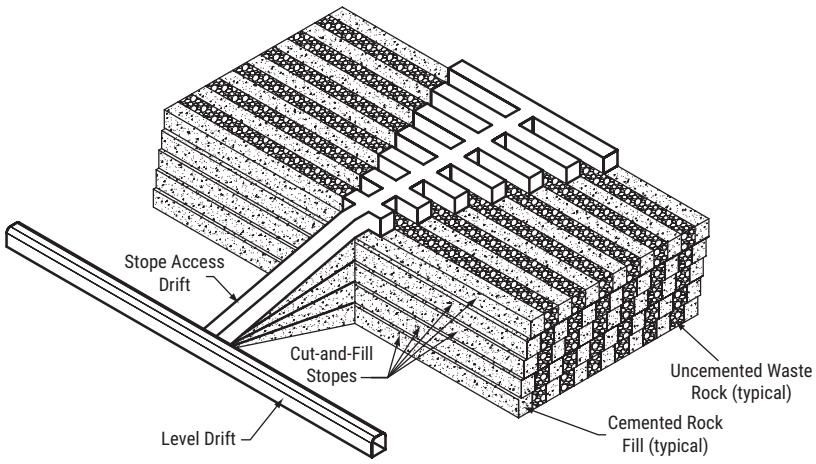


Source: Hamrin 2001

Figure 10.15 Sublevel stoping



Source: Stephan 2011

Figure 10.16 Overhand cut-and-fill stoping of near-vertical vein-type deposit

- **Overhand cut and fill** (Figure 10.16). This method is applied to thin, near-vertical to inclined veins. The ore rock is competent enough to allow a roof span across the vein with minimal support. The hanging wall and footwall rocks are usually weak and require support, hence after the ore is extracted, the remaining opening is backfilled with tailings or waste rock.
- **Underhand cut and fill** (Figure 10.17). This method is applied to a variety of ore body shapes in very weak ore rock. Substantial support with rock bolts and shotcrete is required in all development and production openings. After the ore is extracted, openings are backfilled with cemented sand fill. To minimize ground control costs, subsequent ore extraction then occurs under the cemented sand fill.

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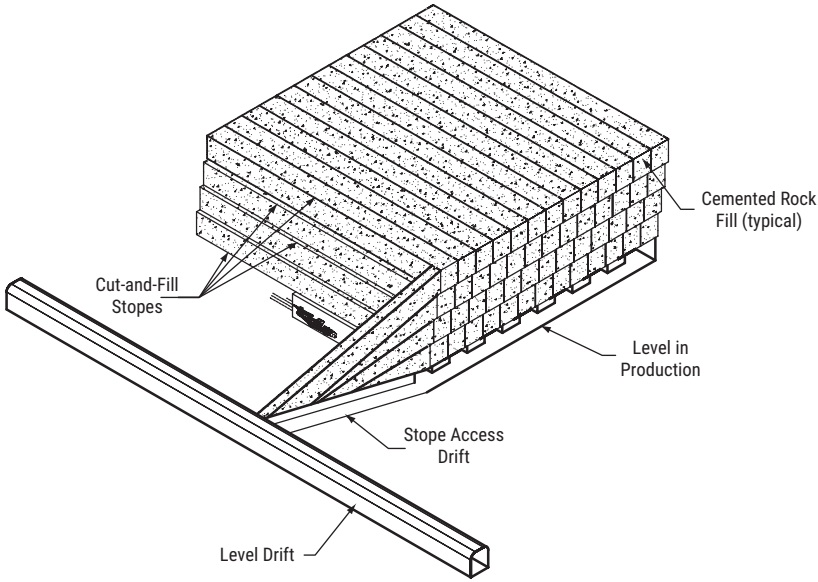

Source: Stephan 2011

Figure 10.17 Underhand cut-and-fill stoping of near-vertical vein-type deposit

Longwall Mining for Coal

Longwall mining (Figures 10.18 and 10.19) is the lowest-cost method for underground coal mining. A typical longwall panel is approximately 300–400 m wide and 2–7 km long, depending on mining conditions. A typical large-scale longwall coal mine may produce about 5–10 million stpy of coal and extract about 15–25 panels over a mine life of about 20 years. The immediate roof rock above the coal seam should collapse readily when the coal seam is extracted. Figure 10.18 shows a typical underground coal mining operation as practiced in the United States, showing a longwall panel and room-and-pillar development for longwall mining.

The major equipment employed on a longwall panel is (1) a shearer for cutting the coal seam, (2) a face conveyor for transporting coal off the longwall face, (3) 200–250 shields for supporting the immediate roof above the longwall face, (4) a crusher and stage loader for crushing the coal as it leaves the longwall face, and (5) a conveyor system for transporting the crushed coal out of the mine. A typical longwall mine will usually deploy two or more continuous miner sections to develop the infrastructure ahead of the longwall panels. The block of coal intended to be extracted by the longwall equipment must be prepared with access entries, the purpose of continuous miner sections. This development work consists of mains with 5–11 entries; head gates with 3 entries; and tailgates, also with 3 entries.

Room-and-Pillar Mining for Coal

Room-and-pillar mining (Figure 10.20) is the other major method used for underground coal mining. Coal mines use a series of mains and submains to access the room-and-pillar panels. The mains and submains provide access for workers and materials, as well as ventilating air, power, water, conveyors for coal transport, and mine dewatering pipelines.

Typical room-and-pillar panels are about 100–200 m wide and consist of 5–13 entries (rooms) and the associated crosscuts. The panel may extend for up to a kilometer or more, depending on mining conditions. Retreat mining, which is the removal of coal pillars in the panels after their development, increases the recovery.

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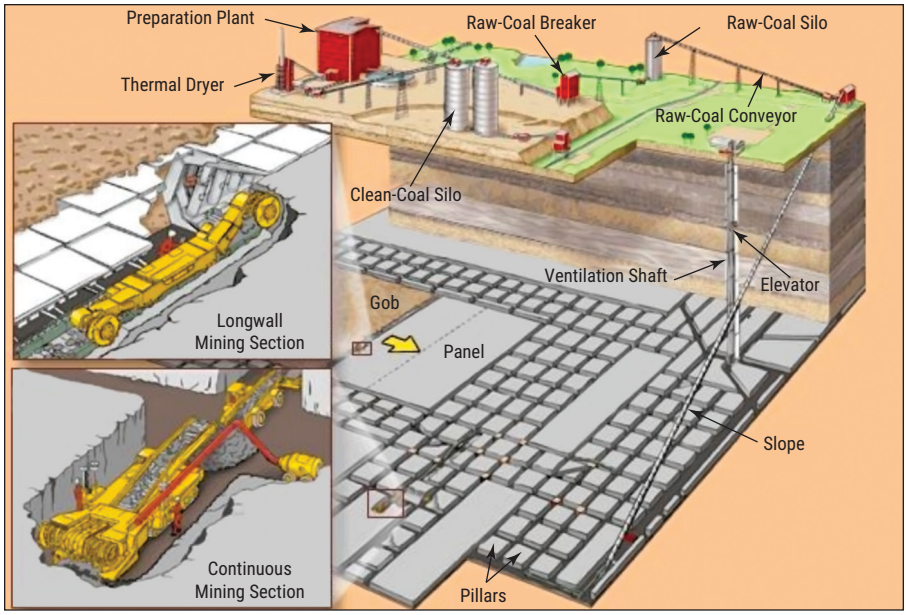
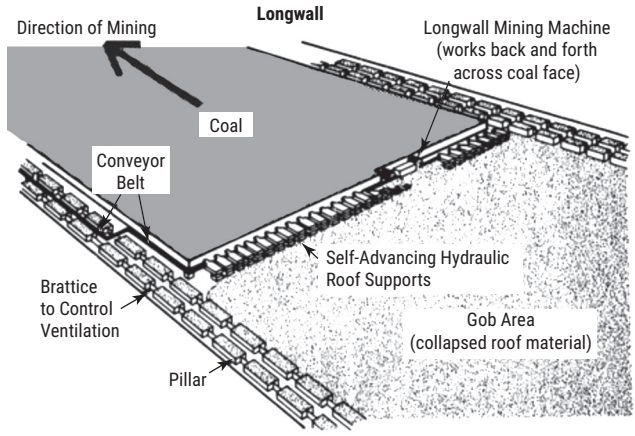


Courtesy of Consol Coal Group

Figure 10.18 Underground coal mining operation with longwall panel and room-and-pillar development for longwall mining



Courtesy of U.S. Energy Information Administration

Figure 10.19 Longwall method applied to coal mining

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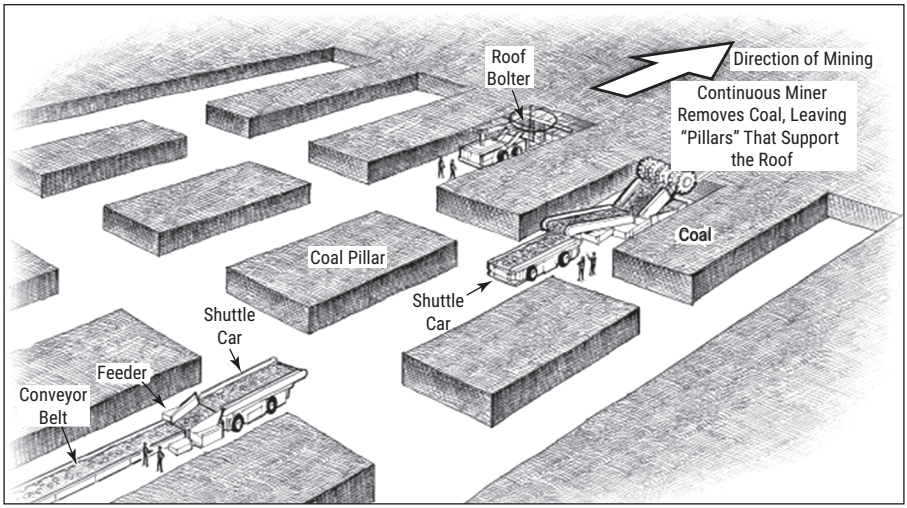

Courtesy of Arch Coal

Figure 10.20 Room-and-pillar method applied to coal mining

A typical room-and-pillar mine may produce 0.5–2 million stpy and have a mine life ranging from a few years to decades or more.

The major equipment employed in a room-and-pillar section is (1) a continuous miner for cutting the coal seam, (2) two shuttle cars for transporting the coal from the face, (3) a roof bolting machine for installing roof support, (4) a feeder breaker for crushing the coal as it leaves the room-and-pillar section, and (5) a conveyor system for transporting the crushed coal out of the mine.

TERMS APPLIED TO MINING METHODS

Adit. A horizontal or near horizontal passage driven from the surface for the working of a mine.

Bench slope angle. The angle from horizontal from the toe of an individual bench to its crest.

Blast casting. A method used in open cast mining to move overburden material in the highwall to its final location in the waste pile by using a larger amount of explosives to decrease the overburden material necessary to move by mechanical earthmover.

Break-even strip ratio. The strip ratio where the cost of removing one unit of spoil or overburden equals the value gained from accessing one unit of ore.

Bucket wheel excavator. An excavating machine that collects material in buckets spaced along the perimeter of a rotating wheel, then dumps the material onto a conveyor system for transport to a dump site at a considerable distance.

Conventional mining. The excavation sequence consisting of (1) drilling blastholes, (2) blasting with explosives, (3) loading the broken rock, and (4) hauling rock from the excavation site in trucks.

Cutoff grade. The ore grade where the value of the metal within the ore equals the cost of mining the ore and waste rock above the ore.

Dilution. The amount of waste rock expressed as a percentage that is mixed in with ore during the extraction process. Minimizing the magnitude of dilution is sometimes desirable, but not necessarily, depending on the economics of the mining method.

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Dragline. An excavating machine that collects material by pulling a bucket toward itself, then elevates the bucket on a boom, then swings the boom to the dump point, then dumps the bucket at a considerable distance.

Drift. A horizontal opening in or near an ore body and parallel to the course of the vein or long dimension of an ore body.

Haul ramp. An inclined travel way for trucks hauling ore, coal, or waste in a surface mine to a crusher or other dump point.

Haul road. A near horizontal travel way for trucks hauling ore, coal, or waste in a surface mine to a crusher or other dump point.

Intermediate bench slope angle. The angle from horizontal from the toe of a pit wall to its crest that includes multiple benches along with one or more working benches or haul ramps.

LHD. Abbreviation for load, haul, dump; a type of vehicle used in underground mining.

Mechanical mining. Refers to the excavation sequence consisting of (1) breaking rock or ore into fragments with a tool, (2) loading the broken rock with a gathering system, and (3) hauling rock from the excavation site usually with a conveyor system. Examples include the continuous miner and tunnel boring machine.

Overall slope angle. The angle from horizontal from the toe of a pit wall to its crest that includes multiple benches but no working benches or haul ramps.

Portal. The surface entrance to a drift, tunnel, adit, or entry.

Pushback. An intermediate stage in open pit mining before the final pit limit is achieved. Mining is done on a working bench in a sequence of intermediate mining stages called *pushbacks*.

Raise. A vertical or inclined opening within a mine driven upward from a level to the level above.

Ramp. An inclined tunnel from the surface to access an underground ore body or deposit.

Rehandle. Waste material in an open cast mine that is moved more than once by the excavator (usually a dragline) to place it into its final location in a waste pile.

Shaft. A vertical or near-vertical excavation from the surface of limited area compared to its depth for accessing a mine.

Shovel (cable or hydraulic). An excavating machine that pushes a bucket into broken material, lifts the bucket with one or more arms, swings the bucket over a truck, then dumps the bucket into the truck bed. A cable shovel uses wire rope to push and lift the bucket, whereas a hydraulic shovel uses hydraulic actuators.

Slope. An inclined entry from the surface to access an underground coal seam.

Stripping ratio. The unit of spoil or overburden that must be removed to gain access to a unit of ore.

Truck. Any wheeled vehicle, usually self-propelled, used to transport ore and waste.

Winze. A vertical or inclined opening within a mine driven downward from a level to the level below.

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CHAPTER

11

Blasting and Explosives

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CHARACTERISTICS OF EXPLOSIVES

The purpose of blasting rock is to get the proper product size distribution for transportation, milling, and/or marketing. Correct blasting can dramatically reduce costs in the comminution circuit. The selection and application of proper explosives is based on the characteristics of the explosive and geological conditions. Variables are of two types: those that can be controlled and those that cannot be controlled (Konya and Walter 1999). Controllable variables are hole diameter and depth, subdrilling depth, stemming, spacing, explosive and detonator choice, delays, and so forth. Variables that cannot be controlled are geology, rock properties, distance from structures, and so on. Different chemical makeups of explosives are shown in Figure 11.1.

GOVERNING AGENCIES AND REGULATIONS

- The U.S. Bureau of Alcohol, Tobacco, Firearms and Explosives (ATF) regulates "importing, manufacturing, dealing, purchasing, usage, storage, or possession" of explosive materials (ATF, n.d.).
  - 18 USC 39, 40; 27 CFR 555
- The U.S. Department of Transportation (DOT) regulates the transportation of explosives on public roads for commercial use.
  - 49 CFR
- The U.S. Environmental Protection Agency (EPA) regulates environmental, record keeping, and emergency planning.
  - 42 USC 82, 103, and 116; 40 CFR 260–270, 302, 355, 370, and 372
- The Mine Safety and Health Administration (MSHA) regulates all use and storage on mine properties.
  - 30 USC 801; 30 CFR 56 Subpart E, 57 Subpart E, 75 Subpart N, and 56 Subpart N
- The Occupational Safety and Health Administration (OSHA) regulates employers with the exception of government agencies.
  - 29 USC 651; 29 CFR 1910 Subpart H and 1926 Subpart U
- The Office of Surface Mining Reclamation and Enforcement (OSMRE) regulates blasting notice, training, logs, blasting delays, noise, and ground vibrations.
  - 30 USC §1201; 30 CFR §780.13, §816.61–§816.68, and Parts 850 and 955
- The Transportation Security Administration (TSA) provides training and performs security threat assessments.
  - 49 USC §5103a; 49 CFR 1572

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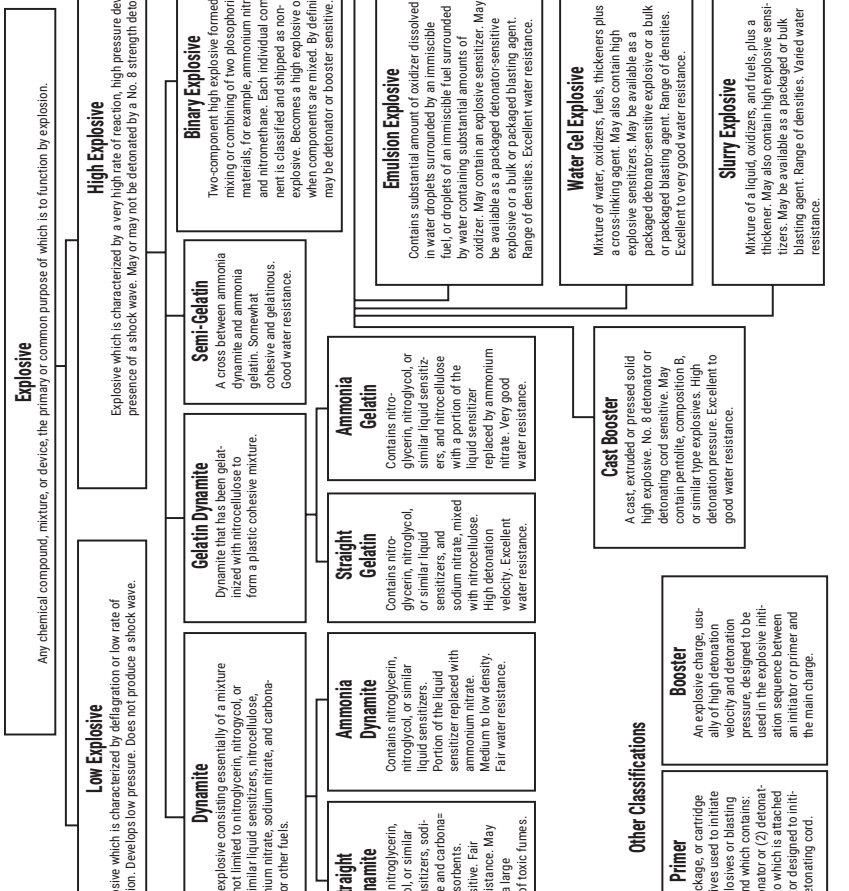
Explosive
Any chemical compound, mixture, or device, the primary or common purpose of which is to function by explosion.
Low Explosive
Explosive which is characterized by deflagration or low rate of reaction. Develops low pressure. Does not produce a shock wave.
High Explosive
Explosive which is characterized by a very high rate of reaction, high pressure development, and the presence of a shock wave. May or may not be detonated by a No. 8 strength detonator when unconfined.
Dynamite
A high explosive consisting essentially of a mixture of but not limited to nitroglycerin, nitroglycol, or other similar liquid sensitizers, nitrocellulose, ammonium nitrate, sodium nitrate, and carbonaceous or other fuels.
Gelatin Dynamite
Dynamite that has been gelatinized with nitrocellulose to form a plastic cohesive mixture.
Semi-Gelatin
A cross between ammonia dynamite and ammonia gelatin. Somewhat cohesive and gelatinous. Good water resistance.
Binary Explosive
Two-component high explosive formed by mixing or combining of two plosophoric materials, for example, ammonium nitrate and nitromethane. Each individual component is classified and shipped as non-explosive. Becomes a high explosive only when components are mixed. By definition may be detonator or booster sensitive.
Blasting Agent
Combination of an oxidizer and a fuel. A high explosive. Cannot be detonated by a No. 8 strength detonator when unconfined. Must be initiated by a booster. Can develop a high rate of reaction, high pressure, and a shock wave.
Straight Dynamite
Contains nitroglycerin, nitroglycol, or similar liquid sensitizers, sodium nitrate and carbonaceous absorbents. Very sensitive. Fair water resistance. May produce a large quantity of toxic fumes.
Ammonia Dynamite
Contains nitroglycerin, nitroglycol, or similar liquid sensitizers. Portion of the liquid sensitizer replaced with ammonium nitrate. Medium to low density. Fair water resistance.
Straight Gelatin
Contains nitroglycerin, nitroglycol, or similar liquid sensitizers, and sodium nitrate, mixed with nitrocellulose. High detonation velocity. Excellent water resistance.
Ammonia Gelatin
Contains nitroglycerin, nitroglycol, or similar liquid sensitizers, and nitrocellulose with a portion of the liquid sensitizer replaced by ammonium nitrate. Very good water resistance.
Emulsion Explosive
Contains substantial amount of oxidizer dissolved in water droplets surrounded by an immiscible fuel, or droplets of an immiscible fuel surrounded by water containing substantial amounts of oxidizer. May contain an explosive sensitizer. May be available as a packaged detonator-sensitive explosive or a bulk or packaged blasting agent. Range of densities. Excellent water resistance.
ANFO
Dry mixture of ammonium nitrate and fuel oil. May also contain solid fuels. Premixed by manufacturer or mixed on site by user. Can be loaded from bulk trucks, poured from bags, or loaded as cartridges/shot bags. Low-density explosive with poor water resistance. Economical and efficient explosive if mixed and used properly.
Water Gel Explosive
Mixture of water, oxidizers, fuels, thickeners plus a cross-linking agent. May also contain high explosive sensitizers. May be available as a packaged detonator-sensitive explosive or a bulk or packaged blasting agent. Range of densities. Excellent to very good water resistance.
Slurry Explosive
Mixture of a liquid, oxidizers, and fuels, plus a thickener. May also contain high explosive sensitizers. May be available as a packaged or bulk blasting agent. Range of densities. Varied water resistance.
Blends
Mixture consisting of water-based explosives or oxidizer matrix and ammonium nitrate or ANFO. Range of densities. Poor to very good water resistance.
Other Classifications
Primer
A unit, package, or cartridge of explosives used to initiate other explosives or blasting agents, and which contains: (1) a detonator or (2) detonating cord to which is attached a detonator designed to initiate the detonating cord.
Booster
An explosive charge, usually of high detonation velocity and detonation pressure, designed to be used in the explosive initiation sequence between an initiator or primer and the main charge.
Cast Booster
A cast, extruded or pressed solid high explosive. No. 8 detonator or detonating cord sensitive. May contain pentolite, composition B, or similar type explosives. High detonation pressure. Excellent to good water resistance.
Source: ISEE 2011

FIGURE 11.1 Descriptive classification of explosives

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EXPLOSIVE PROPERTIES

Explosives are used to propagate rock breakage. Different geological, climatic, product distribution, and availability features help determine the correct explosive to use based on its properties (Table 11.1).

TABLE 11.1 Typical characteristics of commercial explosives

Explosive Type	Grade	Density, g/cc	Velocity of Detonation, ft/s	Energy, RBS*	Water Resistance
Straight dynamite	50% ditching dynamite	1.32	17,400	—	Good
Ammonia dynamite	40%	1.30–1.32	9,800–12,500	146	Fair
	60%	1.28–1.30	12,470–15,300	152	Very good
Semi-gelatin		1.29	13,100		Very good
		1.16	12,600		Very good
		1.26	17,500	149	Very good
		0.94	11,300		Good
Straight gelatin	80%	1.35–1.43	18,700–22,000	201	Excellent
Ammonia gelatin	40%	1.30–1.50	11,000–18,500	146–171	Excellent
	60%	1.34–1.43	17,400–18,900	164	Excellent
	80%	1.34	20,200	199	Excellent
Water gel	Packaged 2–9-in. diameter	1.18–1.52	13,230–19,000	128–183	Very good
Emulsion	Bulk	0.95–1.28	13,100–19,000	103–156	Excellent
	Package HE†	1.10–1.20	13,500–17,000	102–167	Excellent
ANFO‡	Poured	0.77–0.85	9,100–15,100	100	None
	Pneumatic loading	0.85–1.10	9,000–11,000	100–122	None
ANFO/Emulsion mix	20:80	1.05	16,400	138	Poor
	30:70	1.2	17,400	166	Fair
	40:60	1.3	18,100	185	Good
	50:50	1.3	17,900	179	Excellent
	60:40	1.3	17,500	175	Excellent
	70:30	1.3	17,100	169	Excellent
	80:20	1.3	16,700	165	Excellent

* RBS = relative bulk strength.

† HE = high explosive.

‡ ANFO = ammonium nitrate and fuel oil.

SURFACE BLAST DESIGN

Efficient blast design requires the application of sufficient explosive energy, properly distributed in the rock mass, with adequate confinement. Quantifying the overall amount of explosive energy in a blast pattern is done by multiplying the absolute weight strength of the explosive times the mass of explosive contained in the pattern. The distribution and confinement are determined by the blast pattern and delay sequence. Under-confinement of the explosive charges results in flyrock and high airblast. Over-confinement results in poor fragmentation, high ground vibrations, excessive toe, and possible back break.

However, the delay timing sequence chosen can alter the geometric burden and spacing of a blast pattern. As defined, *burden* is the distance from the explosive charge to the nearest free face; *spacing* is the distance between adjacent holes measured perpendicular to the burden.

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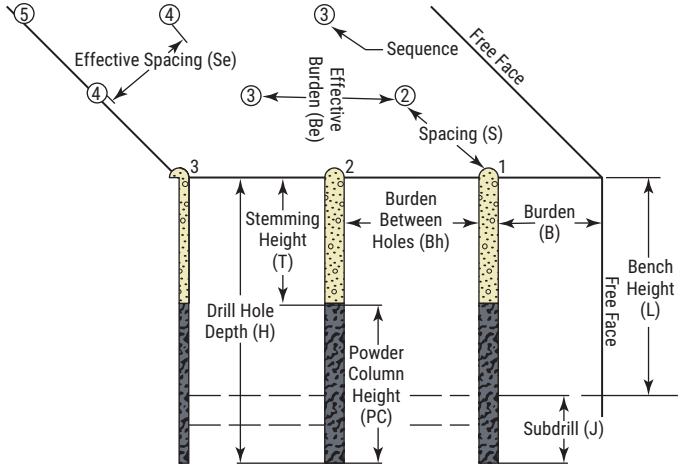

FIGURE 11.2 Blast design nomenclature and blast sequencing for delayed holes

When blasts are delayed so that the boreholes fire in echelon or as in a V-cut, there is a difference between the apparent or geometric burden as drilled on the bench, and the effective or true burden. Likewise, the effective spacing changes with such a delay pattern. The *effective burden* is the distance from the borehole to the nearest free face at the instant the charge is fired. Likewise, the *effective spacing* is the distance between adjacent boreholes perpendicular to the burden as the charges are fired (Figure 11.2).

Empirical methods are included: Following a brief description of variables, these methods for calculating the parameters of a blast pattern are first approximations and need adjustment to meet field conditions. Load factor in pound per foot is determined through calculation using

$$\text{load factor} = 0.34(SG_e)(D_e)^2 \quad \textbf{(EQ 11.1)}$$

where

SG_e = density of explosive, g/cc
D_e = diameter of blasthole, in.

Powder Factor Method

This method requires the use of a known powder factor in pound per cubic yard and calculates the geometry of the blast necessary to attain this powder factor. Conventional application is pound per cubic yard, but other values can be used, especially for coal or other materials with densities different than average rock.

$$T = \frac{24D_e}{12} \quad \textbf{(EQ 11.2)}$$

$$J = \frac{1}{3}\text{stemming} \quad \textbf{(EQ 11.3)}$$

$$C = H + J - T \quad \textbf{(EQ 11.4)}$$

$$W_E = C \times LF \quad \textbf{(EQ 11.5)}$$

$$V_r = \frac{W_E}{Pf} \quad \textbf{(EQ 11.6)}$$

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Rectangular Pattern

$$B = \sqrt{\frac{18 \times V_r}{H}} \quad \text{(EQ 11.7)}$$

$$S = 1.5 \times B \quad \text{(EQ 11.8)}$$

Square Pattern

$$B = \sqrt{\frac{27 \times V_r}{H}} \quad \text{(EQ 11.9)}$$

$$S = B \quad \text{(EQ 11.10)}$$

where

T = stemming, ft
D_e = diameter of blasthole, in.
J = subdrill, ft
C = powder column, ft
H = bench height, ft
W_E = weight of explosive per borehole, lb
LF = load factor, lb/ft (see Equation 11.1)
V_r = volume of rock broken per borehole, yd^3
Pf = powder factor, lb/yd^3
B = burden, ft
S = spacing, ft

Ash Method

The explosive's diameter is known when using the Ash method (Ash 1963), which is based on burden ratios (Table 11.2). The burden ratio corresponds to the hardness of the rock to be shot with the selected explosive. Cubic yards per blasthole is equal to the pounds of explosive per blasthole (Pf = 1 lb/yd^3; see Equation 11.1).

TABLE 11.2 Values of burden ratios, K_b

Explosive Energy	Rock Type		
	Soft	Average	Hard
Low	30	25	20
Average	35	30	25
High	40	35	30

Source: Ash 1963

$$K_b = 30\left(\frac{160}{G_r}\right)^{0.33}\left(\frac{SG_e(VOD^2)}{187 \times 10^6}\right)^{0.33} \quad \text{(EQ 11.11)}$$

$$B = K_b\left(\frac{D_e}{12}\right) \quad \text{(EQ 11.12)}$$

$$T = 0.7B \quad \text{(EQ 11.13)}$$

$$J = 0.3B \quad \text{(EQ 11.14)}$$

spacing = $2B$ for instantaneous holes (ft) (EQ 11.15)

spacing = B for individually delayed holes (ft) (EQ 11.16)

restriction: $1.5B < (L + J) < 4B$ (EQ 11.17)

where

K_b = burden ratio
G_r = density of rock, lb/ft^3
SG_e = density of explosive, g/cc
VOD = velocity of detonation, ft/s
B = burden, ft
D_e = diameter of blasthole, in.
T = stemming, ft
J = subdrill, ft

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Konya Method

The following method (as shown in the following equations) was developed by Konya and Walter (1999).

For adjusting burden ratio,

$$B = 3.15 D_e \sqrt[3]{\frac{SG_e}{SG_r}} \quad \text{(EQ 11.18)}$$

The following equation is ideal for use in the field:

$$B = D_e \times 2\frac{SG_e}{SG_r} \quad \text{(EQ 11.19)}$$

And for using relative energy, use Equation 11.20:

$$B = 0.67 D_e \sqrt[3]{\frac{St_v}{SG_r}} \quad \text{(EQ 11.20)}$$

$$T = 0.7B \quad \text{(EQ 11.21)}$$

$$J = 0.3B \quad \text{(EQ 11.22)}$$

$$S_i = \left(\frac{L + 2B}{3}\right), \text{ where } \frac{L}{B} < 4 \quad \text{(EQ 11.23)}$$

$$S_i = 2B, \text{ where } \frac{L}{B} > 4 \quad \text{(EQ 11.24)}$$

$$S_d = \left(\frac{L + 7B}{8}\right), \text{ where } \frac{L}{B} < 4 \quad \text{(EQ 11.25)}$$

$$S_d = 1.4B, \text{ where} \frac{L}{B} > 4 \quad \text{(EQ 11.26)}$$

where

B = burden, ft
SG_e = density of explosive, g/cc
SG_r = specific gravity of rock, g/cc
St_v = relative bulk strength (100 for ammonium nitrate and fuel oil [ANFO])
T = stemming, ft
J = subdrill, ft
S_i = spacing for instantaneous holes, ft
L = bench height, ft
S_d = spacing for delayed holes, ft

Factors to Modify Burden

- Corrections for multiple rows
  - First two rows (multiply by 1.0)
  - Third and subsequent rows (multiply by 0.9)
  - Corrections for geology
  - Highly fractured (multiply by 1.3)
  - Tight joints (multiply by 1.1)
  - Average rock (multiply by 1.0)
  - Massive intact (multiply by 0.95)

Blast Dynamics Method

The Blast Dynamics method was developed by Floyd (n.d.). The design was derived using the following:

$$\text{When stem length for explosive density} < 0.9 \text{ g/cc, use } D_e\left(\frac{22}{12}\right) \quad \text{(EQ 11.27)}$$

$$\text{When stem length for explosive density} > 0.9 \text{ g/cc, use } D_e\left(\frac{24}{12}\right) \quad \text{(EQ 11.28)}$$

$$J = D_e\left(\frac{7}{12}\right) \quad \text{(EQ 11.29)}$$

$$\rho_e = 0.3405(\text{explosive density}) D_e^2 \quad \text{(EQ 11.30)}$$

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$$C_w = (L + J - T)(\rho_e) \quad \text{(EQ 11.31)}$$

$$C_e = 0.454 C_w (AWS) \quad \text{(EQ 11.32)}$$

$$B = \sqrt{1738\left(\frac{C_e}{CE_d \times \rho_r \times L}\right)} \quad \text{(EQ 11.33)}$$

$$S = 1.15B \quad \text{(EQ 11.34)}$$

Useful Factors

$$E_e = 0.454(AWS) \quad \text{(EQ 11.35)}$$

$$\rho_r = 0.844 SG_r \quad \text{(EQ 11.36)}$$

$$ABS = AWS \times \rho_e \quad \text{(EQ 11.37)}$$

$$RWS = \frac{AWS}{912}\text{cal/g} \quad \text{(EQ 11.38)}$$

$$RBS = \frac{AWS \times \rho_e}{739\text{cal/cc}} \quad \text{(EQ 11.39)}$$

where

D_e = diameter of explosive, in.
J = subdrill, ft
ρ_e = loading density of explosive, g/cc
C_w = charge weight, lb
L = bench height, ft
T = stemming length, ft
C_e = charge energy, kcal/blasthole
AWS = absolute weight strength, cal/g; for ANFO 912, cal/g
CE_d = desired energy factor, kcal/ton
ρ_r = rock density, lb/ft^3
S = spacing, ft
E_e = explosive energy, kcal/lb
SG_r = specific gravity, g/cc
ABS = absolute bulk strength, cal/cc; for ANFO 739, cal/cc
RWS = relative weight strength, cal/g
RBS = relative bulk strength, cal/cc

Energy Factors

- Very weak rock = 100 kcal/ton
- Well-jointed, harder rock = 140 kcal/ton
- Average rock = 180 kcal/ton
- Hard rock = 220 kcal/ton
- Blocky, very hard rock = 250 kcal/ton
- Mine-to-mill blasting = 350 kcal/ton
- Very high energy blasts = 500 kcal/ton
- Upper limit = 1,200 kcal/ton

Considerations

- Best energy distribution is when $D_e = L/10$.
- Maximum blasthole diameter should be $L/5$.
- When $L/B < 2$, the burden is stiff; expect poor fragmentation.
- When $2 < L/B < 3.5$, expect good fragmentation.
- When $L/B > 4$, expect excellent fragmentation.
- The velocity of detonation (VOD) should be greater than the P-wave velocity of the rock.
- The energy factor for the hole should range between 100 and 250 kcal/ton.
- The energy factor should be about 190 kcal/ton.

Stemming Between Deck Charges

On occasion, it is necessary to separate multiple charges in a single borehole, mainly to decrease vibration or flyrock. Inert stemming is placed/filled in between explosive columns, preventing propagation between

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columns. With sensitive explosives or in wet boreholes, the stemming length must be at the upper end of the range (5–20). Under favorable conditions, such as with an insensitive explosive in a dry borehole, the starting point is at the lower end of the range.

$$L = \frac{D_C \times K_S}{12} \quad \text{(EQ 11.40)}$$

where

L = stemming length between charges in a borehole
D_c = explosive charge diameter
K_s = deck stemming ratio (range 5–20)

DETERMINING AMOUNT OF ROCK BROKEN PER BOREHOLE

$$V = \frac{B \times S \times H}{27} \quad \text{(EQ 11.41)}$$

where

V = volume of shot rock per borehole, yd^3
B = burden distance, ft
S = spacing distance, ft
H = bench height, ft (bench height = borehole depth minus any subdrilling)

POWDER FACTOR

The powder factor is the ratio of the explosives used per the unit weight/volume of the rock. Units commonly used in surface mining are pounds of explosive per cubic yard of rock.

$$Pf = \frac{W_e}{V_r} \quad \text{(EQ 11.42)}$$

where

Pf = powder factor, lb/yd^3
W_e = weight of explosives used, lb
V_r = volume of rock broken, yd^3

Powder factor is also expressed as weight of rock per pound of explosive, commonly used in metal mines.

$$Pf = \frac{W_r}{W_e} \quad \text{(EQ 11.43)}$$

where

Pf = powder factor, ton/lb
W_r = weight of rock broken, ton
W_e = weight of explosives used, lb

Effective powder factor is a function of the geology, rock density, type of explosive used, blast pattern, and degree of fragmentation desired. The less confined the blast pattern is, the smaller the powder factor needs to be (Tables 11.3 and 11.4).

TABLE 11.3 Typical powder factors for general surface blasting

Degree of Difficulty in Rock Breakage	Powder Factor, lb/yd^3
Low	0.25–0.40
Medium	0.40–0.75
High	0.75–1.25
Very high	1.25–2.50

Source: Dick et al. 1986

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TABLE 11.4 Relationship of powder factor to seismic velocity

Geology	Seismic Velocity Range, ft/s	Powder Factor, lb/yd^3
Weathered limestone	1,000–2,000	0.20
Weathered porphyry	2,000–3,000	0.60
Rhyolite breccia	3,000–4,000	0.75
Monzonite porphyry	4,000–5,000	1.00
Quartz sericite porphyry	5,000–6,000	1.00
Fresh limestone	5,000–6,000	1.00
Massive jasperoid	6,000–12,000	1.30

Adapted from Heinen and Dimock 1976

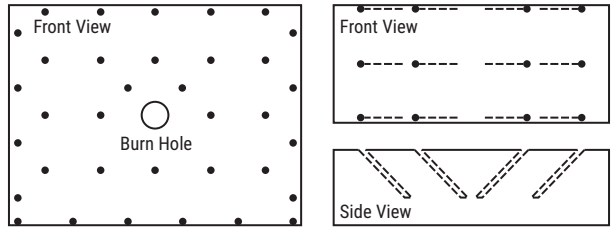

FIGURE 11.3 Burn cut and V-cut for headings

UNDERGROUND BLASTING

Underground blasting uses techniques such as the burn cut and V-cut for headings (Figure 11.3). In a burn cut, a larger hole is drilled to create an additional *free face*. Holes are angled to maximize use of the free face in a V-cut. Overhand and underhand blasts are angled blasts to shoot the roof and floor, respectively.

Powder Factor for Underground Blasting

A heading in an underground mine has only one free face; this type of blasting is much more confined and rock movement is constricted. The powder factor is inversely proportional to the cross-sectional area of the heading, which is the sole free face of the blast (Figure 11.4).

CONTROLLED BLASTING

Controlled blasting refers to the technique used to prevent overbreak or back break to produce a competent finished wall. The most common methods of controlled blasting are presplitting and cushion blasting (summarized in Tables 11.5 and 11.6).

Presplitting involves drilling closely spaced boreholes along the final excavation line, which are loaded lightly and detonated before any adjacent production holes are initiated. The best results are usually obtained when the presplit holes are fired with no delay between them; however, this may not be feasible if vibration concerns exist.

Cushion blasting uses closely spaced boreholes drilled along the final excavation line, loaded lightly and fired after the production blasts have been detonated and the material excavated. Cushion boreholes are normally larger diameter than presplit, and the burden is greater than the spacing.

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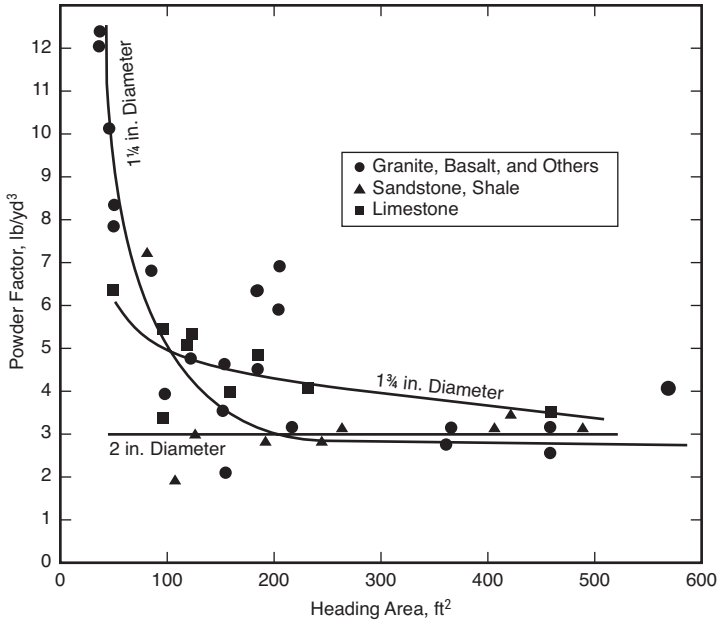

Source: ISEE 2011

FIGURE 11.4 Relationship of powder factor to heading area

TABLE 11.5 Average specification for presplitting blasts

Borehole Diameter, in.	Spacing, ft	Explosive Charge, lb/ft
1.50–1.75	1.00–1.50	0.08–0.25
2.00–2.50	1.50–2.00	0.08–0.25
3.00–3.50	2.50–3.00	0.13–0.50
4.00	3.00–4.00	0.25–0.75

Source: Dick et al. 1986

TABLE 11.6 Average specification for cushion blasting

Borehole Diameter, in.	Spacing, ft	Burden, ft	Explosive Charge, lb/ft
2.0–2.5	3.00	4.00	0.08–0.25
3.0–3.5	4.00	5.00	0.13–0.50
4.0–4.5	5.00	6.00	0.25–0.75
5.0–5.5	6.00	7.00	0.75–1.00
6.0–6.5	7.00	8.00	1.00–1.50

Source: Dick 1973

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HEIGHT OF WATER-RESISTANT EXPLOSIVES TO BUILD OUT OF WATER IN A BOREHOLE

Determining the final height of water when the top of the cartridges are at water level can be performed using the following equations (Dyno Nobel 2009).

$$FH = \frac{H_o \times D_B^2}{D_B^2 - D_E^2} \quad \text{(EQ 11.44)}$$

where

FH = final height of water, ft
H_o = original height of water, ft
D_B = diameter of borehole, in.
D_E = diameter of explosive cartridge, in.

$$N_E = \frac{FH \times 12}{L_E} \quad \text{(EQ 11.45)}$$

where

N_E = number of cartridges needed
FH = final height of water, ft
L_E = length of cartridge, in.

INITIATION SYSTEMS AND BOOSTERS

The International Society of Explosives Engineers (ISEE) offers the following information regarding initiation systems (ISEE 2011):

- Electronic systems are accurate and offer precise timing. Used to improve fragmentation and lower airblast and vibrations, these systems cost more and are more susceptible to failed communications with low power and line interruptions because of high sensitivity.
- Electric systems offer circuit testing ability and were widely used until the introduction of shock tubes in 1970. These systems are susceptible to stray current.
  - Wire resistance

$$R_{\text{total}} = \frac{(L \times R_{1,000\text{ ft}})}{1,000} \quad \text{(EQ 11.46)}$$

where

R_{total} = total wire resistance, Ω (ohm)
L = length of wire used, ft
$R_{1,000\text{ ft}}$ = wire resistance per 1,000 ft, Ω (ohm)

  - Series circuit resistance

$$R_{\text{total}} = \sum_{i=1}^{n} R \quad \text{(EQ 11.47)}$$

where

R_{total} = total wire resistance, Ω (ohm)
R = resistance of each electric detonator and wire, Ω (ohm)
n = total individual elements

  - Parallel circuit resistance

$$R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}} \quad \text{(EQ 11.48)}$$

where

R_{total} = total wire resistance, Ω (ohm)
$R_1, \ldots, R_n$ = resistance of each electric detonator, Ω (ohm)

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- Nonelectric or shock tube systems are the most popular and very durable. They are less susceptible to electric disturbances; however, there is no circuit testing capability.
- Detonating cord systems are resistant to accidental initiation. Reliable for use with shock tubes, they give a high noise level. MSHA requires two paths to each detonator unlike the Dyno Nobel Nonel detonator.
- Cap and fuse systems are considered easy makeup for operations with limited blasting and few holes, although they are imprecise and offer a low accuracy timing system.

Boosters and initiation caps form the primer and deliver energy needed for detonation.

CLASSIFICATION OF EXPLOSIVES BY FUME CHARACTERISTICS

As stated in 30 CFR §15, the total poisonous gases in permissible explosives for use in underground coal mines cannot exceed 155 cm^3/g (2.5 ft^3/lb). Permissible explosives are tested by the National Institute for Occupational Safety and Health (NIOSH) and approved by MSHA.

Other explosives are classified by the Institute of Makers of Explosives (IME) (Table 11.7). The IME classification indicates the amount of poisonous gases produced by an explosive. Only explosives rated as fume class 1, 2, or 3 should be used in confined areas such as tunnels or underground mines.

TABLE 11.7 IME fume classification

Fume Class	Amount of Poisonous Gases per 32 × 203-mm (1¼ × 8-in.) Explosive Cartridge
1	Less than 4.53 L (0.16 ft^3)
2	4.53–9.35 L (0.16–0.33 ft^3)
3	9.35–18.98 L (0.33–0.67 ft^3)

Source: IME 2013

DETONATION PRESSURE

The detonation pressure of an explosive, which is an important factor in determining an explosive's effectiveness as a primer charge or a booster, is calculated as follows:

$$P = (2.32 \times 10^{-7})\rho D^2 \quad \text{(EQ 11.49)}$$

where

P = detonation pressure, kilobar
ρ = specific gravity of explosive
D = detonation velocity, ft/s

Detonation velocity is dependent on the charge diameter and is affected differently by explosives (Figure 11.5).

Loading factor is calculated as

$$LF = 0.34\rho \times d^2 \quad \text{(EQ 11.50)}$$

where

LF = load factor, lb/ft
d = explosive column diameter, in.
ρ = specific gravity of explosive

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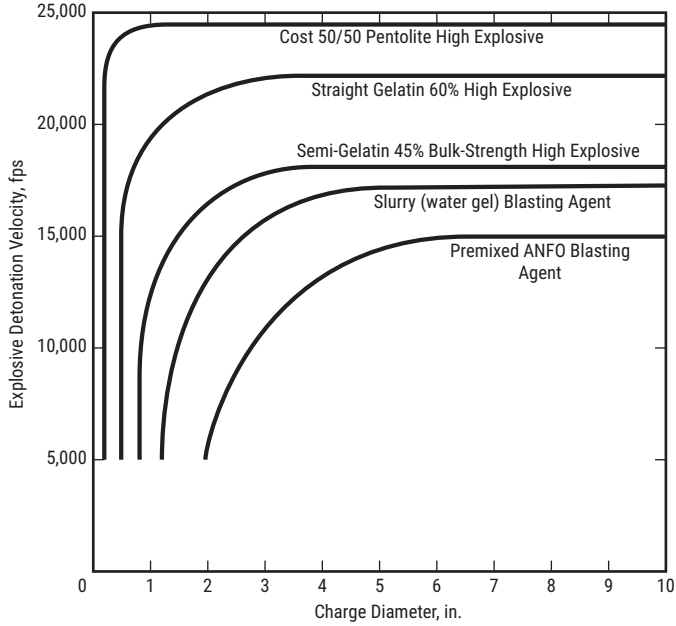

Source: IME 2013

FIGURE 11.5 Effects of charge diameter on detonation velocity

GROUND VIBRATIONS AND AIR CONCUSSION FROM BLASTING

The following subsections give equations for scaled distance and discuss vibration attenuations and peak particle velocity predictions as well as vibration level damage criteria.

Scaled Distance Equations

The scaled distance of a blast, or *scale factor*, is a parameter useful for comparing the seismic effects of blasts of varying charge weights per delay and distances (Kentucky Division of Mining Reclamation 2017; Atlas Powder Company 1987). It is useful for predicting the magnitude of peak particle velocity (PPV).

$$S = \frac{D}{\sqrt{W}} \quad \textbf{(EQ 11.51)}$$

where

S = scaled distance, ft/(lb)$^{1/2}$
D = distance from blast to nearest structure, ft
W = maximum weight of explosives detonated per 8-ms delay, lb

For compliance purposes, the allowable weight per delay can be calculated from the scaled distance equation. The use of specific scaled distances, such as 50, 55, and 65, has been statistically shown to limit peak particle vibrations to specified levels.

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Vibration Attenuations and Peak Particle Velocity Predictions

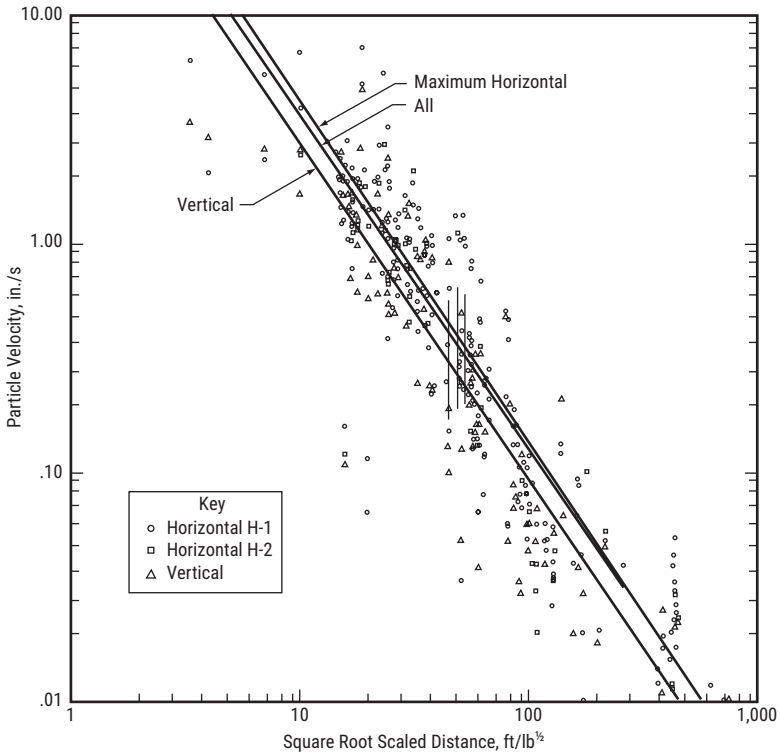
When seismic data are plotted on a log-log graph with PPV on the y-axis and scaled distance on the x-axis, a regression equation can be developed to describe the seismic characteristics of the particular site. Figure 11.6 shows an example of typical plot, and the equation for such lines is in the form

$$V = k \times S^m \quad \text{(EQ 11.52)}$$

where

V = peak particle velocity, in./s
k = site constant equal to the y intercept of the line
S = scaled distance, lb/ft$^{1/2}$
m = site constant equal to the slope of the line

The data used in Figure 11.6 are from many surface coal mines and show the three directional components of ground vibration. The wide data scatter is typical of such plots because of the additional variables not included in scaled distance such as geology, confinement, delay times, and other factors. Some typical site constants are given in Table 11.8.



Source: Siskind et al. 1980

Figure 11.6 Log-log plot of peak particle velocity versus scaled distance

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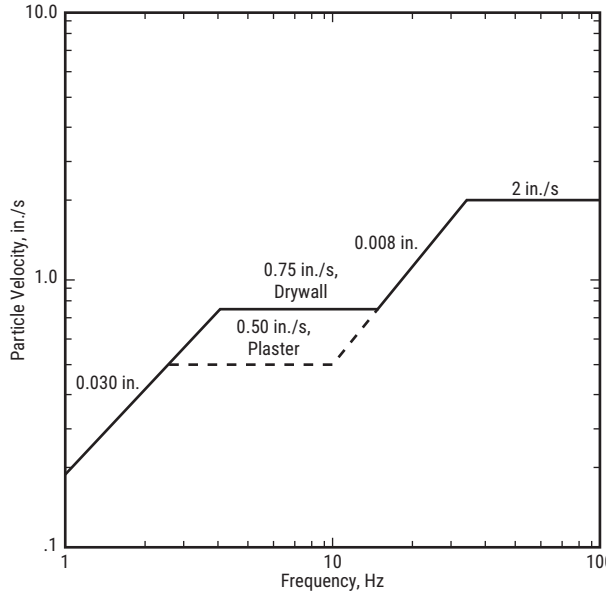
TABLE 11.8 Typical values of site constants

Seismic Channel	Range of *k* Values	Average *k*	Range of *m* Values	Average *m*
Radial	44–135	106	−1.729 to −1.324	−1.454
Transverse	40–106	61	−1.562 to −1.234	−1.368
Vertical	56–335	156	−1.825 to −1.551	−1.642

Source: Siskind et al. 1980



Source: Siskind et al. 1980

FIGURE 11.7 Safe vibration levels of blasting vibrations for residential structures based on both peak particle velocity and frequency

Vibration Level Damage Criteria

The safe blasting vibration criteria in Figure 11.7, which were developed by the U.S. Bureau of Mines, have been generally accepted as the most reliable standards available. Damage potential to residential structures is dependent on both PPV and the frequency of vibration. This curve has been adopted in many regulatory schemes on the federal, state, and local levels, and in many other countries.

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CHAPTER

12

Excavation, Loading, and Material Transport

Frank J. Filas, P.E.

EQUIPMENT PERFORMANCE

General definitions, equations, and factors for determining the performance of crawler and rubber-tired equipment are presented next. Pulling and resistive forces are solved in kilograms and pounds. Convert kilograms to newtons by multiplying by 9.81 m/s^2.

Drawbar Pull

Drawbar pull is the horizontal force available at the drawbar of tracked equipment and is typically determined in field tests conducted on firm, level ground.

$$DBP = \frac{EBP \times EFF \times UCF}{V_{tr}} \quad \text{(EQ 12.1)}$$

where

DBP = drawbar pull, kg (lb)
EBP = engine brake power, kW (hp)
EFF = efficiency in converting engine power to drawbar power, decimal
UCF = unit conversion factor, 102 (m·kg)/(kW·s) or 33,000 (ft·lb)/(hp·min)
V_{tr} = tractor speed, m/s (ft/min)

Rim Pull

Rim pull is the maximum pulling force that the engine can deliver to the tires of the driving wheels of rubber-tired equipment at the point of contact with the ground.

$$RP = \frac{EBP \times EFF \times UCF}{V_{eq}} \quad \text{(EQ 12.2)}$$

where

RP = rim pull, kg (lb)
EBP = engine brake power, kW (hp)
EFF = efficiency in converting engine power to rim pull power, decimal (typically about 0.85 for mechanical drives)
UCF = unit conversion factor, 367 (km·kg)/(kW·h) or 375 (mi·lb)/(hp·h)
V_{eq} = equipment speed, km/h (mi/h)

Traction

Usable drawbar pull and rim pull are limited by the traction of the tracks and drive wheels, respectively, on the road surface.

$$UDBP = CT \times GWT \quad \text{(EQ 12.3)}$$

$$URP = CT \times GWD \quad \text{(EQ 12.4)}$$

where

$UDBP$ = usable drawbar pull, kg (lb)
GWT = gross weight of tractor, kg (lb)
URP = usable rim pull, kg (lb)
GWD = gross weight on drive wheels, kg (lb)
CT = coefficient of traction, decimal (see Table 12.1)

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TABLE 12.1 Coefficient of traction values

Road Surface	Coefficient of Traction	
	Rubber Tires	Tracks
Concrete, new	0.80–1.00	0.45
Concrete, old	0.60–0.80	—
Concrete, wet	0.45–0.80	—
Asphalt, new	0.80–1.00	—
Asphalt, old	0.60–0.80	—
Asphalt, wet	0.30–0.80	—
Gravel, packed and oiled	0.55–0.85	—
Gravel, loose	0.35–0.70	0.50
Gravel, wet	0.35–0.80	—
Rock, crushed	0.55–0.75	0.55
Rock, wet	0.55–0.75	—
Cinders, packed	0.50–0.70	—
Cinders, wet	0.65–0.75	—
Earth, firm	0.55–0.70	0.90
Earth, loose	0.45	0.60
Sand, dry	0.20	0.30
Sand, wet	0.40	0.50–0.55
Snow, packed	0.20–0.55	0.25
Snow, loose	0.10–0.25	—
Snow, wet	0.30–0.60	—
Ice, smooth	0.10–0.25	0.12
Ice, wet	0.05–0.10	—
Coal, stockpiled	0.45	0.60

Source: Hays 1990

Resistance

Rolling resistance is the sum of the forces opposing motion over level terrain for rubber-tired equipment. Grade resistance is the gravitational force that resists movement up a slope (+) and assists movement down a slope (–). Total resistance is the sum of rolling and grade resistance and represents the resisting force that the usable rim pull must exceed before the equipment can move.

$$RR = RRF \times GMW \quad \text{(EQ 12.5)}$$

$$GRF = GCF \times \%\text{grade} \quad \text{(EQ 12.6)}$$

$$GR = GRF \times GMW \quad \text{(EQ 12.7)}$$

$$TR = RR + GR \quad \text{(EQ 12.8)}$$

where

RR = rolling resistance, kg (lb)
RRF = rolling resistance factor, kg/t (lb/st) (see Table 12.2)
GMW = gross machine weight, t (st)
GRF = grade resistance factor, kg/t (lb/st)
GCF = grade conversion factor = 10 kg/t or 20 lb/st
%grade = (± vertical distance/horizontal distance) × 100
GR = grade resistance, kg (lb)
TR = total resistance, kg (lb)

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TABLE 12.2 Rolling resistance factors

Surface	High-Pressure Tires (radial)			Low-Pressure Tires (bias ply)		
	kg/t	lb/st	Percent of Weight	kg/t	lb/st	Percent of Weight
Concrete	15	30	1.5	20	40	2.0
Asphalt	18	36	1.8	24	48	2.4
Packed gravel	22.5	45	2.3	30	60	3.0
Packed earth	30	60	3.0	40	80	4.0
Unplowed earth terrain	75	150	7.5	50	100	5.0
Rutted and uneven earth	105	210	10.5	90	180	9.0
Loose sand and gravel	140	280	14.0	120	240	12.0
Soft, muddy, deeply rutted	175	350	17.5	160	320	16.0
Snow-packed	25	50	2.5	35	70	3.5

Adapted from Drevdahl 1963

Total resistance may also be expressed as effective grade in percent of vehicle weight.

$$EG = \text{grade} + \frac{RRF}{GCF}\% \quad \textbf{(EQ 12.9)}$$

where

EG = effective grade, %

grade = (± vertical distance/horizontal distance) × 100%

RRF = rolling resistance factor, kg/t (lb/st) (see Table 12.2)

GCF = grade conversion factor = 10 kg/t or 20 lb/st

Maximum and Average Speed

The maximum speed that rubber-tired haulage equipment can achieve for a section of road with a specified total resistance can be determined from the manufacturer's specifications (i.e., performance and retarding curves). Calculating an average speed requires allowance for acceleration, deceleration, shifting, and braking, as well as safely negotiating curves, steep downhill grades, and congested areas. Use Table 12.3 to estimate an average speed for a section of road. This table is based on dividing the haul road into segments, with each segment having a relatively uniform total resistance or effective grade. The maximum speed of the truck is determined for each segment based on the manufacturer's specifications and then derated using the factors given in the table.

Altitude Correction

With increasing altitude, there is a corresponding loss in engine power (and drawbar and rim pull) because the air becomes thinner. For naturally aspirated, four-cycle gasoline and diesel engines, a rule-of-thumb reduction of 3% is used for each 305 m (1,000 ft) above the first 305 m (1,000 ft). For two-cycle diesel engines, a reduction of 1% is used for each 305 m (1,000 ft) above the first 305 m (1,000 ft). These general and common-sense rules for reduction do not apply to turbocharged engines, which typically do not experience a loss in power until they reach about 1,500 m (about 5,000 ft) or higher. For high-elevation work, using the altitude derating tables supplied in equipment performance manuals is recommended.

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TABLE 12.3 Factors for converting maximum speed to average speed*

Length of Haul Road Section		Factors for Converting Maximum Speed to Average Speed		
m	ft	Short, Level Hauls 500–1,000-ft Total Length	Unit Starting from Stop	Unit in Motion When Entering Road Section
0–107	0–350	0.20	0.25–0.50	0.50–2.00
107–229	350–750	0.30	0.35–0.60	0.60–0.75
229–457	750–1,500	0.40	0.50–0.65	0.70–0.80
457–762	1,500–2,500		0.60–0.75	0.75–0.80
762–1,067	2,500–3,500		0.65–0.75	0.80–0.85
>1,067	>3,500		0.70–0.85	0.80–0.90

*1. The average speed may be above the maximum speed for a short section if the haulage unit enters the section at high speed.

2. The return time is generally governed by job conditions and safety precautions. If no steep downgrades or operating hazards are present, the following factors apply to top speed, empty: less than 500 feet (favorable = 0.65, average = 0.60, unfavorable = 0.55); more than 500 feet (favorable = 0.85, average = 0.80, unfavorable = 0.75).

3. Recommended average speeds in loading areas for favorable, average, and unfavorable conditions are 16 km/h (10 mi/h), 11.2 km/h (7 mi/h), and 6.4 km/h (4 mi/h), respectively.

4. Recommended maximum downgrade speeds include 40–56 km/h (25–35 mi/h) for 0%–6% grades; 33–40 km/h (21–25 mi/h) for 7%–8% grades; 27–32 km/h (17–20 mi/h) for 9%–10% grades; 21–26 km/h (13–16 mi/h) for 11%–12% grades; and less than 21 km/h (13 mi/h) for grades over 12%.

5. Safety considerations may require the use of slower speeds than listed in this table. Source: Bishop 1968

PRODUCTION CALCULATIONS

General definitions, equations, and factors for determining the production of excavation, loading, and transportation equipment are presented next.

Density and Swell

Density is the mass per unit volume (see Table 2.1 in Chapter 2, which covers material properties, for densities.)

$$\text{loose density } (LD) = \text{mass (kg)}/LCM, \text{ or mass (lb)}/LCY \quad \textbf{(EQ 12.10)}$$

$$\text{bank density } (BD) = \text{mass (kg)}/BCM, \text{ or mass (lb)}/BCY \quad \textbf{(EQ 12.11)}$$

where

LCM = loose cubic meter
LCY = loose cubic yard
BCM = bank cubic meter
BCY = bank cubic yard

Swell is the percentage increase in volume that a material exhibits when removed from its natural state (see Table 2.1 in Chapter 2 for swell factors).

$$\text{swell} = \frac{100\%(LCM - BCM)}{BCM} = \frac{100\%(LCY - BCY)}{BCY} \quad \textbf{(EQ 12.12)}$$

$$\text{swell factor}\,(SF) = \frac{LD}{BD} = \frac{100}{(100 + \text{swell})} \quad \textbf{(EQ 12.13)}$$

Equation 12.13 yields swell factors of less than one. Some references invert this equation, which results in swell factors greater than one. Either method is correct as long as the derived swell factors are used with the appropriate equation.

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Operating Efficiency

The maximum estimated equipment production is adjusted down to account for the periods when equipment is not operating to its full potential. For primary mining and excavating equipment operating under relatively uniform conditions, the efficiency is commonly calculated as

$$E = A \times U \quad \text{(EQ 12.14)}$$

where

E = operating efficiency, decimal
A = availability factor—the portion of scheduled time that the equipment is mechanically and electrically ready to be operated, decimal
U = utilization factor—the portion of available time that the equipment is being operated at full potential, decimal

For equipment operating under more variable conditions or as part of an integrated system, the operating efficiency may be estimated as a function of previous experience with similar job and management conditions.

$$E = JC \times MC \quad \text{(EQ 12.15)}$$

where

E = operating efficiency, decimal
JC = job conditions factor, decimal
MC = management conditions factor, decimal

Bucket and Dipper Fill Factor

The fill factor, denoted here by *FF*, for excavating equipment is the percentage of the bucket or dipper's heaped capacity that actually fills with material, expressed as

$$FF = \frac{LCM(\text{actual})}{LCM(\text{capacity})} = \frac{LCY(\text{actual})}{LCY(\text{capacity})} \quad \text{(EQ 12.16)}$$

Load Cycle Time

The load cycle time T_{cl} is the time required for excavating and loading equipment to complete one cycle of filling the bucket or dipper, swinging or tramming to the dump point, dumping, swinging or tramming to the excavation point, and positioning the bucket or dipper for filling.

Loading Production

Use Equation 12.17 to calculate production for excavating and loading equipment. This equation includes propel (or move) time between production locations as a separate factor. Propel time may also be included as part of the operating efficiency or omitted if it is not a significant consideration. To determine total production, multiply the hourly production rate by the scheduled work hours.

$$P_l = \frac{3{,}600 \times Cb \times SF \times E \times FF \times PT}{T_{cl}} \quad \text{(EQ 12.17)}$$

where

P_l = loading production, *BCM*/h (*BCY*/h)
3,600 = seconds in 1 h (s/h)
Cb = bucket or dipper heaped capacity, *LCM* (*LCY*)
SF = swell factor, ratio of *BCM*/*LCM* (ratio of *BCY*/*LCY*)
E = operating efficiency, decimal
FF = fill factor, decimal
PT = propel time factor, decimal
T_{cl} = load cycle time, s

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Travel Time

Use the formula that follows to calculate equipment travel time.

$$TT = \frac{D}{V_{ave} \times UCF} \quad \text{(EQ 12.18)}$$

where

TT = travel time, min
D = distance, m (ft)
V_{ave} = average speed, km/h (mi/h)
UCF = units conversion factor = 16.7 (m·h)/(km·min) or 88 (ft·h)/(mi·min)

Haulage Cycle Time

The theoretical cycle time for haulage equipment is the sum of the load time, travel time to the dump point, dump or spread time, and return time. The actual or corrected cycle time also includes waiting and expected delays (if these are not already included in the operating efficiency factor).

$$T_{ch} = T_l + TT_o + T_{dp} + TT_r \quad \text{(EQ 12.19)}$$

$$TC_{ch} = T_{ch} + T_w + T_d \quad \text{(EQ 12.20)}$$

where

T_{ch} = theoretical haulage cycle time, min
T_l = equipment load time, min
TT_o = travel time to dump point, min
T_{dp} = dump or spread time, min
TT_r = travel return time, min
TC_{ch} = corrected haulage cycle time, min
T_w = wait time, min
T_d = delay time, min

Haulage Production

Use Equation 12.21 to estimate haulage production. To determine the production in metric or short tons per hour, multiply by the bank density (t/*BCM* or st/*BCY*).

$$P_h = \frac{60 N_h L_h E}{TC_{ch}} \quad \text{(EQ 12.21)}$$

where

P_h = haulage production, *BCM*/h (*BCY*/h)
60 = minutes in 1 h, min/h
N_h = number of haulage units, integer
L_h = haul load, *BCM* (*BCY*)
E = operating efficiency, decimal
TC_{ch} = corrected cycle time, min

EQUIPMENT OPERATING COSTS

Equipment operating costs include labor, electrical power or fuel, preventive maintenance, repairs, and tire replacement (if applicable). General guidelines for estimating some of these costs follow.

Maintenance and Repair

Most pieces of large, nonmobile excavation and haulage equipment have operating lives of 20–30 years with annual maintenance and repair costs ranging between 5% and 15% of installed cost. Use Equation 12.22 and the repair factors listed in Table 12.4 to estimate repair costs. Repair factors vary depending on operating conditions and the age of the equipment. Table 12.4 also presents typical operating lives for mobile equipment. Preventive maintenance costs for mobile equipment—including the labor required for equipment servicing—average between 15% and 25% of the fuel costs. For more information, see Chapter 22, "Maintenance."

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TABLE 12.4 Mobile equipment operating lives and repair factors

Equipment	Normal Operating Life, h	Repair Factor
Wheel loader	7,000–12,000	0.30–1.00
Rear dump truck, mechanical drive	20,000–30,000	0.30–1.00
Rear dump truck, electric drive	30,000–40,000	0.20–0.65
Tractor-trailer truck	20,000–30,000	0.25–0.80
Conventional scraper	12,000–16,000	0.30–1.40
Push-pull scraper	12,000–16,000	0.35–1.45
Elevating scraper	12,000–16,000	0.40–1.60
Track dozer	8,000–18,000	0.16–0.50
Wheel dozer	8,000–12,000	0.20–0.40

Compiled from Hays 1990 and Atkinson 1992

$$Rc = \frac{Fr \times Vd}{10{,}000} \quad \text{(EQ 12.22)}$$

where

Rc = hourly repair cost, $/h
Fr = repair factor, decimal (see Table 12.4)
Vd = depreciable new value of the equipment, $ (exclude tire costs)
10,000 = conversion factor, h

Tires

Tire life is affected by tire type and construction, equipment loads and velocity, operating surface, and tire load and position. Table 12.5 lists the average life of tires used in their correct applications. Estimate hourly tire costs by dividing the cost of the tires by the life of the tires and then adding 15% for repairs.

TABLE 12.5 Tire life

	Job Conditions*			
	A	B	C	D
Average life, operating hours	4,000–5,000	3,300–3,500	2,000–2,500	400–1,500

* A = good road surface with operation over soft, nonabrasive soil or rock.
B = average road surface over soft soil or rock.
C = average road surface over abrasive, medium-hard rock.
D = road surface consisting of hard, angularly fragmented rock.

Source: Atkinson 1992

Fuel and Power Consumption

Use Equation 12.23 to calculate fuel consumption for diesel equipment. This equation is based on an engine fuel consumption of 0.26 kg/kW·h (0.42 lb/hp·h) and a diesel fuel density of 0.85 kg/L (7.1 lb/gal). Engine load factors typically range between 0.25 and 0.75, depending on the equipment type and use level.

$$Cf = EBP \times Fl \times UCF \quad \text{(EQ 12.23)}$$

where

Cf = fuel consumption, L/h (gal/h)
EBP = engine brake power, kW (hp)
Fl = engine load factor, decimal
UCF = unit conversion factor, 0.3 L/kW·h (0.06 gal/hp·h)

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The average power consumption for electrically powered equipment can be found in the manufacturer's equipment specifications. If this is not available, average consumption may be estimated to be about 35% of the total continuous power ratings of the motors. The power consumption per unit of bank material moved generally ranges between 0.45 and 1.2 kW·h/*BCM* (0.35 and 0.93 kW·h/*BCY*) for excavating equipment. Typically, larger equipment is more efficient.

DRAGLINES

Dragline size selection is based on the maximum volume of overburden to be removed per unit time and the required maximum reach of the machine. Given the required stripping rate and reach, estimate the cycle time and determine an approximate bucket capacity by using Equation 12.17 and Tables 12.6 through 12.9. Propel time is included in the utilization factor given in Table 12.9.

TABLE 12.6 Approximate specifications for walking draglines*

Bucket Size		Dumping Radius		Dumping Height		Width over Shoes		Machine Mass	Ballast
m^3	yd^3	m	ft	m	ft	m	ft	t	t
11.5	15	50	165	21	70	14	45	470	115
15	20	58	190	23	75	16	52	550	160
31	40	67	220	26	85	22	72	1,250	200
38	50	79	260	30	100	23	76	1,950	250
46	80	84	275	37	120	27	90	2,900	320
70	90	92	300	41	135	32	105	4,200	340
85	110	92	300	44	145	35	115	5,700	370

* Conversion factor, 1 st (short ton) = 0.9072 t (metric ton). Source: Atkinson 1992

TABLE 12.7 Swell and bucket fill factors for walking draglines

Overburden Conditions	Swell Factor	Bucket Fill Factor
Light blasting	0.81	0.85–0.90
Medium blasting	0.75	0.80–0.90
Heavy blasting	0.71	0.75–0.85
Poor fragmentation	0.69	0.70–0.75

Adapted from Atkinson 1992

TABLE 12.8 Theoretical cycle times (in seconds) for draglines

Bucket Size		Swing Angle, degrees			
m^3	yd^3	90	120	150	180
Up to 15	Up to 19	55	62	69	77
16–26	20–34	56	63	70	78
27–44	35–59	57	64	71	79
45–57	60–74	59	65	72	80
58–92	75–120	60	66	73	81
93–150	121–200	62	69	76	84

Adapted from Atkinson 1992

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TABLE 12.9 Dragline operating efficiency

Availability	Utilization			
	Excellent	Good	Fair	Poor
Excellent	0.84	0.81	0.76	0.70
Good	0.78	0.76	0.71	0.64
Fair	0.72	0.69	0.65	0.60
Poor	0.63	0.61	0.57	0.52

Source: Humphrey 1990

CLAMSHELLS

Clamshells are useful in lifting loose materials vertically from one level to another. They are used to clean ditches and settlement ponds, to load materials from a stockpile into trucks or a hopper, or to unload barges. Use Equation 12.17 to calculate clamshell production.

SHOVELS

When determining the required number and size of shovels for a mine, consider the following 12 factors (Sargent 1990):

1. Ground-bearing pressure
2. Truck size
3. Bench height
4. Tonnage required
5. Blending required
6. Number of ore and waste faces required
7. Fragmentation expected
8. Maintenance facilities
9. Material weights
10. Pit geometry
11. Dipper weight required
12. Infrastructure requirements

Use Tables 12.10 through 12.12, along with Equation 12.17, to estimate shovel production. Average propel time factors are 0.75 for strip mines, 0.85 for open pit mines, 0.90 for sand and gravel pits, and 0.95 for high-face quarries (Atkinson 1992).

TABLE 12.10 Shovel cycle times and fill factors

Bucket Capacity		Average Cycle Time, s*			
m^3	yd^3	E	M	M-H	H
3	4	18	23	28	32
4	5	20	25	29	33
5	6	21	26	30	34
5.5	7	21	26	30	34
6	8	22	27	31	35
8	10	23	28	32	36
9	12	24	29	32	37
11.5	15	26	30	33	38
15	20	27	32	35	40
19	25	29	34	37	42
35	45	30	36	40	45
Average fill factor		0.95–1.0	0.85–0.90	0.80–0.85	0.75–0.80

* E (easy digging) = loose free running material (e.g., sand, small gravel, loose earth, ashes, bituminous coal).
M (medium digging) = partially consolidated materials (e.g., clayey gravel, packed earth, clay, anthracite coal).
M-H (medium-hard digging) = materials requiring some blasting (e.g., weaker rock, gravel w/ boulders, heavy clay).
H (hard digging) = materials requiring heavy blasting (e.g., hard, competent rock). Adapted from Atkinson 1992

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TABLE 12.11 Bench and swing angle corrections

Optimum digging depth, %	40	60	80	100			
Cycle time correction factor	1.25	1.10	1.02	1.00			
Angle of swing, degrees	45	60	75	90	120	150	180
Cycle time correction factor	0.83	0.91	0.95	1.00	1.10	1.19	1.30

Adapted from Atkinson 1992

TABLE 12.12 Shovel and loader operating efficiency

Job Conditions	Management Conditions			
	Excellent	Good	Fair	Poor
Excellent	0.83	0.80	0.77	0.70
Good	0.76	0.73	0.70	0.64
Fair	0.72	0.69	0.66	0.60
Poor	0.63	0.61	0.59	0.54

Source: Atkinson 1992

BACKHOES

Because of its configuration, a backhoe can efficiently excavate material located below its base. Larger backhoes occasionally find use in surface mining applications where the backhoe sits on top of the bench to be loaded. This includes pits having a wet or incompetent floor and narrow pits where trucks can be loaded by backing up to the front and slightly to either side of the backhoe (i.e., a minimal boom swing is required). Calculate backhoe production in the same manner used for shovels.

WHEEL LOADERS

Wheel loaders are used for both loading and load-and-carry applications as well as in other mine support functions. Table 12.13 gives truck-loading cycle times for wheel loaders operating in different types of digging conditions. The wheel loader is usually not used in hard digging conditions.

Wheel loaders are used to load and transport materials for short distances up to about 150 m (490 ft) in various production, stockpiling, and process feed applications. The fixed portion of the load-and-carry cycle time, which consists of positioning, loading, and dumping, typically requires between 30 and 40 seconds per cycle depending on digging conditions, amount of maneuvering required, and dumping procedures. Average tram speeds for load-and-carry applications typically vary from 3 km/h (1.8 mi/h) for short distances

TABLE 12.13 Wheel-loader cycle times

Bucket Capacity		Cycle Time, s*		
m^3	yd^3	E	M	M-H
4.0	5	32	33	41
4.5	6	33	34	42
5.5	7	33	35	44
7.5	10	37	39	51
9.0	12	39	42	56
11.5	15	41	44	60

* E (easy digging) = loose free running material (e.g., sand, small gravel, loose earth, ashes, bituminous coal).
M (medium digging) = partially consolidated materials (e.g., clayey gravel, packed earth, clay, anthracite coal).
M-H (medium-hard digging) = materials requiring some blasting (e.g., weaker rock, gravel w/ boulders, heavy clay).

Adapted from Atkinson 1992

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to more than 15 km/h (9.0 mi/h) for longer distances. Use the manufacturer's performance charts and Table 12.3 to estimate tram speed.

To estimate loader productivity, use Equation 12.17 with propel time *PT* omitted. Fill factors are similar to those listed in Table 12.10 for shovels (i.e., 0.95–1.0 for easy digging, 0.85–0.90 for medium digging, and 0.80–0.85 for medium-hard digging). The maximum bucket capacity of a wheel loader may also be limited by its maximum rated payload (kilograms or pounds). Use Table 12.12 to estimate operating efficiency *E*. For load-and-carry operations, the load cycle time T_{cl} in Equation 12.17 is replaced with the load-and-carry cycle time.

CRAWLER-MOUNTED LOADERS

Because of their reduced mobility, crawler- or track-mounted loaders find fewer appropriate applications in the mining industry than wheel loaders. They do provide an advantage over wheel loaders where the ground is very abrasive, less competent (i.e., soft or wet), or steeply dipping (e.g., decline development). Production calculations for a crawler-mounted unit are similar to those for a wheel unit with longer loading and tram times.

Trucks

Surface haul trucks include the following (Hays 1990):

- Conventional rear dump trucks, which are used in and around open pits, strip mines, and quarries where traction and maneuverability are needed
- Tractor-trailer trucks, which are used for high-speed, long-distance haulage
- Integral bottom dump trucks, which are used for haulage of coal and other soft, free-flowing materials

Truck Loading

Trucks and loading equipment are typically matched in size so that between three and six passes by the loading equipment are required to load each truck. To determine the number of passes required to load a truck to its weight capacity, use Equation 12.24a. For less-dense materials, it may also be necessary to determine the number of passes required to load a truck to its heaped volume capacity (see Equation 12.24b). For estimating purposes, the lower of the two *Np* values is rounded to the next whole integer, denoted by *NP*. Equation 12.25 can then be used to estimate the actual load per truck.

$$Np = \frac{Ctw}{Cb \times FF \times SF \times BD} \quad \text{(EQ 12.24a)}$$

$$Np = \frac{Ctv \times SF}{Cb \times FF \times SF} = \frac{Ctv}{Cb \times FF} \quad \text{(EQ 12.24b)}$$

$$L_h = NP \times Cb \times FF \times SF \quad \text{(EQ 12.25)}$$

where

Np = number of passes, decimal number
NP = number of passes, integer
Ctw = truck capacity, t (st)
Cb = loading bucket heaped capacity, *LCM* (*LCY*)
FF = bucket fill factor, decimal
SF = material swell factor, ratio of *BCM*/*LCM* (ratio of *BCY*/*LCY*)
BD = bank density, t/*BCM* (st/*BCY*)
Ctv = truck heaped capacity, *LCM* (*LCY*)
L_h = truck load, *BCM* (*BCY*)

Truck Spot and Load Time

Spot time is the time needed for the truck to maneuver into position for loading, and load time is the time required for the loading machine to make the number of passes required to load the truck. These times may overlap because the loading machine may perform parts of its work cycle while the truck is being spotted. If the spot time is less than the loading machine cycle time, use Equation 12.26a to determine the combined spot and load time. If the spot time is greater than the loading machine cycle time, use Equation 12.26b (Hays 1990). Spot times for rear dump trucks typically range between 0.3 and 0.6 min, with smaller trucks

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generally being faster. Spot times for tractor-trailer units may range from 0.15 to 1.0 min, depending on the loading method and the need for backing (Bishop 1968).

$$T_l = NP \times T_{cl} \quad \text{(EQ 12.26a)}$$

$$T_l = T_s + (NP - 1)T_{cl} \quad \text{(EQ 12.26b)}$$

where

T_l = truck spot and load time, min
NP = number of passes by the loading machine, integer
T_{cl} = loading machine cycle time, min
T_s = truck spot time, min

Truck Dump Time

Dump time T_{dp} for a rear dump truck is the time required to enter the dump area, turn, back up, raise the body to dump, and lower the body. For a side dump or bottom dump tractor-trailer, the dump time is the time required to pull through or over the dump area, sometimes dumping without stopping. Approximate dump times are 1.0 min for rear dump units and 0.5 min for side and bottom dump units (Hays 1990).

Truck Cycle Time and Production

Calculate truck travel time using the truck's specifications, Table 12.3, and Equation 12.18. To calculate the theoretical truck cycle time T_{ch} and actual or corrected cycle time TC_{ch}, use Equations 12.19 and 12.20, respectively. Use Equation 12.21 to calculate truck fleet production.

Number of Trucks

Use Equation 12.27 to determine the estimated number of trucks needed to service a shovel or loader. Most operations also have spare trucks available in case of breakdowns.

$$N_h = \frac{T_{ch}}{T_l} \quad \text{(EQ 12.27)}$$

where

N_h = number of trucks
T_{ch} = theoretical truck cycle time, min
T_l = truck spot and load time, min

SCRAPERS

Scrapers commonly in use include the following (Hays 1990):

- Conventional scrapers (both single and dual engines), which are used to load a wide range of materials in conjunction with a pusher dozer
- Push-pull scrapers (dual engines), which are used to load a wide range of materials with the ability to load in tandem without a pusher dozer
- Elevating scrapers (both single and dual engines), which are used to self-load easily excavated, fine-grained materials

Scraper Loading and Spreading

Scrapers are generally filled to their heaped capacity with the resulting load calculated using Equation 12.28. The scraper load time T_l and spread time T_{dp} depend on the type of scraper used, material characteristics, pushing or pulling force, and operating conditions. Typical load and spread times for various conditions are presented in Tables 12.14 and 12.15, respectively.

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TABLE 12.14 Scraper load times

Scraper Type	Load Time, min		
	Favorable	Average	Unfavorable
Conventional, push loaded			
Single engine	0.40	0.70	1.00
Dual engines	0.35	0.60	0.90
Conventional, push-pull			
Dual engines*	0.60	0.90	1.50
Elevating			
Single engine	0.60	0.90	1.30
Dual engines	0.45	0.70	1.00

* Time for loading both scrapers.

Source: Hays 1990

TABLE 12.15 Scraper spread times

Scraper Type	Spread Time, min		
	Favorable	Average	Unfavorable
Conventional			
Single engine	0.30	0.60	1.00
Dual engines	0.30	0.50	0.90
Elevating			
Single engine	0.40	0.70	1.10
Dual engines	0.30	0.60	1.00

Source: Hays 1990

$$L_h = Cs \times SF \quad \text{(EQ 12.28)}$$

where

L_h = scraper load, *BCM* (*BCY*)
Cs = scraper heaped capacity, *LCM* (*LCY*)
SF = swell factor, ratio of *BCM*/*LCM* (ratio of *BCY*/*LCY*)

Scraper Cycle Time and Production

To calculate scraper travel time, use the scraper's performance specifications, Table 12.3, and Equation 12.18. Use Equations 12.19 and 12.20, respectively, to determine the theoretical scraper cycle time T_{ch} and the actual or corrected cycle time TC_{ch}. Equation 12.21 may be used to calculate scraper fleet production.

Pusher Dozers

Pusher cycle time T_{pc} is the time required for the pusher dozer to push load the scraper, boost the scraper out of the cut, move to the next scraper, and position the push blade against the scraper. It also includes the average wait time, if any. The pusher cycle time depends on the type of push loading pattern employed, as shown in Table 12.16.

Equation 12.29 gives the theoretical number of scrapers that a pusher dozer can service. Times for other tasks such as ripping, clearing, or salvaging of topsoil need to be included in the pusher cycle time if the dozer is required to perform these operations in addition to pushing.

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TABLE 12.16 Pusher cycle times with no waiting

Pusher Pattern	Pusher Cycle Time, min*		
	Favorable	Average	Unfavorable
Back track loading	0.75	1.25	1.80
Chain loading	0.60	0.95	1.40
Shuttle loading	0.60	0.95	1.40

* For tandem pushers, multiply pusher cycle time by 1.20. Source: Hays 1990

$$N_h = T_{ch}/T_{pc} \quad \text{(EQ 12.29)}$$

where

N_h = number of scrapers, integer
T_{ch} = theoretical scraper cycle time, min
T_{pc} = pusher cycle time with no waiting, min

BULLDOZERS

The track-mounted bulldozer is used in mine production for pushing overburden or product (e.g., ore, coal) up to about 80 m (260 ft), as well as for ripping unconsolidated soil and rock. Track and wheel bulldozers are also used for a wide variety of mine support applications.

Dozer Blades

Common dozer blades include straight, universal, semiuniversal, angling, and cushion blades. The first three blades are used in production dozing with the straight blade having the best penetration and the universal blade the greatest capacity. The blade capacity for dozers is provided in the equipment specifications as *LCM* or *LCY* and can be converted to bank measure for production calculations using Equation 12.30. For side-by-side or slot dozing, increase the capacity by 20%.

$$L_h = Cbd \times SF \quad \text{(EQ 12.30)}$$

where

L_h = dozer load, *BCM* (*BCY*)
Cbd = blade capacity, *LCM* (*LCY*)
SF = swell factor, ratio of *BCM*/*LCM* (ratio of *BCY*/*LCY*)

Bulldozer Cycle Time and Production

Use Equation 12.21 to calculate bulldozer production. The bulldozer cycle time TC_{ch} consists of the time necessary to excavate and push the material the desired distance, spread the material, back up to the starting point, and position the blade for the next pass. When excavating, cut lengths range from 7.6 to 23 m (25 to 75 ft) and load times range from 0.15 to 0.45 min, depending on the type of material involved. Pushing the material is typically done in first or second gear at speeds between 0.8 and 1.7 m/s (2.6 and 5.6 ft/s). Spread times usually range from 0.08 to 0.12 min. Backing up to the beginning of the cut in third gear is typically limited to about 2.2 m/s (7.2 ft/s) for track-mounted units and 4.4 m/s (14.4 ft/s) for wheel units (Hays 1990).

Ripping

To estimate ripper production, use Equation 12.31. The ripper cycle time T_{cr} consists of the time needed to make a single pass while ripping, including the time necessary to turn the dozer to begin the next pass.

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TABLE 12.17 Rolling frictional resistance of mine cars

Resistance		Type of Car	Track Condition
kg/t	lb/st		
3–5	6–10	Large, modern railroad type with excellent bearings, ≥30 st	Excellent
5.0–7.5	10–15	Large, mine type with good bearings, 8-wheel, 15–30 st	Good
10.0–12.5	20–25	Medium size, roller bearings, 5–10 st	Fair to Good
15.0–17.5	30–35	Small, bronze bearings, <5 st	Fair

Adapted from Brantner 1973

During ripping, the dozer is typically operated in first gear at velocities of 25–40 m/min (82–131 ft/min). Turn times are typically between 0.20 and 0.35 min (Hays 1990). If depth of penetration d is left out of the equation, the area ripped per hour (square meter per hour or square yard per hour) can be calculated.

$$P_r = \frac{l \times w \times d \times E \times 60}{T_{cr}} \quad \text{(EQ 12.31)}$$

where

P_r = ripper production, BCM/h (BCY/h)
l = length of the area being ripped, m (yd)
w = width between ripper passes, m (yd)
d = depth of ripper penetration, m (yd)
E = efficiency, decimal
60 = minutes in 1 h, min/h
T_{cr} = ripper cycle time, min

RAIL HAULAGE

Rail haulage is employed in a variety of underground mines and in some large surface mines with long (i.e., more than 4 km), moderately sloped haulage requirements. Haulage grades are usually limited to a maximum of 3% uphill and 4% downhill (Brauns and Orr 1968). In the following equations, pulling and resistive forces are solved for in kilograms and pounds. Convert kilograms to newtons by multiplying by 9.81 m/s^2.

Tractive Resistance

The factors influencing a train's resistance to movement include frictional resistance, grade resistance, track curvature resistance, and acceleration resistance. Frictional resistance for locomotives is usually estimated as 7.5 kg/t (15 lb/st) for antifriction bearings and 10 kg/t (20 lb/st) for bushed bearings. Use Table 12.17 to estimate the frictional resistance of the mine cars, and employ the following equations to estimate grade curve and acceleration resistances. Typical locomotive speeds are between 16 and 32 km/h (10 and 20 mi/h/s) with starting accelerations between 0.16 and 0.32 km/h/s (0.1 and 0.2 mi/h/s; Brantner 1973). Wind resistance is negligible for the speeds employed in most mining applications.

$$CRF = CCF \times C \quad \text{(EQ 12.32)}$$

where

CRF = curvature resistance factor, kg/t (lb/st)
CCF = curvature conversion factor, 0.4 kg/t·degree (0.8 lb/st·degree)
C = track curvature, degrees

$$ARF = ACF \times ACC \quad \text{(EQ 12.33)}$$

where

ARF = acceleration resistance factor, kg/t (lb/st)
ACF = acceleration conversion factor, 31 kg·h·s/t·km (100 lb·h·s/st·mi)
ACC = acceleration, km/h/s (mi/h/s)

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$$TR = LW(FRF_L) + TW(FRF_C) + GTW(GRF + CRF + ARF) \quad \text{(EQ 12.34)}$$

where

TR = total tractive resistance, kg (lb)
LW = locomotive weight, t (st)
FRF_L = locomotive frictional resistance factor, kg/t (lb/st)
TW = gross trailing weight of the cars, t (st)
FRF_C = car frictional resistance factor, kg/t (lb/st; see Table 12.17)
GTW = gross train weight (locomotive and cars), t (st)
GRF = grade resistant factor, kg/t (lb/st; see Equation 12.6)
CRF = curvature resistance factor, kg/t (lb/st; see Equation 12.32)
ARF = acceleration resistance factor, kg/t (lb/st; see Equation 12.33)

Tractive Effort

Tractive effort, defined as the total force delivered by the locomotive to its drive wheels, is a function of its engine power and speed. The net tractive effort (i.e., tractive effort minus tractive resistance) must be greater than zero before the train can move.

$$TE = \frac{(EP - EPa)(EFF \times UCF)}{V} \quad \text{(EQ 12.35)}$$

$$NTE = TE - TR \quad \text{(EQ 12.36)}$$

where

TE = tractive effort of locomotive, kg (lb)
EP = engine power of locomotive, kW (hp)
EPa = engine power to the auxiliaries, kW (hp)
EFF = efficiency in converting engine power to tractive effort, decimal (typically 0.80–0.85)
UCF = unit conversion factor, 367 (km·kg)/kW·h) or 375 (mi·lb)/hp·h)
V = train speed, km/h (mi/h)
NTE = net tractive effort, kg (lb)
TR = total tractive resistance, kg (lb; see Equation 12.34)

Adhesion

Adhesion for locomotives is analogous to traction for crawler and rubber-tired equipment. When the tractive effort exceeds adhesion, the wheels of the locomotive will slip (i.e., the available tractive effort is limited by adhesion). Table 12.18 supplies coefficient of adhesion values for various rail conditions.

$$TE_A = CA \times LW \times UCF \quad \text{(EQ 12.37)}$$

where

TE_A = available tractive effort, kg (lb)
CA = coefficient of adhesion, decimal
LW = locomotive weight, t (st)
UCF = unit conversion factor, 1,000 kg/t (2,000 lb/st)

TABLE 12.18 Coefficient of adhesion values*

Description	Unsanded Rails	Sanded Rails
Clean dry rails, starting and accelerating	0.30	0.40
Clean dry rails, continuous running	0.25	0.35
Clean dry rails, locomotive braking	0.20	0.30
Thoroughly wet rails	0.18	0.25
Greasy, moist rails	0.15	0.20
Dry, snow-covered rails	0.11	0.15

* For cast-iron wheels on steel rails, reduce listed values by 20%. Adapted from Brantner 1973

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Acceleration

Acceleration of the train depends on the available net tractive effort.

$$T_a = \frac{TCF}{NTE}(V_2 - V_1) = \frac{V_2 - V_1}{ACC} \quad \text{(EQ 12.38)}$$

$$D_a = \frac{DCF}{NTE}(V_2 - V_1) = \frac{UCF \times T_a \times (V_2 - V_1)}{2} \quad \text{(EQ 12.39)}$$

where

T_a = acceleration time, s
D_a = acceleration distance, m (ft)
TCF = time conversion factor, 27 s·h·kg/km (95.6 s·h·lb/mi)
DCF = distance conversion factor, 6 m·kg·h/km (70 ft·lb·h/mi)
NTE = net tractive effort, kg (lb)
V_1 = initial velocity, km/h (mi/h)
V_2 = final velocity, km/h (mi/h)
ACC = acceleration, km/h/s (mi/h/s)
UCF = units conversion factor, 0.278 m·h/km·s (1.47 ft·h/mi·s)

Braking

A deceleration rate D of 0.5 km/h/s (0.3 mi/h/s) is normally acceptable for bringing a train to a stop, although a higher rate may be needed for emergencies (Brantner 1973). Braking time and distance depends on the available retarding force (Equation 12.40). To find stopping time and distance, substitute deceleration for acceleration or available retarding force F_r for NTE in Equations 12.38 and 12.39, respectively (Ramani 1990). Consistent with other equations in this chapter, uphill grades are considered positive and downhill grades, negative in Equation 12.40.

$$F_r = BE + (TW \times FRF_C) + (GTW \times GRF) \quad \text{(EQ 12.40)}$$

where

F_r = available retarding force, kg (lb)
BE = braking effort, kg (lb)
TW = gross trailing weight of the cars, t (st)
FRF_C = car frictional resistance factor, kg/t (lb/st; see Table 12.17)
GTW = gross train weight (locomotive and cars), t (st)
GRF = grade resistance factor, kg/t (lb/st; see Equation 12.6)

Rail Haulage Cycle Time and Production

To calculate haulage travel time, use Equation 12.18 and an estimated average locomotive velocity. The average velocity will depend on the NTE available, as well as on acceleration/deceleration requirements. Use Equations 12.19 and 12.20, respectively, to calculate the theoretical haulage cycle time and corrected cycle time. Because loading and dumping times vary substantially, they must be estimated on a case-by-case basis. Use Equation 12.21 to calculate rail haulage production.

Track

Rail weight depends on the weight per wheel of the locomotive and cars, with the minimum rail weight equal to about 5.5 kg/m for each metric ton (10 lb/yd for each short ton) of weight on a wheel. The track gauge may vary from 46 to 168 cm (18 to 66 in.) with 107 cm (42 in.) being considered standard gauge in the United States. Wood ties should have a length of about twice the track gauge. Spacing of ties is dependent on rail use and ground conditions and may vary from 40 cm (16 in.) on main lines, with soft beds to 1 m (3.3 ft) or more in production areas.

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BELT CONVEYORS

Belt conveyors, which are used in a wide variety of mine and plant applications, are the most common type of conveyor. Maximum belt inclinations vary from 15 to 25 degrees, depending on the material characteristics.

Conveyor Production Capacity

Use Equation 12.41 to calculate the production capacity of conveyors. Conveyors are typically designed for peak production requirements. For belt conveyors, the average cross-sectional area depends on belt width, surcharge angle, lump size, and troughing angle. Belt speed is dependent on belt width and material properties. To estimate the optimum belt width and speed for a given production capacity and material density, use Tables 12.19 through 12.21 and Figures 12.1 and 12.2 in an iterative process. These tables and figures apply to conveyors up to about 600 m (about 2,000 ft) long with 20- or 35-degree troughing angles and horizontal or inclined configurations (Duncan and Levitt 1990).

$$P_c = A \times LD \times V_c \times UCF \qquad \textbf{(EQ 12.41)}$$

where

P_c = conveyor production capacity, t/h (stph)
A = average cross-sectional area of material on the conveyor, m^2 (ft^2)
LD = density of material (loose), kg/m^3 (lb/ft^3)
V_c = conveyor speed, m/s (ft/min)
UCF = unit conversion factor, 3.6 t·s/h·kg (0.03 st·min/h·lb)

TABLE 12.19 Recommended maximum belt speeds

Material Being Conveyed	Belt Speeds		Belt Width	
	m/s	ft/min	mm	in.
Grain or other free-flowing, nonabrasive material	2.5	500	450	18
	3.6	700	600–750	24–30
	4.1	800	900–1,050	36–42
	5.1	1,000	1,200–2,400	48–96
Coal, damp clay, soft ores, overburden and earth, fine-crushed stone	2.0	400	450	18
	3.0	600	600–900	24–36
	4.1	800	1,050–1,500	42–60
	5.1	1,000	1,800–2,400	72–96
Heavy, hard, sharp-edged ore, coarse-crushed stone	1.8	350	450	18
	2.5	500	600–900	24–36
	3.0	600	>900	>36
Feeder belts, flat or troughed, for feeding fine, nonabrasive, or mildly abrasive materials from hoppers and bins	0.3–0.5	50–100	Any width	

Source: Conveyor Equipment Manufacturers Association 2014

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TABLE 12.20 **Cross-sectional area of 20-degree troughed belt**

Belt Width*		Cross Section of Load, m^2							Cross Section of Load, ft^2						
		Surcharge Angle							Surcharge Angle						
mm	in.	0°	5°	10°	15°	20°	25°	30°	0°	5°	10°	15°	20°	25°	30°
450	18	0.008	0.010	0.012	0.014	0.016	0.017	0.019	0.089	0.108	0.128	0.147	0.167	0.188	0.209
600	24	0.016	0.019	0.023	0.026	0.030	0.033	0.037	0.173	0.209	0.246	0.283	0.320	0.359	0.399
750	30	0.026	0.032	0.037	0.043	0.048	0.054	0.060	0.284	0.343	0.402	0.462	0.522	0.585	0.649
900	36	0.039	0.047	0.055	0.064	0.072	0.080	0.089	0.423	0.509	0.596	0.684	0.774	0.866	0.960
1,050	42	0.055	0.066	0.077	0.088	0.100	0.112	0.124	0.588	0.708	0.828	0.950	1.074	1.201	1.332
1,200	48	0.073	0.087	0.102	0.117	0.132	0.148	0.164	0.781	0.940	1.099	1.260	1.424	1.592	1.765
1,350	54	0.093	0.112	0.131	0.150	0.169	0.189	0.210	1.002	1.204	1.407	1.613	1.822	2.037	2.258
1,500	60	0.116	0.139	0.163	0.187	0.211	0.236	0.261	1.249	1.501	1.753	2.009	2.270	2.537	2.812
1,800	72	0.170	0.204	0.238	0.272	0.308	0.344	0.381	1.826	2.192	2.560	2.933	3.312	3.701	4.102
2,100	84	0.233	0.280	0.327	0.374	0.423	0.472	0.524	2.513	3.014	3.519	4.030	4.551	5.085	5.635
2,400	96	0.307	0.369	0.430	0.493	0.556	0.621	0.689	3.308	3.967	4.631	5.302	5.986	6.687	7.411

* Three equal rolls standard edge distance = 0.055 b + 0.9 in. (b = belt width).

Source: Conveyor Equipment Manufacturers Association 2014

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TABLE 12.21 **Cross-sectional area of 35-degree troughed belt**

Belt Width*		Cross Section of Load, m^2							Cross Section of Load, ft^2						
		Surcharge Angle							Surcharge Angle						
mm	in.	0°	5°	10°	15°	20°	25°	30°	0°	5°	10°	15°	20°	25°	30°
450	18	0.013	0.015	0.016	0.018	0.020	0.021	0.023	0.144	0.160	0.177	0.194	0.212	0.230	0.248
600	24	0.026	0.029	0.032	0.035	0.038	0.041	0.044	0.278	0.309	0.341	0.373	0.406	0.440	0.474
750	30	0.042	0.047	0.052	0.057	0.062	0.067	0.072	0.455	0.506	0.557	0.609	0.662	0.716	0.772
900	36	0.063	0.070	0.077	0.084	0.091	0.098	0.106	0.676	0.751	0.826	0.903	0.980	1.060	1.142
1,050	42	0.087	0.097	0.107	0.117	0.126	0.137	0.147	0.940	1.044	1.148	1.254	1.361	1.471	1.585
1,200	48	0.116	0.129	0.141	0.154	0.168	0.181	0.195	1.248	1.385	1.523	1.662	1.804	1.949	2.099
1,350	54	0.149	0.165	0.181	0.198	0.215	0.232	0.250	1.599	1.774	1.950	2.128	2.309	2.494	2.686
1,500	60	0.185	0.205	0.226	0.246	0.267	0.289	0.311	1.994	2.211	2.429	2.651	2.876	3.107	3.345
1,800	72	0.271	0.300	0.330	0.359	0.390	0.421	0.453	2.913	3.229	3.547	3.869	4.197	4.532	4.879
2,100	84	0.372	0.412	0.453	0.494	0.536	0.578	0.623	4.007	4.440	4.876	5.317	5.766	6.226	6.701
2,400	96	0.490	0.543	0.596	0.650	0.705	0.761	0.819	5.274	5.842	6.415	6.994	7.584	8.189	8.812

* Three equal rolls standard edge distance = 0.055 b + 0.9 in. (b = belt width).

Source: Conveyor Equipment Manufacturers Association 2014

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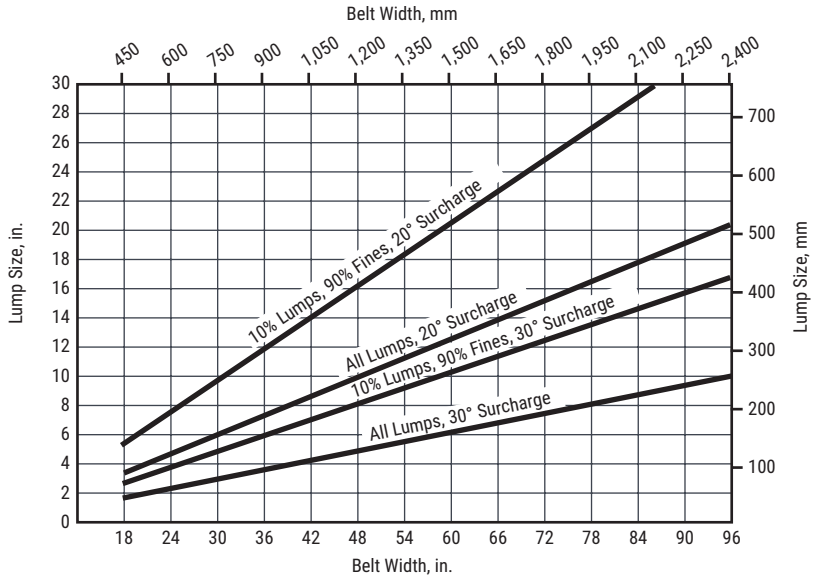
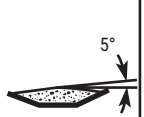
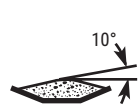
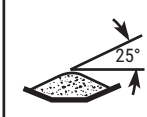
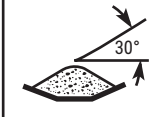


Source: Conveyor Equipment Manufacturers Association 2014

FIGURE 12.1 Belt width necessary for a given lump size

Very Free Flowing	Free Flowing	Average Flowing		Sluggish
5° Angle of Surcharge	10° Angle of Surcharge	20° Angle of Surcharge	25° Angle of Surcharge	30° Angle of Surcharge
				
0°–19° Angle of Repose	20°–29° Angle of Repose	30°–34° Angle of Repose	35°–39° Angle of Repose	≥40° Angle of Repose
Material Characteristics				
Uniform size, very small rounded particle, either very wet or very dry, such as dry silica sand, cement, wet concrete, etc.	Rounded, dry polished particles, of medium weight, such as whole grain and beans	Irregular, granular or lumpy materials of medium weight, such as anthracite coal, cottonseed meal, clay, etc.	Typical common materials such as bituminous coal, stone, most ores, etc.	Irregular, stringy, fibrous, interlocking material, such as wood chips, bagasse, tempered foundry sand, etc.

Source: Conveyor Equipment Manufacturers Association 2014

FIGURE 12.2 Flowability, angle of surcharge, and angle of repose

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TABLE 12.22 Weight per unit length of belt and idlers in kilograms (pounds)

Belt Width		Material Density, kg/m³ (lb/ft³)							
mm	(in.)	800	(50)	1,600	(100)	2,400	(150)	3,200	(200)
450	(18)	17.9	(12)	20.8	(14)	25.3	(17)	25.3	(17)
600	(24)	23.8	(16)	28.3	(19)	34.2	(23)	34.2	(23)
750	(30)	29.8	(20)	35.7	(24)	43.2	(29)	43.2	(29)
900	(36)	41.7	(28)	52.1	(35)	61.0	(41)	71.4	(48)
1,050	(42)	50.6	(34)	62.5	(42)	72.9	(49)	87.8	(59)
1,200	(48)	61.0	(41)	75.9	(51)	102.7	(69)	114.6	(77)
1,350	(54)	71.4	(48)	86.3	(58)	116.1	(78)	132.4	(89)
1,500	(60)	89.3	(60)	104.2	(70)	129.5	(87)	147.3	(99)
1,800	(72)	110.1	(74)	123.5	(83)	168.2	(113)	193.5	(130)
2,100	(84)	151.8	(102)	189.0	(127)	221.7	(149)	245.5	(165)
2,400	(96)	174.1	(117)	212.8	(143)	269.4	(181)	269.4	(181)

Source: Conveyor Equipment Manufacturers Association 2014

Belt Conveyor Power Requirements

Belt conveyors require power to (1) drive the empty conveyor, (2) elevate the material, and (3) convey the material horizontally. Use Equation 12.42 and Table 12.22 to estimate power requirements.

$$PR_c = (R1 \times V_c) + (R2 \times H) + (R3 \times P_c) \quad \textbf{(EQ 12.42)}$$

where

PR_c = total conveyor power required, W (hp)
$R1$ = power rate to drive empty conveyor, W/0.5 m/s (hp/100 ft/min; see Table 12.22)
V_c = conveyor belt speed, m/s (ft/min)
$R2$ = power rate to elevate material, W/m (hp/ft)
H = height of lift, m (ft)
$R3$ = power rate to convey material horizontally, W/25 kg/s (hp/100 stph)
P_c = conveyor production capacity, kg/s (stph)

HOISTING

The two types of hoists in common use are the drum hoist and the friction or Koepe hoist. Drum hoists consist of one or two steel or cast-iron hoisting drums on which the hoisting rope is stored. The other end of the hoisting rope is connected to the conveyance (i.e., either a skip or cage). In friction hoists, the hoist rope passes over a wheel or sheave with a conveyance on each end of the rope or with a conveyance on one end and a counterweight on the other end.

Hoist Cycle Time

Use Equation 12.43 to calculate the cycle time for loading, hoisting, and dumping (Edwards 1992). Maximum hoist speeds are typically between 2 and 20 m/s (6.6 and 66 ft/s); acceleration and deceleration rates can be as high as 1.2 m/s^2 (4 ft/s^2), but are typically closer to 0.75 m/s^2 (2.5 ft/s^2). Mining laws and regulations may also place limits on maximum accelerations and speeds. Typical combined dump and load times are about 20 s.

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$$TC_H = \frac{V_{max}}{ACC} + \frac{H}{V_{max}} - \left[\left(\frac{V_{max}}{2}\right)\left(\frac{1}{ACC} + \frac{1}{RET}\right)\right] + \frac{V_{max}}{RET} + T_{dl} \quad \text{(EQ 12.43)}$$

where

TC_H = hoist cycle time, s
H = hoisting distance, m (ft)
V_{max} = maximum hoisting speed, m/s (ft/s)
ACC = skip acceleration, m/s^2 (ft/s^2)
RET = skip retardation or deceleration, m/s^2 (ft/s^2)
T_{dl} = combined time to load and dump the skip, s

For a single skip system, calculate the time to lower the skip by excluding T_{dl} in Equation 12.43.

Skips

Mine skips include the Kimberley or overturning skip, the swing-out body skip, and the bottom dump skip. To estimate the approximate weight of the empty skip, use Equation 12.44 (Edwards 1992).

$$WS = 0.5(L_h) + SWC \quad \text{(EQ 12.44)}$$

where

WS = weight of empty skip, kg (lb)
L_h = skip payload, kg (lb)
SWC = skip weight constant, 680 kg (1,500 lb)

Hoisting Ropes

Hoisting ropes include round-strand, flattened-strand, and locked-coil ropes. Each has different internal construction, weight per foot, and breaking strength characteristics. Table 12.23 lists the properties for the most common rope dimensions. Mining laws and regulations typically require a minimum load strength for the hoisting rope(s) equal to a safety factor multiplied by the maximum suspended load. Safety factors between 4 and 10 are common.

Hoisting Capacity

Use Equation 12.21 to estimate the hoisting capacity of production skips. Convert cycle time to minutes. The haul load is typically given in metric tons (short tons) rather than bank measure. The operating efficiency (E) for hoists averages about 0.7.

Hoist Power Requirements

The running power requirements (i.e., hoisting at constant velocity) for a hoist motor are given in Equation 12.45. When designing hoist power systems, the power is normally calculated for each phase of hoisting (e.g., acceleration, constant speed, deceleration) to determine the peak accelerating power and root-mean-square power required.

$$PR_h = \frac{TWS \times V_{max}}{UCF \times EFF} \quad \text{(EQ 12.45)}$$

where

PR_h = approximate hoist motor power required, kW (hp)
TWS = total weight of loaded skip including rope, kg (lb)
V_{max} = maximum hoisting speed, m/s (ft/s)
UCF = unit conversion factor, 102 (m·kg)/(kW·s) or 550 (ft·lb)/(hp·s)
EFF = gear efficiency in converting engine power, decimal (typically about 0.85)

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TABLE 12.23 Wire rope data*

Rope Diameter		18 × 7 Independent Wire Rope Core Sinking (round)				6 × 27 Fiber Core Flattened Strand (triangular)				Locked Coil			
		Weight		Breaking Strength		Weight		Breaking Strength		Weight		Breaking Strength	
cm	in.	k /m	lb/ft	1,000 kg	1,000 lb	k /m	lb/ft	1,000 kg	1,000 lb	kg/m	lb/ft	1,000 kg	1,000 lb
1.9	¾	1.65	1.11	25.05	55.23	1.44	0.97	25.9	57	1.95	1.31	31.5	69.4
2.2	⅞	2.11	1.42	33.58	74.04	1.98	1.33	35.4	78	2.75	1.85	42.6	94.0
2.5	1	2.74	1.84	43.48	95.85	2.59	1.74	46.7	103	3.65	2.45	55.9	123.2
2.9	1⅛	3.47	2.33	54.98	121.21	3.30	2.22	59.9	132	4.58	3.08	71.1	156.8
3.2	1¼	4.17	2.80	67.43	148.65	4.08	2.74	73.9	163	5.58	3.75	87.4	192.6
3.5	1⅜	5.03	3.38	81.39	179.44	4.94	3.32	89.4	197	6.74	4.53	105.7	233.0
3.8	1½	5.98	4.02	96.40	212.53	5.88	3.95	106.6	235	7.81	5.25	126.0	277.8
4.1	1⅝	6.98	4.69	112.95	249.00	6.93	4.66	125.2	276	9.29	6.24	147.3	324.8
4.4	1¾	8.36	5.62	135.39	298.47	7.99	5.37	144.7	319	10.85	7.29	170.7	376.4
4.8	1⅞	9.20	6.18	150.23	331.19	–	–	–	–	12.59	8.46	196.1	432.4
5.1	2	10.54	7.08	170.46	375.79	–	–	–	–	14.39	9.67	223.5	492.8
5.4	2⅛	12.19	8.19	197.25	434.85	–	–	–	–	–	–	–	–
5.7	2¼	13.62	9.15	220.24	485.54	–	–	–	–	–	–	–	–

* Data is for standard tensile wire with a nominal breaking load of 120 lt/in.^{2} (long tons per square inch).

Data from Wire Rope Industries, as appears in Edwards 1992

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SELECTED UNDERGROUND EQUIPMENT

The common types of equipment used in underground coal and hard-rock production operations are discussed next.

Continuous Miners

Continuous miners are used to excavate coal and other soft materials such as trona and potash in underground room-and-pillar operations. Cutting heads are typically between 2.4 and 3.7 m (8 and 12 ft) wide. Most rooms and cross-cuts are between 14 and 24 ft wide (i.e., two passes of the machine are required in each face). Although the continuous miner is capable of excavating between 13.6 and 32.7 t/min (15 and 36 st/min), production is primarily controlled by the mine plan, face dimensions, roof conditions, amount of reject material, efficiency in moving between production faces, the number of shuttle cars and average haulage cycle time, equipment availability (including main line haulage), and scheduled production hours per shift. Depending on these factors, average production rates of clean coal can vary between 20 and 130 t/h (22 and 143 stph).

Shuttle Cars

Shuttle cars are wheel-mounted units used to haul coal from the continuous miner to a loading point (typically a belt feeder). They may be powered by cable-reel electric, battery, or diesel, and their capacities typically range between 6.8 and 10.9 t (7.5 and 12 st). Shuttle cycle times vary according to haul road conditions and the length of haul. A typical cycle time for a 91- to 122-m (300- to 400-ft) haul consists of loading time at 60 s, hauling to feeder at 75 s, unloading time at 30 s, and tramming to face at 75 s (Breithaupt 1982).

Longwall Systems

To estimate productions for shearer-type longwall systems, use Equation 12.46 (adapted from Peng and Chiang 1992). Haulage speeds of shearers typically range between 12 and 18 m/min (40 and 60 ft/min). The listed values for operating efficiency are for the shearer only. These values include stopping or reversing at each end of the face and some stoppage during cutting operations. If production delays are also expected as a result of conveyor, roof support, or other ancillary operations, the operating efficiency should be accordingly reduced. Moving the longwall system between panels typically requires 1–2 weeks and must be included in any long-term production calculations. Longwall panels are typically 120–300 m (400–1,000 ft) wide and 900–5,500 m (3,000–18,000 ft) long.

$$P_{she} = 60 \times Hc \times S \times V_s \times BD \times LC \times E \qquad \textbf{(EQ 12.46)}$$

where

P_{she} = shearer production, t/h (stph)
60 = minutes in 1 h, min/h
Hc = mining height, m (ft)
S = cutting web or width, m (ft)
V_s = shearer haulage speed, m/min (ft/min)
BD = bank density of coal, 1.2–1.4 t/m^3 (0.037–0.044 st/ft^3)
LC = loading coefficient of the shearer, decimal (typically 0.90–0.95)
E = operating efficiency of the shearer, decimal (typically 0.70–0.90)

Slushers

A slusher consists of a scraper pulled by wire ropes over a series of rotating drums. Slushers are used to load and transport ore over a short distance, generally between 7.5 and 150 m (25 and 500 ft). Typical scraper capacities range from 0.25 *LCM* (0.33 *LCY*) for 0.7-m (28-in.) widths up to 2.8 *LCM* (3.7 *LCY*) for 2.1-m (84-in.) widths. Rope speeds are typically between 46 and 107 m/min (150 and 350 ft/min) with the higher speeds used for finer materials, smooth bottoms, and longer hauls (Rhoades 1982).

Load-Haul-Dump Units

A load-haul-dump (LHD) unit is a wheel-mounted, bidirectional, articulated machine equipped with four-wheel drive and a bucket. LHD units are typically powered by diesel, with a low and narrow profile for underground use. Capacities range from about 1 t (1.1 st) to more than 20 t (22 st). Use Equation 12.21 to estimate productivity for the load, haul, and dump operation with the operating efficiency (E) ranging from

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0.75 (severe job conditions) to 0.92 (excellent job conditions). Use Equations 12.18–12.20 to estimate cycle times, with the combined time for loading, maneuvering, and dumping ranging between 0.8 and 1.4 min (depending on operating conditions). Tram speeds range from 5 to 16 km/h (3 to 10 mi/h) depending on haulageway clearances (Stevens and Acuna 1982).

Mine Trucks

Underground mine trucks include tip dumpers, telescoping dumpers, and push-plate dumpers (Stevens 1982). Tip dumpers require headroom to raise the truck bed during dumping; telescoping and push-plate dumpers dump out the back. Underground trucks are usually articulated and may be two- or four-wheel drive. Use Table 12.3 to estimate haulage speeds, but underground safety considerations dictate that maximum speeds be reduced by at least one gear and that average speeds be estimated at the low end of the range of values. To estimate travel and haulage cycle times, use Equations 12.18 through 12.20, and employ Equation 12.21 to estimate haulage production. Operating efficiencies and load, maneuver, and dump times are approximately the same as for LHD units discussed earlier.

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CHAPTER

13

Haul Roads

John E. Feddock, P.E. and Lyall Workman, P.E.

HAUL TRUCK SPECIFICATIONS

Haul road design parameters are based on the specifications of the largest vehicle that uses the road on a regular schedule. In most cases, this is termed an *off-highway truck*. Off-highway haul truck dimensions and weights greatly vary over a range of sizes (Table 13.1). Repair facility bays, fueling stations, and haul road design must account for this variation in size. A mine replacing existing trucks with larger units must consider modifying its facilities to accommodate the larger units and the associated costs.

STOPPING DISTANCE

Stopping distance is the sum of the distance traveled during brake reaction time and the distance required to decelerate the vehicle. A vehicle should be capable of stopping within the sight distance. When designing a road segment, use Figures 13.1, 13.2, and 13.3 to determine the stopping distance for a vehicle with a particular gross vehicle weight (GVW).

To determine stopping distances for different speeds and grades, Kaufman and Ault (1977) developed an empirical formula that is based on SAE International stopping distance limitations:

$$SD = \frac{1}{2}gt^2 \sin\theta + V_o t + \left[\frac{gt \sin\theta + V_o}{2g(U_{min} - \sin\theta)} \right]^2 \quad \text{(EQ 13.1)}$$

where

SD = stopping distance, m (ft)
g = gravitational acceleration, 9.81 m/s^2 (32.2 ft/s^2)
t = elapsed time between driver's perception of the need to stop and the actual occurrence of frictional contact at the wheel brakes, s
θ = angle of descent, degrees
U_{min} = coefficient of friction at the tire-to-road contact area, dimensionless
V_o = vehicle speed at time of perception, m/s (ft/s)

The factor t is actually the sum of two times t_1 and t_2. The component t_1 is the elapsed time for brake reaction caused by pressure buildup in the brake system once the brake pedal has been depressed and the brake mechanism activates to exert a retarding force on the wheels. Stopping time varies with truck GVW. For example, for trucks of 100,000–200,000 lb, a value of 1.5 seconds may be used. For trucks with weights between 200,000 and 400,000 lb, this increases to 2.75 seconds and increases to 4.5 seconds for trucks with a GVW greater than 400,000 lb. For the very large trucks often used today (GVW greater than 1,000,000 lb), the time may be more than 4.5 seconds.

The variable t_2 is the driver's reaction time, which is the time between the driver's perception of the need to brake and when the individual actually begins to depress the brake pedal. A reasonable time for this is considered to be 1.5 seconds, but the reaction time can be affected by driver fatigue or poor visibility for example.

Work by Kaufman and Ault substituting values into the equation for U_{min} determined an average value of 0.3. The factor U_{min} can be evaluated using the following expression:

$$U_{min} = \frac{V^2}{2gS} \quad \text{(EQ 13.2)}$$

where

V = SAE test velocity of 8.94 m/s (32.2 km/h)
g = gravitational acceleration, 9.81 m/s^2
S = stopping distance computed by subtracting 8.94 × t_1 from the SAE recommended stopping distance

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TABLE 13.1 Summary of off-highway, rigid-frame truck specifications by gross vehicle weight (GVW)

Specification	Truck Sizes, GVW, lb										
	150,000–300,000				300,000–600,000		700,000–1,000,000		>1,000,000		
Capacity											
Metric tons	36	44	53	62	92	133	183	226	290	318	363
Short tons	39	49	59	68	102	147	202	249	320	350	400
Tires	18.00R33	21.00R33	24.00R35	24.00R35	27.00R49	33.00R51	37.00R57	40/00R57	53/80R63	56/80R63	56/80R63
Wheel load (per tire)											
Kilograms	11,869	13,683	17,123	18,635	27,442	41,579	54,053	64,334	86,938	95,028	103,948
Pounds	26,167	30,167	37,750	41,083	60,500	91,667	119,167	141,833	191,667	209,500	229,167
Turning radius											
Meters	20	21	27	27	25	33	30	33	32	39	42
Feet	67	70	87	87	83	109	99	107	106	127	138
Width											
Meters	5	5	6	6	7	7	8	8	9	9	10
Feet	16	16	19	19	22	23	25	27	30	30	32
Height											
Meters	4	4	5	5	5	6	6	7	8	8	8
Feet	14	14	15	15	17	19	21	22	25	26	25
Estimated operator eye level											
Meters	3	3	3	3	4	5	5	5	6	6	6
Feet	11	11	11	11	14	15	16	16	20	19	20
Maximum speed											
Kilometers per hour	74	80	68	68	68	56	58	61	60	64	68
Miles per hour	46	50	42	42	42	35	36	38	37	40	42
Weight, empty											
Kilograms	35,460	37,888	49,312	50,103	72,412	116,482	141,249	159,755	231,332	252,651	260,815
Pounds	78,175	83,528	108,715	110,458	159,641	256,800	311,400	352,200	510,000	557,000	575,000
Gross vehicle weight											
Kilograms	71,214	82,100	102,739	111,810	164,654	249,476	324,318	386,007	521,631	570,165	623,689
Pounds	157,000	181,000	226,500	246,500	363,000	550,000	715,000	851,000	1,150,000	1,257,000	1,375,000

Adapted from Caterpillar, Inc. 2017

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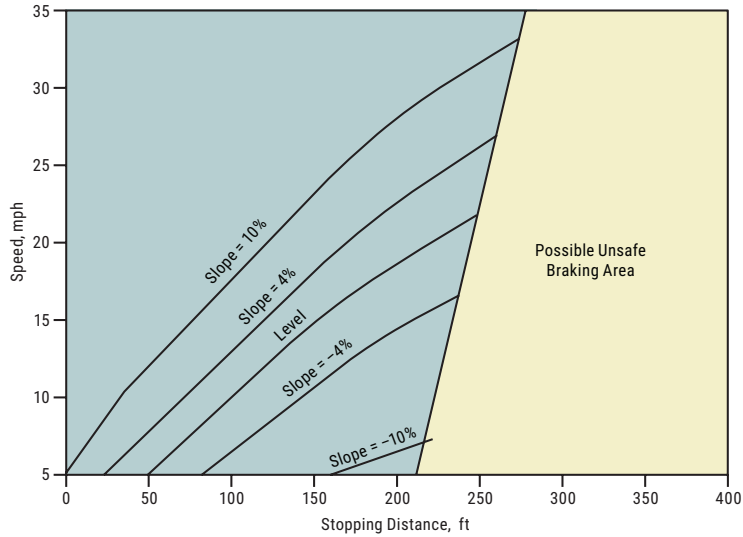
The preceding equations do not allow for brake overheating and fading. If the brakes are activated for a longer interval, the amount of heat generated will cause a reduction in the brakes' ability to slow or stop the vehicle. Based on work by the British Columbia Department of Mines, it has been determined that stopping distances should not exceed 61 m (200 ft); however, in some cases, test vehicles exceeded 200 ft while executing a safe stop.

The preceding equations are empirical and provide only rough estimates of stopping distances. Before beginning to construct the haul road, the equipment manufacturer should be consulted to determine service brake performance for the vehicle to be used.

Using the preceding equations, Kaufman and Ault constructed charts of stopping distance versus vehicle speed. Figure 13.1 is a chart for trucks with a GVW in excess of 400,000 lb. The near vertical line on the chart reflects the 200-ft stopping distance, increasing with speed to reflect the increasing distance traveled during the operator reaction time.

In this case, the brake reaction time is assumed to be 4.5 seconds. The "possible unsafe braking area" on the chart is the area where the overheating of the brakes may render them ineffective. Again, equipment manufacturers should be consulted for the specific brake performance of a given truck. This chart may not be applicable to the very large trucks often used today.

Figures 13.2 and 13.3 show stopping distances for vehicles with 100,000–200,000 lb and 200,000–400,000 lb GVW, respectively.



Source: Feddock 2000; Kaufman and Ault 1977

FIGURE 13.1 Stopping distance for greater than 400,000 lb GVW

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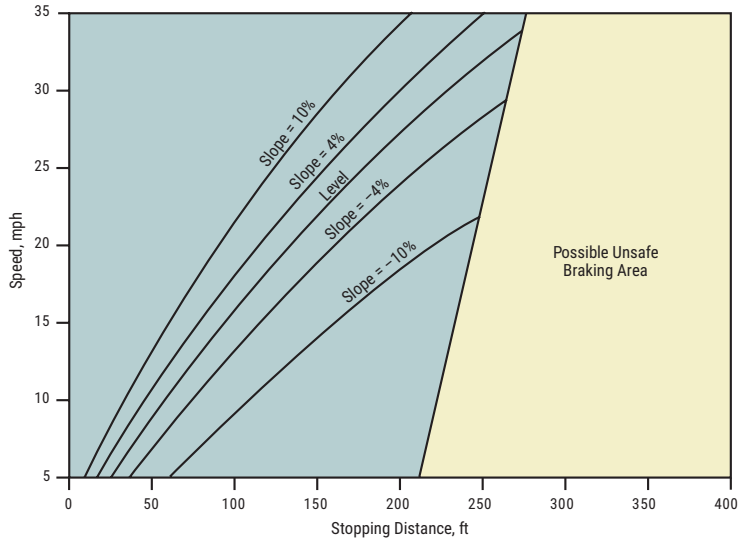
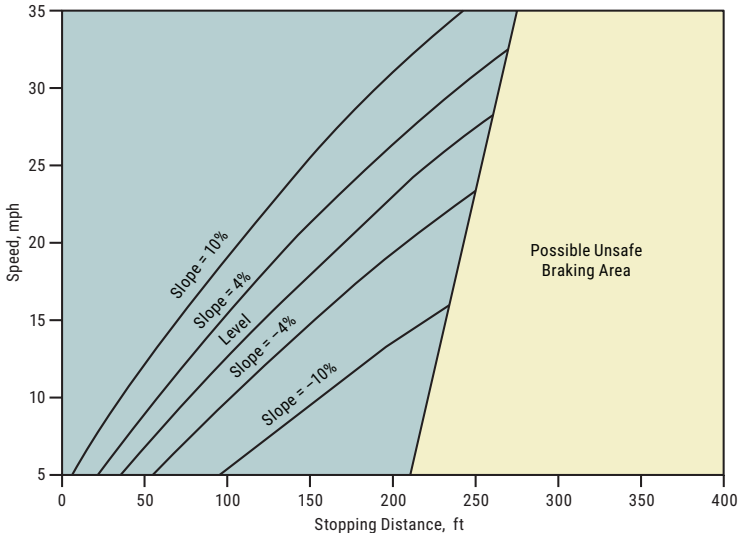


Source: Feddock 2000; Kaufman and Ault 1977

FIGURE 13.2 Stopping distance for 100,000–200,000 lb GVW



Source: Feddock 2000; Kaufman and Ault 1977

FIGURE 13.3 Stopping distance for 200,000–400,000 lb GVW

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SAFE DISTANCE BETWEEN TRUCKS

Safe distance depends on the driver's reaction time (usually estimated at 1.5 seconds), the gradient of the road, the road surface, and an allowance distance (usually 16.5 ft or 5 m). The estimated safe distance d_b between trucks can be calculated from the following equations (Atkinson 1992):

$$d_b = \frac{2.0v}{3.6} + \frac{v^2}{254(C_t \pm i)} + 5.0\,(\text{m}) \qquad \textbf{(EQ 13.3)}$$

$$d_b = \frac{v}{1.08} + \frac{v^2}{91.5(C_t \pm i)} + 16.5\,(\text{ft}) \qquad \textbf{(EQ 13.4)}$$

where

C_t = coefficient of traction
i = steepest haul road gradient as a fraction
v = vehicle speed, km/h (mph)

ROAD WIDTH

Haul roads must be wide enough for safe operation and operational efficiency. Road widths vary widely across the mining industry because the size of equipment utilized. Therefore, the best design criteria for road width is the size of the vehicles involved.

For straight road segments, road designs often account for clearance on either side of the vehicle by using one-half the truck width. When the road is multilane, the clearance between trucks in adjacent lanes is often shared. This criteria for road width does not include the width of any safety berms that are required. The height of safety berms must be equal to the axle height of the largest equipment using the road. For the large trucks in use today, the base width of the berm may add a significant width in the overall design of the road.

Using the preceding criteria, road width can be calculated using Equation 13.5 (Tannant and Regensburg 2001). This formula produces Table 13.2, which gives the factor the truck width must be multiplied by to determine the running width of the road for the number of lanes.

$$W = (1.5L + 0.5)X \qquad \textbf{(EQ 13.5)}$$

where

W = width of running surface, m (ft)
L = number of lanes
X = vehicle width, m (ft)

With technological advancements, haul trucks continue to evolve into larger and more powerful units. Therefore, guidelines for appropriate clearance may differ in some cases. For example, the blind spot off the right-hand corner of the cab for a driver operating a 400-st (short ton) haul truck may be as much as 80 ft. This may require more clearance to the right-hand side of the truck. The clearance between trucks may be wider and approach a full truck width. Consequently, when designing a haul road, the size of the largest equipment must be considered and typical designs modified if appropriate.

TABLE 13.2 **Width for straight regular grade haul roads**

Number of Lanes	Factor × Maximum Vehicle Width
1	2.0
2	3.5
3	5.0
4	6.5

Source: Atkinson 1992

In the case of switchbacks and other sharp curves, road widths should be increased to account for the front and rear overhang of the vehicle and the extra clearance needed to navigate curves. Table 13.3 is a guide for increasing the running width on sharp curves as a percent increase in width for a given radius and for different trucks. As the radius of curvature increases, the required road width decreases.

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TABLE 13.3 Width for haul roads with sharp curves

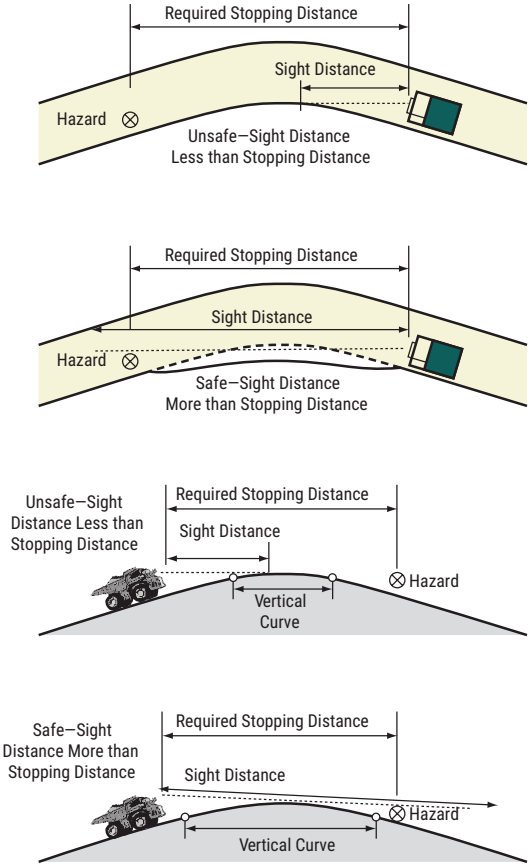
Vehicle	Percent of Table 13.2 (inside radius)*		
	6 m (20 ft)	45 m (150 ft)	60 m (200 ft)
Rear-dump and unitized trucks	125	118	110
Articulated trucks	155	135	115

* Percentages for other radii may be interpolated.

Source: Atkinson 1992

SIGHT DISTANCE

The driver's sight distance must be sufficient to enable a vehicle traveling at a given speed to stop before reaching a hazard that is 0.5 ft high above the road surface. The 0.5-ft specification is used to account for obstructions such as a prostate person, animal, or other low-profile obstruction on the road. Figures 13.4 and 13.5 show line-of-sight curves and layback requirements, respectively; Table 13.4 gives the estimated height of the driver's eye.



Source: Thompson 2011

FIGURE 13.4 Horizontal and vertical curve line of sight

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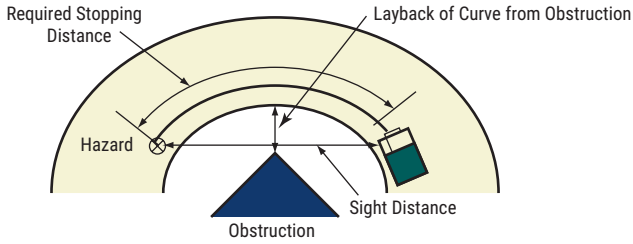

Source: Thompson 2011

FIGURE 13.5 Layback requirements on obscured horizontal curve

TABLE 13.4 Estimated height of driver's eye

Gross Vehicle Weight Range, lb	Rigid-Frame Rear Dump Truck, ft
150,000–300,000	11
300,000–600,000	14.5
700,000–1,000,000	16
>1,000,000	20

Data from Caterpillar, Inc.

The sight distance on vertical curves is measured by the chord length between points of tangency. Vertical curve lengths are necessary to provide adequate stopping distance (i.e., equal to sight distance) and may be computed as follows:

If S is greater than L_e,

$$L_s = 2S - \frac{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}{A} \quad \text{(EQ 13.6)}$$

If S is less than L_e,

$$L_s = \frac{AS^2}{100\left(\sqrt{2H_1} + \sqrt{2H_2}\right)^2} \quad \text{(EQ 13.7)}$$

where

L_e = existing curve length, ft
L_s = safe curve length, ft
A = algebraic difference in grades, %
S = stopping distance, ft
H_1 = height of eye of driver, ft
H_2 = height of object above road surface, 0.5 ft

Some modern vehicles may use a combination of radar and camera systems in front and back of the unit to assist in object detection.

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GRADIENT

For rear dump trucks in open pit mines, gradients often range between 8% and 10%. Surface coal mines often use large tractor-trailer bottom dump trucks to haul coal. These have less gradability and the gradients will typically be 5%–6%. Articulated trucks can generally pull more grade than rigid-frame units. Table 13.5 tabulates typical haul road gradients.

From a production and economics point of view, there is a trade-off between a shallow grade, longer road distance, and higher speed versus a steeper grade, shorter distance, and lower speed. For a given truck, one can optimize grade versus speed in order to optimize cycle times.

TABLE 13.5 Haul road gradients for most situations*

Sustained maximum	8%–15%
Usually optimum	≈8%
For trolley-assisted trucks	≤12%

* 150-ft sections at ≤2% gradient should be included for every 1,500–1,800 ft of severe gradient. Source: Atkinson 1992

SUPER ELEVATION

Haul road curves for vehicle speeds at or above 10 mph should be super elevated. The amount of super elevation depends on the radius of curvature and the speed of the truck. Table 13.6 is a guide of the required super elevation to eliminate lateral forces.

TABLE 13.6 Super elevation for haul road curves (in percent grade), to provide no lateral tire force

Radius, ft	Truck Speed, mph					
	10	15	20	25	30	35
50	13%	–	–	–	–	–
100	7%	15%	–	–	–	–
150	4%	10%	–	–	–	–
200	3%	8%	13%	–	–	–
300	2%	5%	9%	14%	–	–
500	1%	3%	5%	8%	12%	16%
700	1%	2%	4%	6%	9%	12%

Source: Tannant and Regensburg 2001

The portion of the haulageway that is transformed from a normal cross-slope section into a super elevated section is termed the *runout length*. One-third of the runout length should be in the curve and two-thirds on the tangent (Table 13.7).

TABLE 13.7 Maximum rates of super elevation change on tangentså

Truck speed, mph	15	20	25	30	<40
Super elevation change/100 yd of tangent, in./yd	9.35	9.00	8.28	7.20	5.76

Adapted from Atkinson 1992

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CONSTRUCTION MATERIAL FACTORS

Rolling resistance is an important factor in determining truck productivity and cost. Both tons hauled and fuel consumption are adversely affected by an increase in rolling resistance. Rolling resistance is the amount of draw bar or tractive effort required to overcome the retarding effect between the tires and ground caused by tire penetration into the ground, internal power train friction, and tire flexing under load. Rolling resistance can be equated to equivalent haul road resistance. To calculate the effective road grade,

$$\text{effective grade} = \text{rolling resistance (\%)} + \text{grade resistance (\%)} \quad \textbf{(EQ 13.8)}$$

Uphill grade resistances are usually expressed as positive and downhill grades as negative. An empirical equation for estimating rolling resistance (RR) is given as (Caterpillar 2017)

$$RR(\%) = 2\% + 1.5\% \text{ per inch of tire penetration} \quad \textbf{(EQ 13.9)}$$

For efficient equipment operation, the importance of keeping haul roads well maintained cannot be overemphasized (Table 13.8).

TABLE 13.8 Rolling resistance for various road surfaces

Ground Surface	Rolling Resistance, Equivalent Percent Haul Road Grade
Gravel, well compacted, dry, free of loose material	2.0
Gravel, not firmly compacted but dry	3.0
Gravel, loose	10.0
Mud with firm base	4.0
Mud with soft spongy base	16.0
Sand, loose	10.0
Snow, packed	2.5
Dirt, smooth, hard, dry, well maintained, free of loose material	2.0
Dirt, soft, unplowed, poorly maintained	4.0
Dirt, soft plowed	8.0
Dirt, deeply rutted	16.0

Source: Bucyrus-Erie Company 1979

OTHER CONSIDERATIONS

The road designer should consider the following points:

- Avoid introducing sharp horizontal curves at or near the crest of a hill.
- Design sections of a haulage road with long tangents and constant grades.
- Avoid intersections near crests and sharp horizontal curves.
- Adequate road drainage should have a cross-sectional slope of ¼-in. to ½-in. drop per foot of road width.

Subbase design is shown in Figures 13.6 and 13.7 in both SI and Imperial units.

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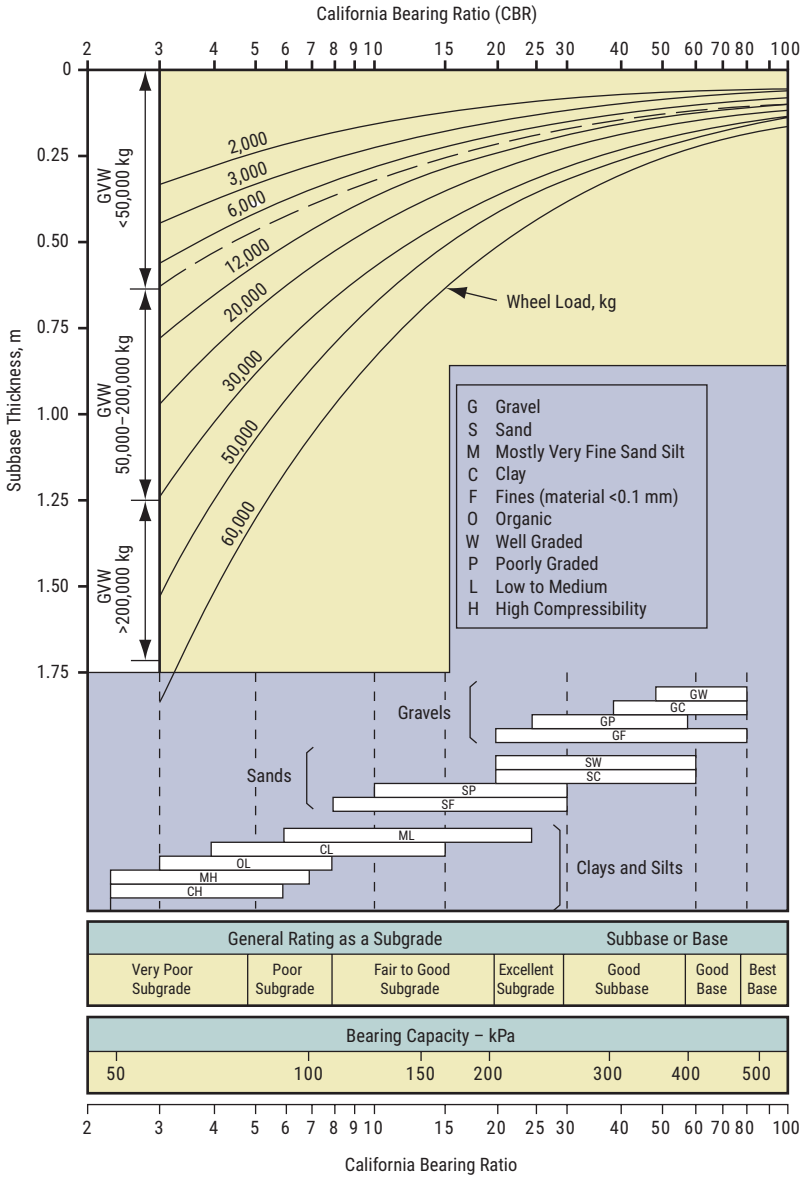

Source: Kaufman and Ault 1977

FIGURE 13.6 Haul road subbase design, SI units

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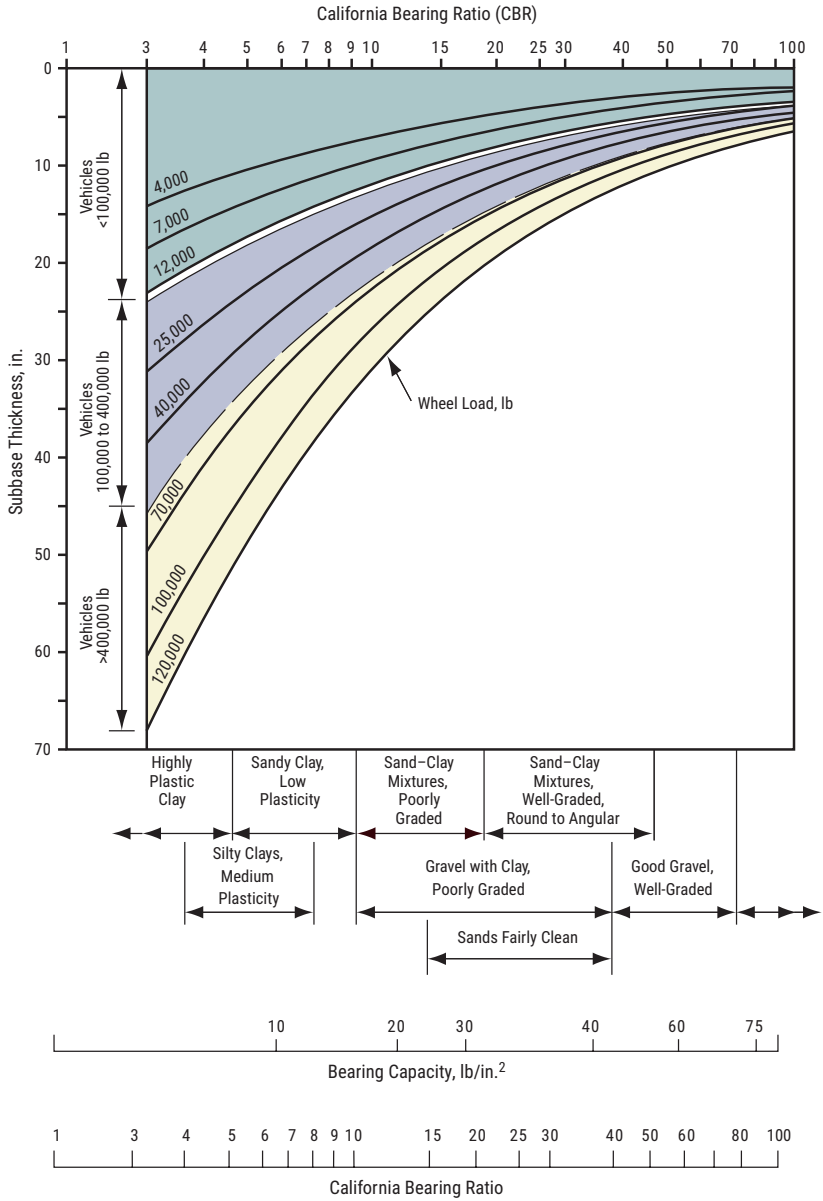
California Bearing Ratio (CBR)
1 3 4 5 6 7 8 9 10 15 20 25 30 40 50 70 100
Subbase Thickness, in.
0 10 20 30 40 50 60 70
Vehicles <100,000 lb
Vehicles 100,000 to 400,000 lb
Vehicles >400,000 lb
4,000
7,000
12,000
25,000
40,000
70,000
100,000
120,000
Wheel Load, lb
Highly Plastic Clay
Sandy Clay, Low Plasticity
Sand–Clay Mixtures, Poorly Graded
Sand–Clay Mixtures, Well-Graded, Round to Angular
Silty Clays, Medium Plasticity
Gravel with Clay, Poorly Graded
Good Gravel, Well-Graded
Sands Fairly Clean
10 20 30 40 60 75
Bearing Capacity, lb/in.2
1 3 4 5 6 7 8 9 10 15 20 25 30 40 50 60 80 100
California Bearing Ratio

Source: Kaufman and Ault 1977

FIGURE 13.7 Haul road subbase design, Imperial units

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CHAPTER

14

Ground Control and Support

James D. Newman, P.E., Thomas M. Barczak, and Alan A. Campoli, P.E.

INTRODUCTION

Generally rock masses are not homogenous and isotropic, but are layered and jointed; for this reason, the ground control requirements can vary considerably throughout a mine. While relying heavily on trial and error, ground control programs are tempered with common sense and experience. For example, in a track drift, one might start with 4-in. sections of steel with which to build sets. If that proves inadequate, 6-in. or 8-in. sections may be used for ground control. Similarly, in one part of a mine, 6-ft-long rock bolts may be used to hold the back/roof, but a change to 8-ft-long bolts may be required in another part of the mine.

Table 14.1 shows some important properties that can be used to design effective ground control programs. See Chapter 2 on material properties for additional information.

TABLE 14.1 Strength properties of rock and coal

Rock or Coal Type*	Uniaxial Compressive Strength, psi			Uniaxial Tensile Strength, psi			Modulus of Elasticity, ksi†			Poisson's Ratio		
	From	To	Mean	From	To	Mean	From	To	Mean	From	To	Mean
Basalt	6,090	51,475	21,750	290	4,060	1,885	2,320	14,645	7,685	0.13	0.38	0.22
Dolerite	32,915	46,255	40,600	1,740	3,770	2,900	8,700	13,050	10,150	0.15	0.29	0.20
Gneiss	10,585	49,300	23,055	435	3,045	2,030	2,320	14,935	8,410	0.10	0.40	0.22
Granite	4,350	46,980	24,070	435	5,655	1,740	1,450	10,730	6,525	0.10	0.39	0.23
Limestone	6,960	30,450	14,790	290	5,800	1,740	145	13,340	6,960	0.08	0.39	0.25
Norite	42,050	47,270	43,210	2,175	3,625	2,900	13,050	15,950	14,500	0.21	0.26	0.24
Quartzite	29,000	44,080	36,540	2,465	4,060	3,625	10,150	15,225	13,050	0.11	0.25	0.16
Sandstone	5,800	25,955	13,920	435	1,015	725	1,450	6,670	3,190	0.10	0.40	0.24
Shale	5,220	24,940	13,775	290	725	435	1,450	6,380	4,060	0.10	0.19	0.14
Pittsburgh coal	2,088	4,307	3,219	276	464	363	218	537	464	–	–	0.37
Pocahontas No. 3 coal	2,639	2,828	2,741	–	–	–	348	392	377	–	–	–
Herrin No. 6 coal	1,450	2,045	1,653	–	–	–	450	551	508	–	–	0.42

* Rock specimens are 4.25 in. high and 2.125 in. in diameter; coal specimens are 34-in. cubes.

† One ksi equals 1,000 psi.

Source: Bieniawski 1984

ROCK PROPERTIES CLASSIFICATION

Many different tests are used in determining rock properties and their classifications. With jointed varying ground conditions, there are tests to help determine the best ground control measures for the tested areas. The field tests in this chapter are rock quality designation (RQD), rock mass rating (RMR), and coal mine roof rating (CMRR). RQD, developed by Deere and Deere (1989), is a percent drill core recovery of 100 mm (4 in.) or larger pieces and is determined using Equation 14.1 and classified in Table 14.2.

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TABLE 14.2 RQD classification system

Rock Quality	RQD, %	Approximate Description of Rock
Excellent	90–100	Intact rock
Good	75–90	Massive, moderately jointed
Fair	50–75	Blocky and seamy
Poor	25–50	Shattered, very blocky and seamy
Very poor	0–25	Crushed

$$\text{RQD} = \frac{\text{sum of pieces} \geq 100\ \text{mm}\ (4\ \text{in.})}{\text{total drill run length}} \times 100(\%) \quad \textbf{(EQ 14.1)}$$

While at the South African Council for Scientific and Industrial Research, Bieniawski developed RMR. The method takes into account RQD and rock properties as well as the conditions of the surrounding rock. The values of the six parameters that are significant for RMR are

1. Uniaxial compressive strength of rock material,
2. RQD,
3. Spacing of discontinuities,
4. Condition of discontinuities,
5. Groundwater conditions, and
6. Orientation of discontinuities.

Details on each of these parameters are found in Table 14.3.

Using the sum of the ratings calculated for RMR from Table 14.3, excavation size and suggested supports can be determined. Dependent on the RMR for the area, ranging from very good rock to very poor rock, a guideline for supporting excavated areas can be determined. Very competent rock is less likely to require additional supports, while very weak rock requires multiple support techniques. Table 14.4 is an example of these rules of thumb for a horseshoe, 10-m wide, vertical stress less than 25 MPa, drilled and blasted tunnel.

CMRR was developed by the U.S. Bureau of Mines (Molinda and Mark 1994) as an engineering tool. CMRR can be calculated from roof falls, overcasts, and highwall exposures using only simple field tests and observations (Molinda and Mark 1994). The method is similar to the RMR; however, it is more geared toward coal measure rocks and does not require core samples. Although core samples can be used to determine the strength rating. CMRR is found using the following equations:

$$IDR = CR + SP \quad \textbf{(EQ 14.2)}$$

$$UR = IDR_L + MDA + S_r + MS_r \quad \textbf{(EQ 14.3)}$$

where

CR = cohesion–roughness rating (see Table 14.5)
SP = spacing–persistence rating (see Table 14.6)
IDR = individual discontinuity rating
IDR_L = the lowest individual discontinue rating (It is recommended to take three samples for each test.)
MDA = multiple discontinuity adjustment (see Table 14.7)
S_r = strength rating (see Figure 14.1 and Table 14.7)
MS_r = moisture sensitivity rating (see Table 14.7)
UR = unit rating (CMRR is determined using the thickness weighted average of the unit ratings for the length of the bolt.)

CMRR is used as a guide in determining adequate support system ratings and entry widths (Figure 14.2). It is supported by numerous case studies and is also used in determining stability (safety) factors in the Analysis of Longwall Pillar Stability (ALPS) software application (Mark 1999):

$$\text{ALPS SF}(R) = 2.00 - 0.016\ \text{CMRR} \quad \textbf{(EQ 14.4)}$$

where

APLS SF = ALPS stability (safety) factor
CMRR = coal mine roof rating

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TABLE 14.3 RMR classification system*

A. Classification Parameters and Their Ratings

	Parameter		Range of Values						
1	Strength of intact rock material†	Point-load strength index	>10 MPa	4–10 MPa	2–4 MPa	1–2 MPa	For this low range, uniaxial compressive tests are preferred		
		Uniaxial compressive strength	>250 MPa	100–250 MPa	50–100 MPa	25–50 MPa	5–25 MPa	1–5 MPa	<1 MPa
	Rating		15	12	7	4	2	1	0
2	Drill core quality RQD		90%–100%	75%–90%	50%–75%	25%–50%	<25%		
	Rating		20	17	13	8	3		
3	Spacing of discontinuities		>2 m	0.6–2 m	2–600 mm	60–200 mm	<60 mm		
	Rating		20	15	10	8	5		
4	Condition of discontinuities (see E)		Very rough surfaces; not continuous, no separation; unweathered wall rock	Slightly rough surfaces; separation <1 mm; slightly weathered walls	Slightly rough surfaces; separation <1 mm; highly weathered walls	Slickensided surfaces or gouge <5 mm thick or separation 1–5 mm; continuous	Soft gouge >5 mm thick or separation >5 mm; continuous		
	Rating		30	25	20	10	0		
5	Groundwater	Inflow per 10-m tunnel length, L/min	None	<10	10–25	25–125	>125		
		(joint water press)/(major principle σ)	0	<0.1	0.1–0.2	0.2–0.5	>0.5		
		General conditions	Completely dry	Damp	Wet	Dripping	Flowing		
	Rating		15	10	7	4	0		

B. Rating Adjustment for Discontinuity Orientations (see F)

Strike and dip orientations		Very favorable	Favorable	Fair	Unfavorable	Very unfavorable
Ratings	Tunnels and mines	0	−2	−5	−10	−12
	Foundations	0	−2	−7	−15	−25
	Slopes	0	−5	−25	−50	−60

C. Rock Mass Classes Determined from Total Ratings

Rating	100 ← 81	80 ← 61	60 ← 41	40 ← 21	<21
Class number	I	II	III	IV	V
Description	Very good rock	Good rock	Fair rock	Poor rock	Very poor rock

(continues)

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TABLE 14.3 RMR classification system* (continued)

D. Meaning of Rock Classes

Parameter	Range of Values				
Class number	I	II	III	IV	V
Average stand-up time	20 years for 15-m span	1 year for 10-m span	1 week for 5-m span	10 hours for 2.5-m span	30 minutes for 1-m span
Cohesion of rock mass, kPa	>400	300–400	200–300	100–200	<100
Friction angle of rock mass, degrees	>45	35–45	25–35	15–25	<15

E. Guidelines for Classification of Discontinuity Conditions

Discontinuity length (persistence) Rating	<1 m 6	1–3 m 4	3–10 m 2	10–20 m 1	>20 m 0
Separation (aperture) Rating	None 6	<0.1 mm 5	0.1–1.0 mm 4	1–5 mm 1	>5 mm 0
Roughness Rating	Very rough 6	Rough 5	Slightly rough 3	Smooth 1	Slickensided 0
Infilling (gouge) Rating	None 6	Hard filling <5 mm 4	Hard filling >5 mm 2	Soft filling <5 mm 2	Soft filling >5 mm 0
Weathering Rating	Unweathered 6	Slightly weathered 5	Moderately weathered 3	Highly weathered 1	Decomposed 0

F. Effect of Discontinuity Strike and Dip Orientation in Tunneling

Strike Perpendicular to Tunnel Axis		Strike Parallel to Tunnel Axis	
Drive with dip: Dip 45°–90°	Drive with dip: Dip 20°–45°	Dip 45°–90°	Dip 20°–45°
Very favorable	Favorable	Very favorable	Fair
Drive against dip: Dip 45°–90°	Drive against dip: Dip 20°–45°	Dip 0°–20°, irrespective of strike	
Fair	Unfavorable	Fair	

* Some conditions are mutually exclusive. For example, if infilling is present, the roughness of the surface will be overshadowed by the influence of the gouge. In such cases, use A.4 directly.

† See Figure 14.1A.

Source: Hoek et al. 1998

STABILITY FACTOR

Stability factors are based on correct classification of failed case histories in the Analysis of Retreat Mining Pillar Stability (ARMPS) and ALPS database and are used to calculate stability factors for increased longevity of openings. The value of a stability factor considered acceptable for a design is usually established from previous experience with successful designs. A factor of safety of 1.3 is generally considered adequate for a temporary mine opening; whereas a value of 1.5–2.0 may be required for a *permanent* excavation, such as an underground crusher station (Hoek et al. 1998). For coal mines, recommended stability factors are outlined in Table 14.8.

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TABLE 14.4 RMR system guidelines for excavation and support in rock tunnels*

Shape: Horseshoe; Width: 10 m; Vertical Stress: Below 25 MPa; Construction: Drilling and Blasting Rock

Rock Mass Classification	Excavation	Support		
		Rock Bolts (20 mm diameter, fully grouted)	Shotcrete	Steel Sets
Very good rock I RMR: 80–100	Full face; 3 m advance	Generally no support required except for occasional spot bolting		
Good rock II RMR: 61–80	Full face; 1.0–1.5 m advance; complete support 20 m from face	Locally bolts in crown 3 m long, spaced 2.5 m with occasional wire mesh	50 mm in crown where required	None
Fair rock III RMR: 41–60	Top heading and bench 1.5–3 m advance in top heading; begin support after each blast; complete support 10 m from face	Systematic bolts 4 m long, spaced 1.5–2 m in crown and walls with wire mesh in crown	50–100 mm in crown and 30 mm in sides	None
Poor rock IV RMR: 21–40	Top heading and bench 1.0–1.5 m advance in top heading; install support concurrently with excavation 10 m from face	Systematic bolts 4–5 m long, spaced 1–1.5 m in crown and walls with wire mesh	100–150 mm in crown and 100 mm in sides	Light to medium ribs spaced 1.5 m apart where required
Very poor rock V RMR: <20	Multiple drifts; 0.5–1.5 m advance in top heading; install support concurrently with excavation; shotcrete as soon as possible after blasting	Systematic bolts 5–6 m long, spaced 1–1.5 m in crown and walls with wire mesh; bolt invert	150–200 mm in crown, 150 mm in sides, and 50 mm on face	Medium to heavy ribs spaced 0.75 m apart with steel lagging and forepoling if required; close invert

* Table should be used as a guide only.

Source: Bieniawski 1979

TABLE 14.5 CMRR cohesion–roughness rating*

Roughness	Strong Cohesion	Moderate Cohesion	Weak Cohesion	Slickenside
Jagged	35	29	24	10
Wavy	35	27	20	10
Planar	35	25	16	10

* See Figure 14.1B

Source: Molinda and Mark 1994

TABLE 14.6 CMRR spacing–persistence rating

Persistence, m (ft)	>1.8 m (>6 ft)	0.6–0.8 m (2–6 ft)	20–60 cm (8–24 in.)	6–20 cm (2.5–8 in.)	<6 cm (<2.5 in.)
0–0.9 (0–3)	35	30	24	17	9
0.9–3 (3–10)	32	27	21	15	9
3–9 (10–30)	30	25	20	13	9
>9 (>30)	30	25	20	13	9

Source: Molinda and Mark 1994

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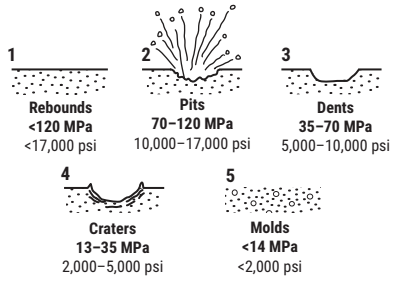
TABLE 14.7 CMRR multiple discontinuity adjustment and strength and moisture sensitivity ratings

Multiple Discontinuity Adjustment		Strength Rating		Moisture Sensitivity Rating	
Two Lowest Discontinuity Ratings Both Lower Than	Adjustment	Strength Number (see Figure 14.1)	Rating	Moisture Sensitivity	Rating
30	−5	1	30	Not sensitive	0
40	−4	2	22	Slightly sensitive	−3
50	−2	3	15	Moderately sensitive	−10
		4	10	Severely sensitive	−25
		5	5		

Source: Molinda and Mark 1994



A. Scale for comparing ball peen hammer Indentation to the compressive strength of rock

	Cohesion	Roughness	Spacing	Persistence Lateral/Vert
1	Strong (>7)*	Jagged	>1.8 m >6 ft	0–0.9 m 0–3 ft
2	Moderate (4–7)	Wavy	0.6–1.8 m 2–6 ft	0.9–3 m 3–10 ft
3	Weak (1–3)	Planar	20–60 cm 8–24 in.	3–9 m 10–30 ft
4	Slickensided (0)		6–20 cm 2.5–9 in.	>9 m >30 ft
5			<6 cm <2.5 in.	

*Chisel blows necessary to split bedding.

B. Unit discontinuities

Source: Molinda and Mark 1994

FIGURE 14.1 (A) CMRR strength, (B) unit discontinuities for determination of rating

ROCK AND ROOF BOLTING

The natural back/roof and walls/ribs may require mechanical support. The three ways that roof and rock bolting anchor are suspension, beam building, and mechanical interlocking. Some bolts are tensioned on installation and are considered active support. Other bolts are installed and provide support as the roof/back sags and are known as *passive support* (Table 14.9). Roof bolting is the required means of permanent roof support for underground coal mines (USC 1977). Table 14.10 offers common-sense rules for rock bolts.

STANDING ROOF SUPPORTS

Conventional Wood Cribs

Conventional wood cribs are constructed by stacking alternating layers of timber on top of one another, creating an *open* structure, and are considered a constant yielding standing support. (Figure 14.3 shows the four basic types of performance from standing roof support systems.) The number of contact points within each layer indicate the name (i.e., two timbers per layer creates a four-point crib; Figure 14.4). The capacity and stiffness of the support are governed by the contact area, compressive strength, and elastic modulus of the wood material. The crib should be constructed from timbers with similar material properties. A single weak layer can control the support performance to a large degree. The stability is influenced by the aspect ratio. The aspect ratio should be maintained between 2.5 and 5. Studies by the U.S. Bureau of Mines recommend an optimum aspect ratio of 4.3 (Barczak and Gearhart 1994).

Table 14.11 gives standing support specifications.

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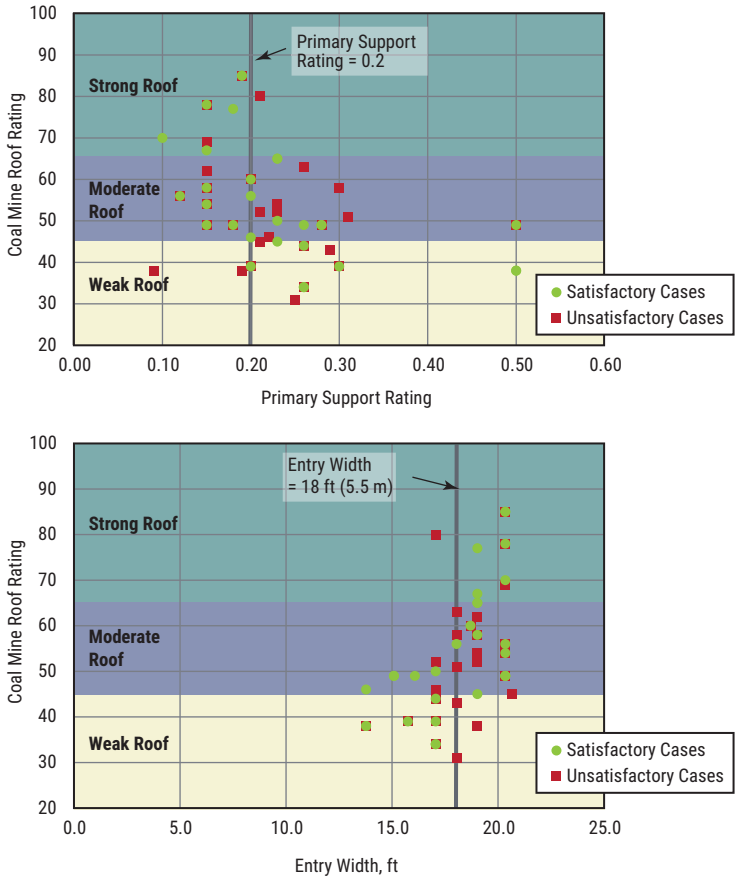

Source: Mark 2013

FIGURE 14.2 (A) Cases of CMRR and primary support rating, (B) CMRR and entry width cases

TABLE 14.8 Coal mine recommended stability factors

	Depth (H)	Weak and Intermediate Roof Strength	Strong Roof
Pillar stability factor	H < 650 ft	1.5	1.5
	650 ft ≤ H ≤ 1,250 ft	1.5 – [H-650]/1,000	1.4 – [H-650]/1,000
	1,250 ft ≤ H ≤ 2,000 ft	0.9	0.8
Barrier pillar stability factor	H ≥ 1,000 ft	≥2.0	≥1.5* (≥2.0†)
	H < 1,000 ft	No recommendation	—

* Nonbump-prone ground.

† Bump-prone ground.

Source: NIOSH 2013

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TABLE 14.9 Roof bolt characteristics by type

Type	Bolt Material	Support Type	Anchoring Mechanism	Advantages	Disadvantages
Grouted rebar	Deformed bar	Passive	Resin	High pullout resistance, corrosion resistance	Dual components, cost
Conventional	Smooth bar	Active	Expansion shell	Single component, cost	Anchor slippage, corrosion, tension loss
Cable	Wrapped wire	Passive	Cement or resin	High pullout resistance, elasticity	Cost, dual components
Friction	Split or expanding tube	Active	Tube deformations	Ease of installation	Low pullout resistance, corrosion

Source: M.L. Thomson, Fosroc, Inc., Georgetown, KY, personal communication

TABLE 14.10 Typical rock-bolt rules of thumb

Parameter	Empirical Rule
Minimum length	Greatest of (a) 2 × bolt spacing (b) 3 × thickness of critical and potentially unstable rock blocks* (c) For elements above the spring line: Spans <6 m: 0.5 × span Spans between 18 and 30 m: 0.25 × span Spans between 6 and 18 m: interpolate between 3 and 4.5 m (d) For elements below the spring line: Height <18 m: as (c) Height >18 m: 0.2 × height
Maximum spacing	Least of (a) 0.5 × bolt length (b) 1.5 × width of critical and potentially unstable blocks* (c) 2.0 m†
Minimum spacing	0.9–1.2 m
Minimum average confining pressure at yield point of elements‡	Greatest of (a) Above spring line: Either pressure = vertical rock load of 0.2 × opening width or 40 kN/m^2 (b) Below spring line: Either pressure = vertical rock load of 0.1 × opening height or 40 kN/m^2 (c) At intersections: 2 × confining pressure determined in (a) and (b)§

* Where joint spacing is close and the span relatively large, the superposition of two reinforcement patterns may be appropriate (e.g., long heavy elements on wide centers to support the span; and shorter, lighter bolts on closer centers to stabilize the surface against raveling).

† Greater spacing than 2.0 m makes attachment of surface support elements (e.g., weld mesh or chain link mesh) difficult.

‡ Assuming the elements behave in a ductile manner.

§ This reinforcement should be installed from the first opening excavated prior to forming the intersection. Stress concentrations are generally higher at intersections, and rock blocks are free to move toward both openings.

Adapted from U.S. Army Corps of Engineers 1980

Interlocking Engineered Wood Crib Supports

Notched timber supports are designed to improve the efficiency of the timber usage by creating full contact with the timber layers. They are a constant yielding standing support (Figures 14.3 and 14.5). This is achieved by notching the timbers near their end. In addition to providing full contact, which significantly increases the support strength and stiffness, the notching also enhances stability. The improved efficiency of the timber application also allows a much smaller and lighter timber to be used to provide equivalent

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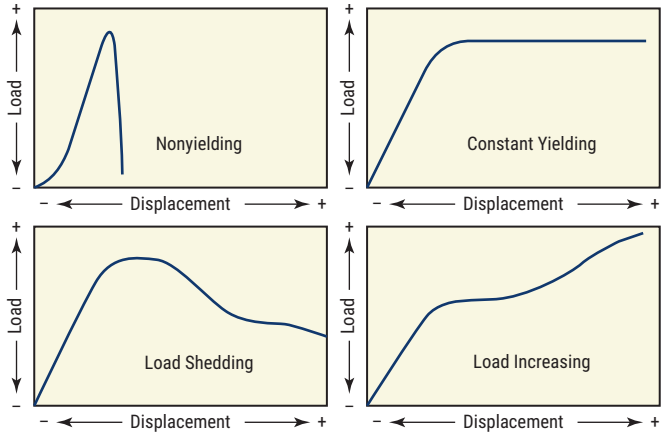
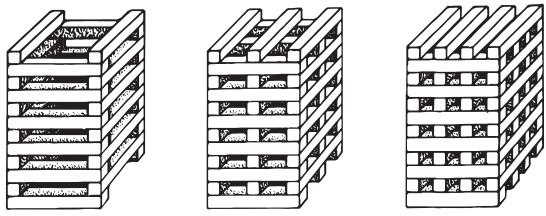


Source: Barczak 2005

FIGURE 14.3 Standing roof support system performance



Source: Barczak and Gearhart 1994

FIGURE 14.4 Wood cribs with four-point, nine-point, and sixteen-point configuration

or greater capacity than that of a conventional wood crib. The support is manufactured in a number of different timber sizes.

Engineered Timber Post Supports

Timber posts fail in buckling at loads that are proportional to their slenderness ratio. They have very limited yielding with failure occurring at about 1 in. of displacement. Cross-grain cap boards are often used to prevent punching into the roof. The cap boards also add some yielding as the cross-grain structure is softer and lower strength than the end-grain orientation of the post body.

Controlled yielding without buckling of the prop can be achieved by angled cuts into the bottom of the prop, creating a tapered solid core that progressively crushes during load application. Steel bands are added to provide confinement of the wedge cuts and aid in stability to the central core during crushing. This type of support provides support capacities similar to that of conventional wood cribbing. These props can also be arranged in clusters to provide elevated capacities. These supports must be used with a head board and foot board to perform properly.

Yieldable Cementitious Supports

Most concrete materials have high compressive strength but are weak in tensile and shear strength and exhibit brittle failure that is not conducive to using this material to construct a yieldable roof support

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TABLE 14.11 Standing support specifications

Support Type	Loading Stiffness, st (short ton)/in.	Peak Load, st	Displaced Peak Load, in.	Yield Characteristics	Yield Range, in.	Residual at 6 in., st	Residual at 10 in., st	Work at 6 in., short ton × inch	Work at 10 in., short ton × inch
Conventional wood cribs									
4-pt wood crib	16	54	3.3	Strain hardening	20	69	84	286	597
9-pt wood crib	37	118	3.2	Strain hardening	20	147	168	770	1,421
Engineered notched wood cribs									
3.5 × 6 × 24-in. timbers	63	102	2.4	Strain hardening	20	125	136	789	1,321
3.5 × 6 × 30-in. timbers	72	264	3.0	Strain hardening	20	149	160	948	1,576
3.5 × 6 × 42-in. timbers	97	365	3.4	Strain hardening	20	196	208	1,267	2,087
Engineered timber props									
8.5-in. wedge prop	24	62	6.0	Constant	12	62	46	266	469
10-in. wedge prop	87	72	0.8	Constant	12	76	81	433	770
8.5-in. tri-cluster prop	198	195	4.0	Constant	18	172	143	1,020	1,575
10-in. tri-cluster prop	612	198	0.5	Constant	18	231	275	1,341	2,184
Yieldable cementitious supports									
30-in. pumpable (calcium sulfoaluminate [CSA] grout)	469	202	0.7	Strain softening	9	127	N/A	791	N/A
30-in. pumpable (portland grout)	373	242	1.6	Strain softening	14	118	86	951	1,355
Aerated cement supports									
18-in. aerated support	24	68	2.1	Constant yield	22	59	73	310	552
24-in. aerated support	54	95	1.8	Constant yield	22	89	99	465	829
30-in. aerated support	101	135	1.9	Constant yield	22	137	137	707	1,267
Yielding steel props									
Mechanical wedge prop	51	58	6.5	Constant yield	22	58	55	313	536
Rigid (limited yielding) steel props									
100-ton "spindle" prop	1,002	240	0.3	Non-yielding	N/A	N/A	N/A	N/A	N/A
55-ton "spindle" prop	276	57	0.3	Non-yielding	N/A	N/A	N/A	N/A	N/A
Sand-filled prop	22	52	2.3	Non-yielding	N/A	N/A	N/A	N/A	N/A
Hydraulic functioning props									
Internal yielding hydraulic prop	119	50	0.5	Constant yield	17	51	53	298	507

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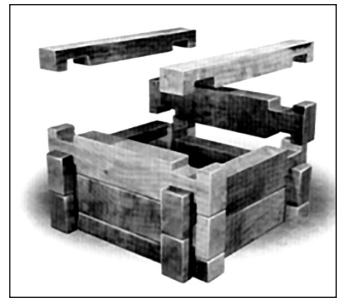

Source: Barczak 2005

FIGURE 14.5 Engineered wood crib

system. However, innovative concepts have been developed to provide yieldable cementitious support systems. One approach is to utilize low-strength, crushable, aerated concrete contained within a thin-walled steel container. The support provides controlled yielding by progressive wrinkling (folding) of the steel container as convergence occurs while at the same time providing containment and confinement to the aerated cementitious material inside. This approach is known as a *can* and is a constant yielding standing support (Figure 14.3, constant yielding)

Another approach to developing a yieldable cementitious support is to fabricate the support in place in the mine by pumping the cementitious material from the surface through a borehole to the installation site. These supports are generally referred to as *pumpable cementitious supports*. These supports are filled with a cementitious grout into a collapsible, containment bag that is hung from the mine roof at the installation site. The support capacity is controlled by the material strength and the confinement provided by the bag, especially following the peak loading whereby the bag works to control the bulking of the failed cementitious grout inside the bag (Figure 14.3, load shedding).

Rigid (Limited Yielding/Load Shedding) Steel Props

The goal in rigid steel prop designs is to provide for props that can be extended to furnish a wide range of height adjustment as efficiently as possible. The loading is controlled by the slenderness ratio of the steel pipe and the steel properties. Typically, they are designed to provide loading of 50–100 tons without buckling. Height adjustment is provided by a collar and threaded spindle, providing 12–18 in. of height adjustment.

Yielding Steel Props

Yielding is generally provided in steel props by progressive movement of some form of a mechanical arrangement along the length of a steel pipe or column of some sort, creating a form of *friction/permanent deformation* prop. One approach is to use two telescopic sections that are extended to the mine roof and floor by pumping water or some other fluid into the lower cylinder section. Once the cylinder is pressurized, a locking collar is set between the interacting cylinder sections, transforming the prop into a mechanical yielding system as the collar's downward movement from the applied loading progressively produces controlled permanent deformation of the bottom cylinder.

Hydraulic/Water Props

Another approach to provide a yielding prop is to use some form of a low-cost hydraulic prop. Water is pumped into the cylinder to extend the prop much like any hydraulic cylinder. Once the setting action is completed, a check valve in the bottom of the cylinder maintains the setting force. As the roof loading increases and overcomes the setting force, the captured water in the prop increases in pressure, functioning just like a hydraulic cylinder. The piston in the cylinder incorporates a yield valve, which controls the pressure in the cylinder. Once the yield pressure is reached, water is relieved through the piston and into the hollow section of tubing on the opposite end of the prop.

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PILLARS

Pillar load is determined by the stresses and the width of the entry.

$$L_p = S_v\,(W + W_E)/W \qquad \text{(EQ 14.5)}$$

where L_p is the average vertical load on the pillar, S_v is the in situ vertical stress, and W_E is the entry width.

Tributary Area Method

The tributary area method is a simple and conservative approach for calculating pillar load (Bieniawski 1992; Berkhimer 2011). Several assumptions must be made:

- Loading is only by vertical pressure.
- Each pillar supports the column of rock over an area that is the sum of the cross-sectional area of the pillar plus a portion of the room area.
- Load is uniformly distributed over the cross-sectional area of the pillar. Research has shown the following:
  - Stress is not evenly distributed over the cross section of a pillar. The maximum stress occurs at the corners formed by the intersection of the three orthogonal planes, namely two sidewalls of the pillar and the roof or the floor.
  - Stress on the pillars increases with percentage extraction until the critical width of the pillar is reached.
  - Stress distribution in pillars depends on the ratio of pillar width to pillar height.

For rectangular pillars, find the pillar stress using (Bieniawski 1992)

$$Sp = \frac{1.1\,H(w + B)(L + B)}{w \times L} \qquad \text{(EQ 14.6)}$$

For width-to-height ratios, the pillar strength is determined using the Bieniawski (1984) formula:

$$\sigma_p = \sigma_1\left[0.64 + 0.36\left(\frac{w}{h}\right)\right] \qquad \text{(EQ 14.7)}$$

For the extraction ratio of the mined-out area to the total area, use

$$e = 1 - \left[\frac{w}{(w + B)}\right]\left[\frac{L}{(L + B)}\right] \qquad \text{(EQ 14.8)}$$

The following is a simplified in situ vertical stress equation:

$$S_v = 1.1\left(\frac{\text{psi}}{\text{ft}}\right)H\ (\text{ft}) \qquad \text{(EQ 14.9)}$$

where

Sp = pillar stress, psi
H = depth below surface, ft
w = pillar width, ft
L = pillar length, ft
B = entry width, ft
σ_p = pillar strength, psi
σ_1 = in situ coal strength, psi
h = pillar height, ft
e = extraction ratio

Analysis of Retreat Mining Pillar Stability

Figure 14.6 shows section layout parameters used in ARMPS. Figures 14.7 and 14.8 show load schematic and abutment stress distribution, respectively.

SUBSIDENCE

Tables 14.12 and 14.13 show angle of draw examples and residual subsidence duration over longwall mines, respectively. Figures 14.9, 14.10, and 14.11 show various safety zones.

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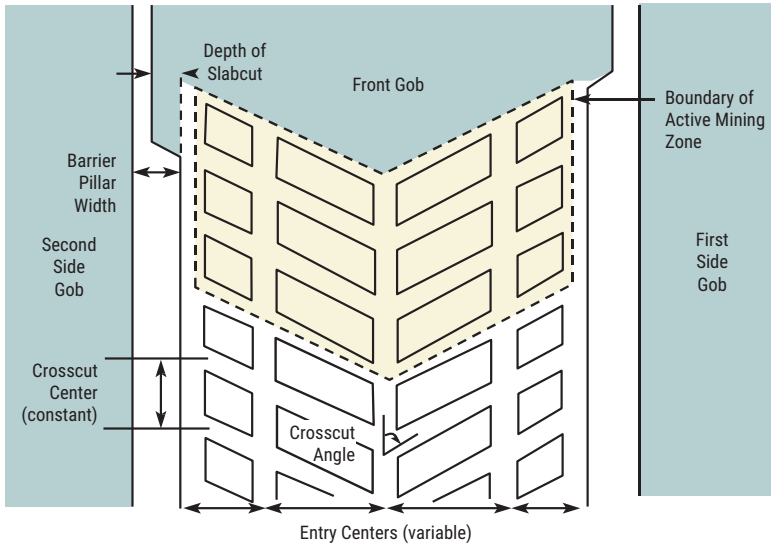
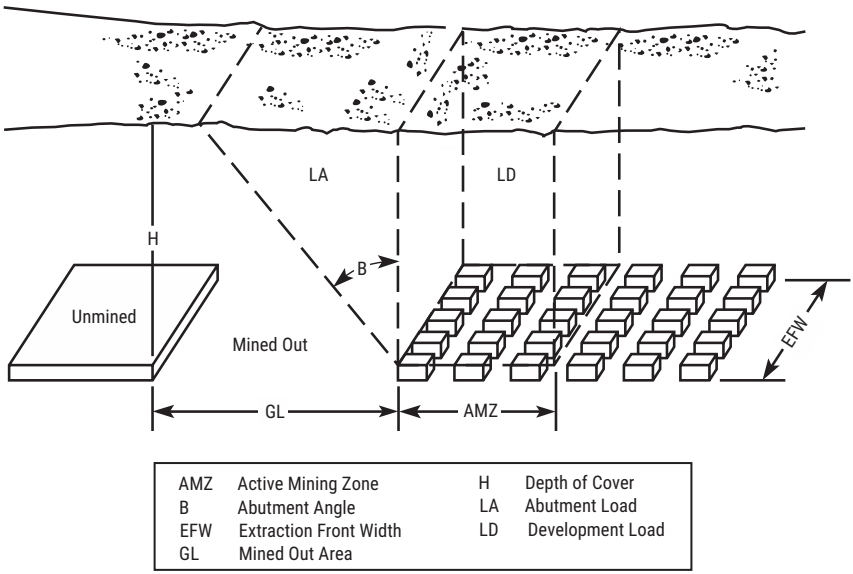


Source: Mark 2010

FIGURE 14.6 ARMPS section layout parameters



Source: Mark 1997

FIGURE 14.7 Schematic of loads

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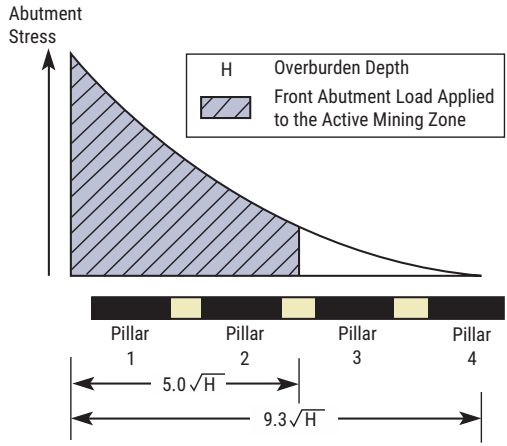

Source: Mark 1997

FIGURE 14.8 Distribution of abutment stress

TABLE 14.12 Examples of angle of draw

Country	Angle of Draw, degrees
Australia (Newcastle coalfield)	35
Austria (Tertiary coal basin)	41–42
France (North)	35
Germany	30–45
India (Pench and Kanhan coalfields)	41
Japan (Kuho II mine)	40–50
Netherlands (Limburg coalfield)	35–45
Russia	30
Slovakia (Ostrava-Karvina coalfield)	25–39
South Africa	11
United Kingdom	25–35
United States (Central)	0–8.5
United States (East)	15–27
United States (West)	12–16

Adapted from Bell and Donnelly 2006, as appears in Harrison 2011

TABLE 14.13 Duration of residual subsidence over longwall mines

Country/Coalfield	Residual Subsidence Duration	Reference
United Kingdom	Several months to 3–6 years (strong overburden)	Orchard and Allen 1974
United Kingdom	2–4.5 years	Collins 1977
Germany	1 year (Cretaceous overburden), 2 years (sandstone overburden)	Brauner 1973
USSR	2 years (shallow mines), 4–5 years (deep mines, >400 m)	Brauner 1973
United States/Illinois	12 months	Hood et al. 1983
India	16 months	Singh and Singh 1998

Source: Harrison 2011

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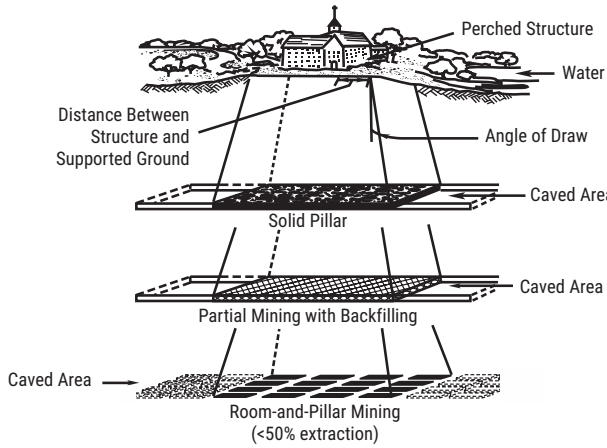
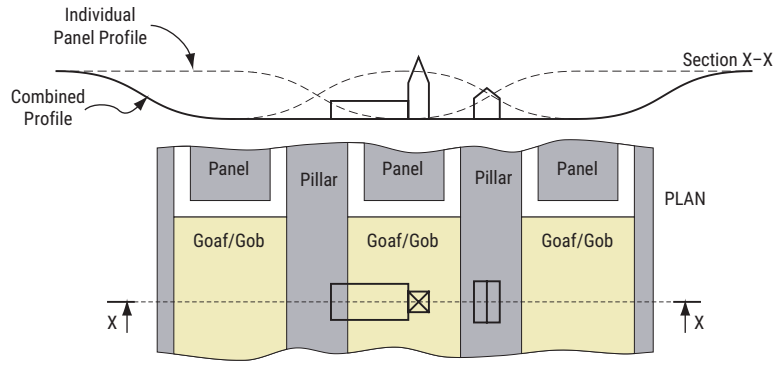


Source: Harrison 2011

FIGURE 14.9 Protective zones for surface structures



Source: Harrison 2011

FIGURE 14.10 Pillar widths between adjacent extraction zones sized to give uniform vertical movement

SLOPE STABILITY

Usually a factor of safety/stability (FS) of 1.3 is considered adequate for slopes that are required to stand for a short time; for long-term stability, a value of 1.5 is desirable (Bise 2003).

The most common form of slope failure in coal and hard-rock mines is planar failure. Planar failure can result when a competent block of rock lies along a planar discontinuity that dips with regard to the slope. When the dip angle is greater than the peak friction angle for the discontinuity surface, the block tends to slide.

The safety factor for planar failure can be approximated with the following equation (see Figure 14.12 for illustration of variables) (Bise 2003):

$$FS = cA + \frac{\left[\left(W(\cos\psi_p) - U - V(\sin\psi_p)\tan\phi\right]\right.}{W(\sin\psi_p) + V(\cos\psi_p)} \quad \textbf{(EQ 14.10)}$$

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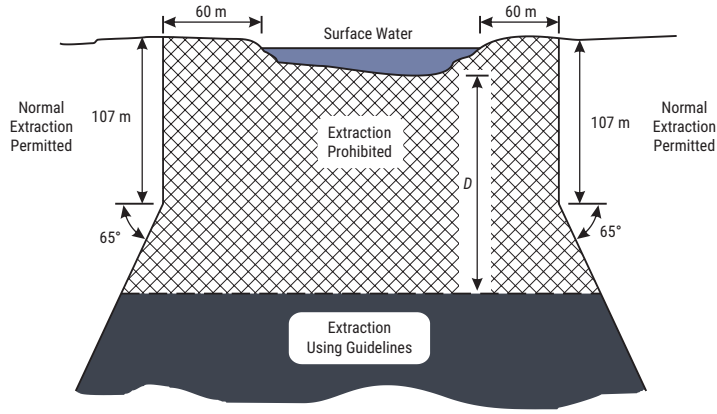


Depth of Safety Zone
Total Extraction: $D > 60t$
Room and Pillar: $D >$ max $(5s, 10t)$

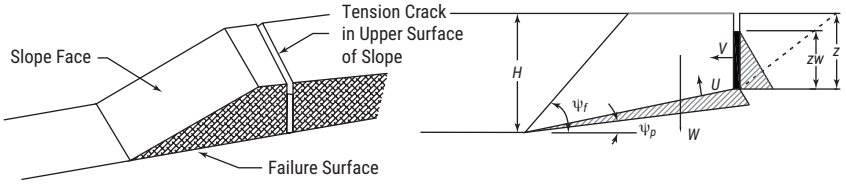
Principal Extraction Guidelines
Total Extraction: Total extraction thickness < $60D$
Horizontal extensile strain at bed of water body < 8.75×10^{-3}
Room and Pillar: Pillar dimensions must satisfy

$$D > \left[\frac{552}{\sqrt{t}} + 20\left(\frac{w}{t}\right)^2\right] \Big/ \left[4.921\left(1 + \frac{s}{w}\right)^2\right]$$

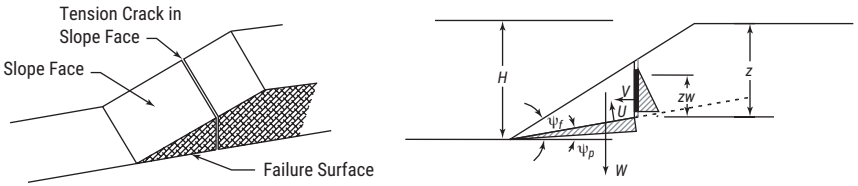
where
t = extraction thickness, m
s = room span, m
w = pillar width, m

Adapted from Babcock and Hooker 1977, as appears in Harrison 2011

FIGURE 14.11 Safety zone beneath a body of surface water



Geometry of Slope with Tension Crack in Upper Slope Surface



Geometry of Slope with Tension Crack in Slope Face

Adapted from Hoek and Bray 1977

FIGURE 14.12 Planar failure

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$$V = \frac{bH^2}{2}(\cot\psi_p - \cot\psi_p) \quad \textbf{(EQ 14.11)}$$

Simplified, it is as follows:

$$FS = \frac{c + \sigma_{normal}\tan\phi}{\sigma_{shear}} = \frac{\text{resisting force}}{\text{driving stress}} \quad \textbf{(EQ 14.12)}$$

where

FS = factor of safety/stability
c = cohesion, lb/ft^2
A = length of slip surface
= $(H - z)(\text{cosec}\ \psi_p)$, ft^2
U = water pressure uplift
= $0.5(\gamma_w)(z_w)\ (H - z)\ (\text{cosec}\ \psi_p)$, lb
V = force from water in tension crack = $0.5(\gamma_w)(z_w)(\sin\psi_p)$, lb
W = weight of block, lb
ψ_p = angle of discontinuity plane, degrees
ϕ = friction angle, degrees
γ_w = density of water (62.4 lb/ft^3)
b = slice width, ft (assume 1-ft-wide slice)
σ_{normal} = normal stress
σ_{shear} = shear stress

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CHAPTER

15

Ventilation

William J. Francart, P.E. and Kelvin K. Wu, P.E.

AIR COMPOSITION, DENSITY, AND PSYCHROMETRY

Tables 15.1, 15.2, and 15.3 show air composition; barometric pressure, temperature, and air density at different altitudes; and dew point temperatures, respectively. Figure 15.1 is a sea-level psychrometric chart. Air is primarily made up of nitrogen and oxygen with small amounts of carbon dioxide and argon and other rare gases as can be seen in Table 15.1.

TABLE 15.1 Air composition

Gas	Percent by Volume	Percent by Weight
Nitrogen	78.09	75.55
Oxygen	20.94	23.12
Carbon dioxide	0.04	0.06
Argon, other rare gases	0.93	1.27

Adapted from Hartman et al. 1997

TABLE 15.2 Barometric pressure, temperature, and air density at different altitudes

Altitude Above or Below Sea Level, *Z* ft	Barometric Pressure *Pb*		At Constant $t = 70°F$		At Varying *t* and *Z*	
	Pounds per Square Inch	Inches Mercury	Relative Air Density	Air Density, *w* lb/ft^3	Air Temperature, °F	Air Density, lb/ft^3
−1,000	15.23	31.02	1.037	0.0778	73.8	0.0771
−500	14.94	30.47	1.018	0.0764	71.9	0.0761
0	14.70	29.92	1.000	0.0750	70.0	0.0750
500	14.42	29.38	0.981	0.0736	68.1	0.0740
1,000	14.16	28.86	0.964	0.0723	66.1	0.0730
1,500	13.91	28.33	0.947	0.0710	64.2	0.0719
2,000	13.66	27.82	0.930	0.0698	62.3	0.0709
2,500	13.41	27.31	0.913	0.0685	60.4	0.0698
3,000	13.16	26.81	0.896	0.0672	58.4	0.0687
3,500	12.92	26.32	0.880	0.0660	56.5	0.0676
4,000	12.68	25.84	0.864	0.0648	54.6	0.0666
4,500	12.45	25.36	0.848	0.0636	52.6	0.0657
5,000	12.22	24.89	0.832	0.0624	50.7	0.0648
5,500	11.99	24.43	0.816	0.0612	48.8	0.0638
6,000	11.77	23.98	0.799	0.0599	46.9	0.6628
6,500	11.55	23.53	0.786	0.0590	45.0	0.0619
7,000	11.33	23.09	0.774	0.0580	43.0	0.0610
7,500	11.12	22.65	0.758	0.0568	41.0	0.0600
8,000	10.91	22.22	0.739	0.0554	39.0	0.0590
8,500	10.70	21.80	0.728	0.0546	37.1	0.0581
9,000	10.50	21.38	0.715	0.0536	35.2	0.0573
9,500	10.30	20.98	0.701	0.0526	33.3	0.0564
10,000	10.10	20.58	0.687	0.0515	31.3	0.0555
10,500	9.90	20.18	0.674	0.0506	29.4	0.0546
11,000	9.71	19.75	0.661	0.0496	27.5	0.0538
11,500	9.52	19.40	0.648	0.0486	25.5	0.0529
12,000	9.34	19.03	0.636	0.0477	23.6	0.0521
12,500	9.15	18.65	0.624	0.0468	21.6	0.0513
13,000	8.97	18.29	0.611	0.0458	19.7	0.0505
13,500	8.80	17.93	0.599	0.0449	17.7	0.0496
14,000	8.62	17.57	0.587	0.0440	15.8	0.0488
14,500	8.45	17.22	0.576	0.0432	13.9	0.0480
15,000	8.28	16.88	0.564	0.0423	12.0	0.0473

Source: Madison 1949

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TABLE 15.3 Temperature of dew point in degrees Fahrenheit*

Air Temperature (t)	Vapor Pressure (e)	Depression of Wet-Bulb Thermometer ($t - t'$)													
		1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0
20	0.103	17	13	8	2	−5	−18								
21	0.108	18	14	10	4	−3	−14	−42							
22	0.113	19	15	11	6	−1	−10	−29							
23	0.118	20	16	12	8	+1	−7	−22							
24	0.124	21	18	14	9	3	−4	−17							
25	0.130	22	19	15	11	5	−2	−12	−36						
26	0.136	23	20	16	12	7	±0	−9	−26						
27	0.143	24	21	18	14	9	+3	−5	−19						
28	0.150	25	22	19	15	11	5	−2	−14	−45					
29	0.157	26	24	20	17	12	7	±0	−9	−29					
30	0.164	27	25	22	18	14	9	+3	−5	−20					
31	0.172	29	26	23	20	16	11	5	−2	−14	−50				
32	0.180	30	27	24	21	17	13	8	+1	−9	−29				
33	0.187	31	28	25	22	19	15	10	8	−5	−20				
34	0.195	32	29	27	24	20	16	12	6	−2	−14	−50			
35	0.203	33	30	28	25	22	18	14	8	+1	−8	−28			
36	0.211	34	31	29	26	23	20	15	11	4	−4	−19			
37	0.219	35	32	30	27	24	21	17	13	7	−1	−12	−44		
38	0.228	36	33	31	28	26	23	19	14	9	+3	−7	−25		
39	0.237	37	34	32	29	27	24	21	16	12	6	−3	−16		
40	0.247	38	35	33	31	28	25	22	18	14	8	+1	−10	−35	
41	0.256	39	37	34	32	29	26	23	20	16	11	4	−5	−21	
42	0.266	40	38	35	33	30	28	25	21	17	13	7	−1	−13	−59
43	0.277	41	39	36	34	31	29	26	23	19	15	10	+3	−7	−28
44	0.287	42	40	38	35	32	30	27	24	21	17	12	6	−2	−17
45	0.298	43	41	39	36	34	31	29	26	22	19	14	8	+2	−9
46	0.310	44	42	40	37	35	32	30	27	24	20	16	11	5	−4
47	0.322	45	43	41	39	36	34	31	28	25	22	18	13	8	±0
48	0.334	46	44	42	40	37	35	32	30	27	23	20	15	10	+4
49	0.347	47	45	43	41	39	36	34	31	28	25	21	17	13	7
50	0.360	48	46	44	42	40	37	35	32	29	27	23	19	15	9
51	0.373	49	47	45	43	41	39	36	34	31	28	25	21	17	12
52	0.387	50	48	46	44	42	40	37	35	32	29	26	23	19	14
53	0.402	51	49	47	45	43	41	39	36	34	31	28	24	21	16
54	0.417	52	50	49	47	44	42	40	38	35	32	29	26	23	19
55	0.432	53	52	50	48	46	43	41	39	36	34	31	28	24	21
56	0.448	54	53	51	49	47	45	43	40	38	35	32	29	26	23
57	0.465	55	54	52	50	48	46	44	42	39	36	34	31	28	24
58	0.482	56	55	53	51	49	47	45	43	40	38	35	32	29	26
59	0.499	57	56	54	52	50	48	46	44	42	39	37	34	31	28
60	0.517	58	57	55	53	51	49	48	45	43	41	38	35	32	29
61	0.536	59	58	56	54	52	51	49	46	44	42	39	37	34	31
62	0.555	60	59	57	55	54	52	50	48	46	43	41	38	35	32
63	0.575	61	60	58	56	55	53	51	49	47	45	42	40	37	34
64	0.595	62	61	59	58	56	54	52	50	48	46	44	41	38	36
65	0.616	63	62	60	59	57	55	53	51	49	47	45	43	40	37
66	0.638	64	63	61	60	58	56	54	53	51	48	46	44	42	39
67	0.661	65	64	62	61	59	57	56	54	52	50	48	45	43	40
68	0.684	67	65	63	62	60	58	57	55	53	51	49	47	44	42
69	0.707	68	66	64	63	61	60	58	56	54	52	50	48	46	43
70	0.732	69	67	66	64	62	61	59	57	55	53	51	49	47	45
71	0.757	70	68	67	65	63	62	60	58	57	55	53	51	49	46
72	0.783	71	69	68	66	65	63	61	60	58	56	54	52	50	48
73	0.810	72	70	69	67	66	64	62	61	59	57	55	53	51	49
74	0.838	73	71	70	68	67	65	64	62	60	58	56	54	53	50
75	0.866	74	72	71	69	68	66	65	63	61	60	58	56	54	52
76	0.896	75	73	72	70	69	67	66	64	62	61	59	57	55	53
77	0.926	76	74	73	71	70	68	67	65	64	62	60	58	56	54
78	0.957	77	75	74	72	71	69	68	66	65	63	61	59	58	56
79	0.989	78	76	75	73	72	70	69	67	66	64	62	61	59	57
80	1.022	79	77	76	75	73	72	70	69	67	65	64	62	60	58

* Pressure = 29.0 in.

Source: Marvin 1915

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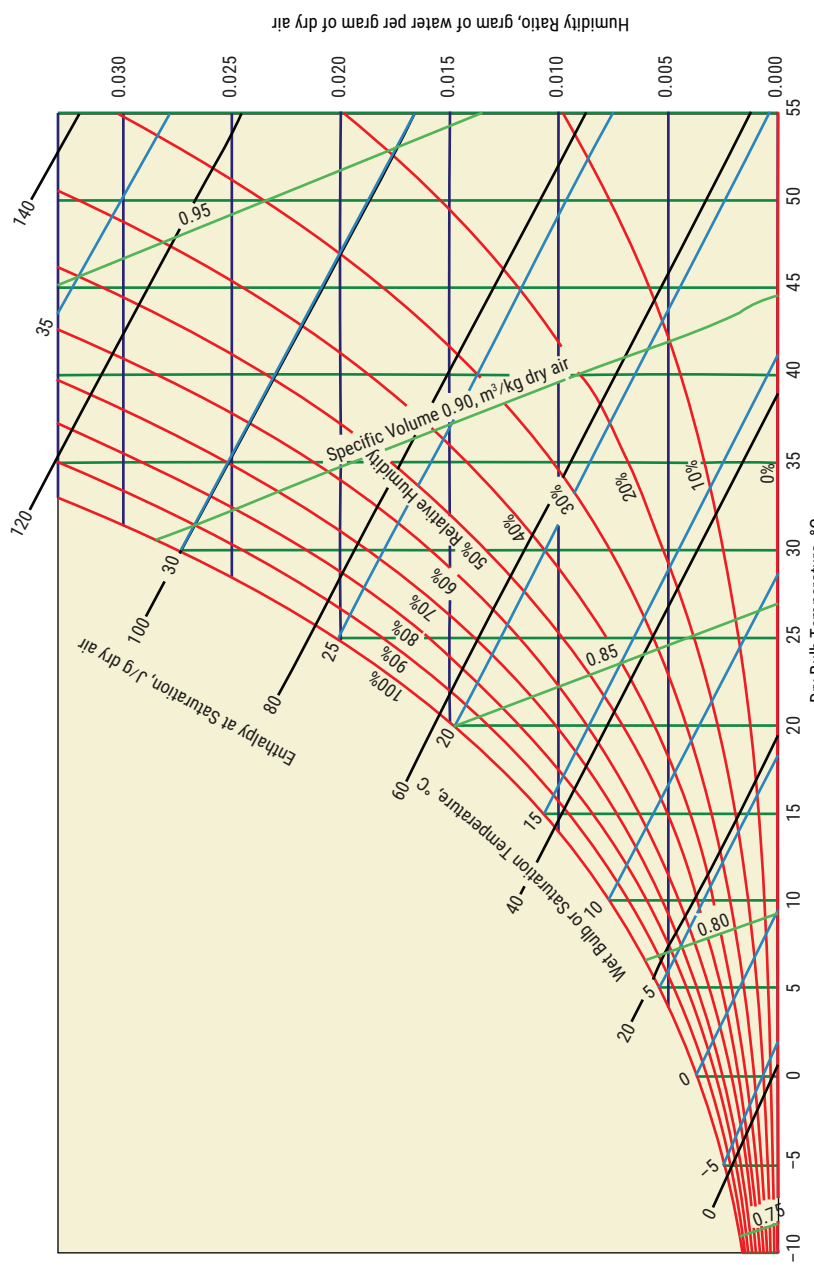

Courtesy of A. Ogawa

FIGURE 15.1 Psychrometric chart, sea level (barometric pressure 101.325 kPa)

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The following equations are used for determining air composition, density, and psychrometry (adapted from Ramani 1992):

$$w = B/0.287(273 + t_d) \quad \text{(EQ 15.1a; SI units)}$$

$$w = \frac{1.327B}{460 + t_d} \quad \text{(EQ 15.1b)}$$

where

w = density, kg/m^3 (lb/ft^3)
B = barometric pressure, kPa (in. mercury)
t_d = dry-bulb temperature, °C (°F)

To correct for water vapor (adapted from Ramani 1992),

$$w = \frac{B - 0.378f}{0.287(273 + t_d)} \quad \text{(EQ 15.2a; SI units)}$$

$$w = \frac{1.327(B - 0.378f)}{460 + t_d} \quad \text{(EQ 15.2b)}$$

where f = vapor pressure at the dew point temperature in kilopascals or inches of mercury.

GAS LAWS

The following equations are adapted from Hartman et al. (1997). See Chapter 4 on physical science and engineering for more information.

$$p_1 v_1 = p_2 v_2 \text{ constant temperature} \quad \text{(EQ 15.3)}$$

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \text{constant pressure} \quad \text{(EQ 15.4)}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{(EQ 15.5)}$$

where

p = absolute pressure
v = specific volume
T = absolute temperature, K (°R)

AIRFLOW

The following equations are used to determine airflow quantity (adapted from Hartman et al. 1997), velocity head, and friction and shock losses. The velocity head and friction loss equations are adapted from Ramani (1992). See Chapter 4 for more information.

Quantity

$$Q = V A \quad \text{(EQ 15.6)}$$

where:

Q = quantity, m^3/s (ft^3/min)
V = velocity, m/s (ft/min)
A = area, m^2 (ft^2)

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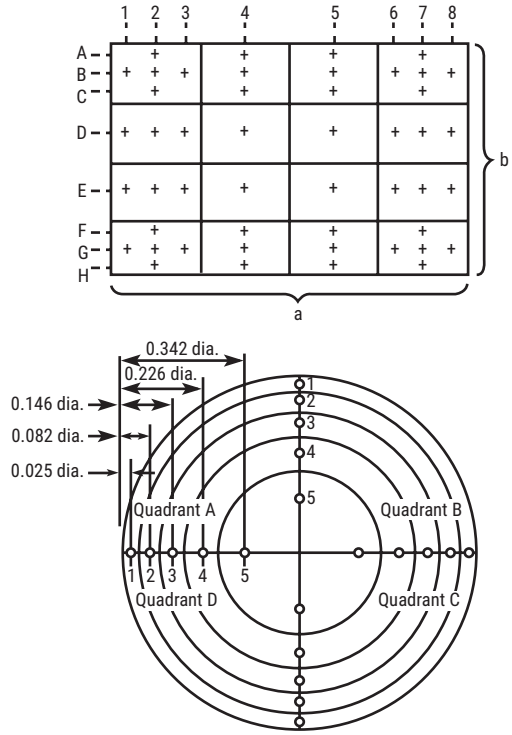

Source: Ramani 1992

FIGURE 15.2 Measurement points for fixed traversing in rectangular and circular airways

Velocity Head

Figure 15.2 shows measurement points for fixed traversing in rectangular and circular airways.

$$H_v = wV^2/2g \qquad \textbf{(EQ 15.7a; SI units)}$$

$(w/g = kg/m^3)$

$$H_v = w(V/1{,}098)^2 \qquad \textbf{(EQ 15.7b)}$$

where

H_v = velocity head, Pa (in. water)
V = velocity, m/s (ft/min)
W = density, kg/m^3 (lb/ft^3)

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TABLE 15.4 Friction factors for mine airways

Type of Airway	Value of K × 10^{10}					
	Straight			Curved		
	Clean	Slightly Obstructed	Moderately Obstructed	Clean	Slightly Obstructed	Moderately Obstructed
Smooth lined	25	28	34	31	39	43
Unlined rock-bolted	43	49	61	62	68	74
Timbered	67	75	82	85	87	90

Source: McElroy 1935

TABLE 15.5 Friction factors for coal mine airways*

	Intake		Return		Belt		Cribbed	
	SI	Imperial	SI	Imperial	SI	Imperial	SI	Imperial
Average value	0.008	40	0.009	47	0.011	57	0.068	366
Maximum value	0.012	62	0.011	61	0.018	95	0.144	777
Minimum value	0.005	26	0.006	31	0.005	24	0.045	244
Standard deviation	0.002	12	0.002	10	0.006	34	0.025	136
Number of measurements	23		15		5		7	

* Atkinson friction factor, kg/m^3 (lbf·min^2/ft^4 × 10^{-10}). Source: Prosser and Wallace 1999

TABLE 15.6 Friction factors for metal mine airways*

	Level Drift		Ramp		Alimak Raise		Bored Raise		Beltway		TBM† Drift	
	SI	Imperial	SI	Imperial	SI	Imperial	SI	Imperial	SI	Imperial	SI	Imperial
Average value	0.0088	47	0.0116	62	0.0113	61	0.0047	25	0.0140	75	0.0044	24
Maximum value	0.0128	69	0.0174	94	0.0158	85	0.0070	38	0.0166	90	0.0056	30
Minimum value	0.0047	26	0.0070	38	0.0087	47	0.0023	12	0.0123	66	0.0034	18
Standard deviation	0.0024	13	0.0030	17	0.0030	18	0.0015	8	0.0018	10	0.0011	6
Number of measurements	40		20		5		10		5		3	

* Atkinson friction factor, kg/m^3 (lbf·min^2/ft^4 × 10^{-10}). Source: Prosser and Wallace 1999

† TBM = tunnel boring machine.

Friction Loss

Tables 15.4, 15.5, and 15.6 show friction factors for mine, coal mine, and metal mine airways, respectively.

$$H_f = KLOV^2/A \quad \text{(EQ 15.8a; SI units)}$$

$$H_f = KLOV^2/5.2A \quad \text{(EQ 15.8b)}$$

where

H_f = friction loss in airway, Pa (in. water)
L = length of airway, m (ft)
O = perimeter of airway, m (ft)
V = velocity of air, m/s (ft/min)
A = cross-sectional area of airway, m^2 (ft^2)
K = friction factor, kg/m^3 (lb·min^2/ft^4; from Table 15.4)

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$$Hf = RQ^2 \quad \text{(EQ 15.9)}$$

where

R = resistance, N-s^2/m^8 (in.-min^2/ft^6)
Q = quantity of air, m^3/s (ft^3/min)

Shock Losses in Roadways

When airflow in a roadway changes direction, meets an intersection, or changes a cross-sectional area, a shock loss occurs, which can be expressed as an equivalent length of roadway to simplify resistance and pressure losses. Table 15.7 gives examples of how to calculate changes in a roadway's condition to equivalent length. For example, a 100-m straight roadway with a single sharp 90-degree turn will have the equivalent resistance as a 120-m straight roadway; if that same 90-degree turn is rounded, the equivalent resistance is a 101-m roadway.

TABLE 15.7 Equivalent lengths for various sources of shock loss

Source	Meters	Feet	Source	Meters	Feet
Bend, acute, round	1	3	Contraction, gradual	1	1
Bend, acute, sharp	45	150	Contraction, abrupt	3	10
Bend, right, round	1	1	Expansion, gradual	1	1
Bend, right, sharp	20	70	Expansion, abrupt	6	20
Bend, obtuse, round	1	1	Splitting, straight branch	10	30
Bend, obtuse, sharp	5	15	Splitting, deflected branch (90°)	60	200
Doorway	20	70	Junction, straight branch	20	60
Overcast	20	65	Junction, deflected branch (90°)	10	30
Inlet	6	20	Mine car or skip (20% of airway area)	30	100
Discharge	20	65	Mine car or skip (40% of airway area)	150	500

Source: Hartman et al. 1997

FANS

Fan laws should not be confused with the similar looking pump laws (Table 15.8). Fan laws are for compressible flow, and pump laws are for incompressible flow. Pump laws use a constant blade tip speed. In a pump law question, if the pump diameter is doubled, then the revolutions per minute will automatically be halved to keep the blade tip traveling at the same speed. Blade tip speed is not held constant in fan laws.

To determine quantity, use the following equation:

$$Q \propto nD^3,\ Q = k_q nD^3,\ or\ Q_2 = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3 Q_1 \quad \text{(EQ 15.10)}$$

TABLE 15.8 Fan laws

Variance in Performance Characteristics	Law 1, with Speed Change, n (D and w constant)	Law 2, with Size Change, D (w and n constant)	Law 3, with Specific Weight Change, w (n and D constant)
Quantity, Q	Directly	As cube	Constant
Head, H_s or H_t	As square	As square	Directly
Power, P_a or P_m	As cube	Fifth power	Directly
Efficiency, n	Constant	Constant	Constant

Source: Hartman et al. 1997

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Head is found with the following equation:

$$H \propto n^2 D^2 w,\ H = k_h n^2 D^2 w,\ or\ H_2 = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 \frac{w_2}{w_1} H_1 \quad \textbf{(EQ 15.11)}$$

Power is determined using the following equation:

$$P = n^3 D^5 w,\ P = k_p n^3 D^5 w,\ or\ P = \left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 \frac{w_2}{w_1} P \quad \textbf{(EQ 15.12)}$$

where

Q = air quantity, m^3/s (ft^3/min)
n = fan speed, rpm
H = fan head, Pa (in. water)
P = horsepower, kW (hp)
w = density of air, kg/m^3 (lb/ft^3)
D = fan diameter, m (ft)
k_q, k_h, k_p = numerical constants

Horsepower Requirements

The following equations are adapted from Ramani (1992).

$$AHP = H_L Q/1{,}000 \quad \textbf{(EQ 15.13a; SI units)}$$

$$AHP = H_L Q/6{,}350 \quad \textbf{(EQ 15.13b)}$$

where

AHP = air, kW (hp)
H_L = total head loss, Pa (in. water)

NATURAL VENTILATING PRESSURE

The following equations, adapted from Hartman et al. (1997), are used to determine natural ventilating pressure.

$$H_n = 0.255BL\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \quad \textbf{(EQ 15.14)}$$

where

H_n = natural ventilating pressure, Pa (in. water)
B = average absolute pressure, Pa (in. mercury)
L = vertical height of air columns, m (ft)
T_1, T_2 = average absolute temperatures of the air columns, K (°R)

$$H_n = 3.484\ BL\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \quad \textbf{(EQ 15.15a; SI units)}$$

$$H_n = 0.255\ BL\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \quad \textbf{(EQ 15.15b)}$$

where

H_n = natural ventilating pressure, Pa (in. water)
B = average absolute pressure at the mid-point of the column, Pa (in. mercury)
L = vertical height of air columns, m (ft)
T_1, T_2 = average absolute temperatures of the air columns for the downcast (T_1) and the upcast (T_2), K (°R)

PARALLEL FLOW

Equation 15.16, adapted from Hartman et al. (1997), and Equation 15.17, adapted from Kingery (1960), are used to determine parallel flow. See Chapter 4 for more information.

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$$Q_n = Q\sqrt{\frac{R_{eq}}{R_n}} \quad \text{(EQ 15.16)}$$

where

Q_n = quantity in particular entry n
Q = total quantity of common entries
R_{eq} = equivalent resistance of common entries
R_n = resistance of entry n

$$R_2 = \frac{R_1}{N_2} \quad \text{(EQ 15.17)}$$

where

R_1 = the original resistance for n_1 number of entries
R_2 = the resistance for n_2 number of entries
N = the ratio n_2/n_1

NATURAL SPLITTING

The formulas for both natural and controlled splitting are adapted from Kingery (1960).

Formula for Potential Air Splitting

$$F_n = A_n\sqrt{\frac{A_n}{L_n O_n}} \quad \text{(EQ 15.18)}$$

where

A_n = cross-sectional area of airway, ft^2
L_n = length of airway n, 1,000-ft units
O_n = perimeter of airway n, ft
F_n = the potential (no units) for split n

and

$$Q_n = \left(\frac{F_n}{F_1 + F_2 + \cdots F_n}\right)Q_{\text{total}} \quad \text{(EQ 15.19)}$$

where Q_n is the quantity in the nth split in cubic feet per minute.

CONTROLLED SPLITTING

Following is the regulator formula adapted from Kingery (1960).

Regulator Formula

$$A = 1.24\,\frac{Q}{\sqrt{H}} \quad \text{(EQ 15.20a; SI units)}$$

$$A = 40\frac{Q}{\sqrt{H}} \quad \text{(EQ 15.20b)}$$

where

A = area of the opening, m^2 (ft^2)
Q = quantity of air in the split in 100,000 cfm, m^3/s
H = head loss through regulator, Pa (in. water)

MINE GASES

Table 15.9 shows the gases most commonly found in mines with their hazards and threshold limits, as well as detection methods. Figure 15.3 depicts explosibility of mine gases determined by the U.S. Bureau of Mines (Zabetakis et al. 1959).

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TABLE 15.9 Classification of gases most commonly found in subsurface openings

Name	Symbol	Molecular Weight (based on C^{12})	Density at 20°C and 100 kPa kg/m^3	Density Relative to Dry Air	Gas Constant, J/kg °C	Primary Sources in Mines	Smell, Color, Taste	Hazards	Guideline Threshold Limit Values (TLVs)	Detection Methods*	Flammability Limits in Air, %
Dry air	–	28.966	1.1884	1	287.04	–	None	–	–	–	–
Oxygen	O_2	31.999	1.3129	1.1047	259.83	Air	None	Oxygen deficiency; may cause explosive mixtures with reactive gases	>19.5	Electrochemical, paramagnetic, flame lamp	–
Nitrogen	N_2	28.015	1.1493	0.9671	296.80	Air, strata	None	Inert	–	By difference	–
Methane	CH_4	16.04	0.6581	0.554	518.35	Strata	None	Explosive; layering	1%:isolate electricity 2%:remove personnel	Catalytic oxidation, thermal conductivity, optical, acoustic, flame lamp	5–15
Carbon dioxide	CO_2	44.00	1.805	1.519	188.96	Oxidation of carbon, fires, explosions, internal combustion (IC) engines, blasting, respiration	Slight acid taste and smell	Promotes increased rate of respiration	Time-weighted average (TWA) = 0.5%; short-term exposure limit (STEL) = 3.0%	Optical, infrared	–
Carbon monoxide	CO	28.01	1.149	0.967	296.8	Fires, explosions, IC engines, blasting, spontaneous or incomplete combustion of carbon compounds	None	Highly toxic; explosive	TWA = 0.005%; STEL = 0.04%	Electrochemical, catalytic oxidation, semiconductor, infrared	12.5–74.2
Sulfur dioxide	SO_2	64.06	2.628	2.212	129.8	Oxidation of sulfides, acid water on sulfide ores, IC engines	Acid taste, suffocating smell	Very toxic; irritant to eyes throat and lungs	TWA = 2 ppm; STEL = 5 ppm	Electrochemical, infrared	–

(continues)

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TABLE 15.9 **Classification of gases most commonly found in subsurface openings** (continued)

Name	Symbol	Molecular Weight (based on C^{12})	Density at 20°C and 100 kPa kg/m^3	Density Relative to Dry Air	Gas Constant, J/kg °C	Primary Sources in Mines	Smell, Color, Taste	Hazards	Guideline Threshold Limit Values (TLVs)	Detection Methods*	Flammability Limits in Air, %
Nitric oxide	NO	30.01	1.231	1.036	277.1	IC engines, blasting, welding	Irritant to eyes, nose and throat	Oxidizes rapidly to NO_2	TWA = 25 ppm	Electrochemical, infrared	–
Nitrous oxide	N_2O	44.01	1.806	1.519	188.9	IC engines, blasting, welding	Sweet smell	Narcotic (laughing gas)	TWA = 50 ppm	Electrochemical	–
Nitrogen dioxide	NO_2	46.01	1.888	1.588	180.7	IC engines, blasting, welding	Reddish brown, acidic smell and taste	Very toxic; throat and lung irritant; pulmonary infections	TWA = 3 ppm; ceiling: 5 ppm	Electrochemical, infrared	–
Hydrogen sulfide	H_2S	34.08	1.398	1.177	244.0	Acid water on sulfides, stagnant water, strata, decomposition of organic materials	Odor of bad eggs	Highly toxic; irritant to eyes and respiratory tracts; explosive	TWA = 10 ppm; STEL = 15 ppm	Electrochemical, semiconductor	4.3–45.5
Hydrogen	H_2	2.016	0.0827	0.0696	4124.6	Battery charging, strata, water on incandescent materials, explosions	None	Highly explosive	–	Catalytic oxidation	4–74.2
Radon	Rn	≈222	9.108	7.66	37.45	Uranium minerals in strata	None	Radioactive; decays to radioactive particles	1 working level (WL) and 4 WL-months per year	Radiation detectors	–
Water vapor	H_2O	18.016	0.739	0.622	461.5	Evaporation of water, IC engines, respiration, spon. com. and other fires	None	Affects climatic environment	–	Psychrometers, dielectric effects	–

* In addition to the methods listed, most of these gases can be detected by gas chromatography and stain tubes.

Source: McPherson 2009

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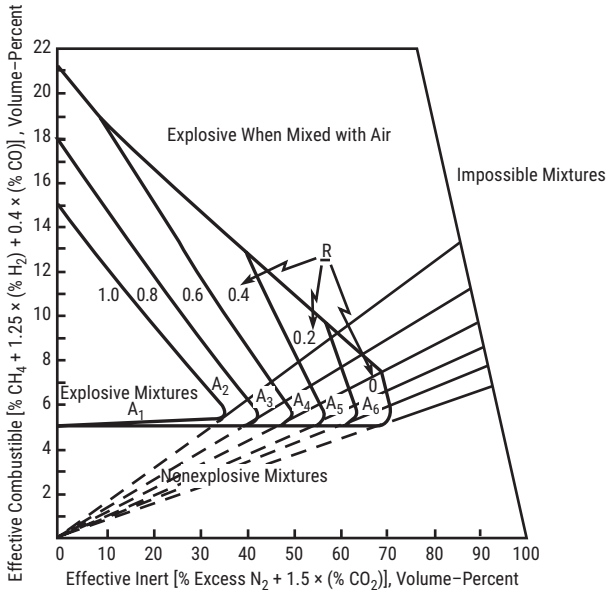

Source: Zabetakis et al. 1959

FIGURE 15.3 Explosibility of mine gases

REGULATORY REQUIREMENTS

Table 15.10 offers key ventilation requirements for underground mines.

TABLE 15.10 Key ventilation requirements for underground coal mines

Last open crosscut, minimum	9,000 cfm
Each working face, minimum	3,000 cfm
Longwall intake, minimum	30,000 cfm
Mean entry air velocity, minimum	60 fpm
Trolley entry air velocity, maximum	250 fpm
Belt entry air velocity when using atmospheric monitoring system (fire detection), minimum	50 fpm
Ventilation of diesel engine exhausts	Nameplate quantity
Ventilation of multiple diesel exhausts	Sum of nameplate quantities
Gas quantities	
Oxygen, minimum	19.5%
Carbon dioxide, maximum	0.5%
No harmful quantities of noxious or poisonous gases	Threshold limit values
Actions for excessive methane in working places and intake airways:	
1.0% CH_4 or more	Changes or adjustments to less than 1.0%; cut off power to energized face equipment.
1.5% CH_4 or more	Withdraw miners (except those adjusting ventilation); cut off power to endangered area of the mine.

Adapted from 30 CFR 75 (1997)

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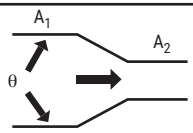
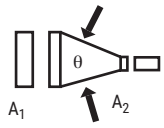
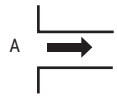
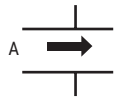
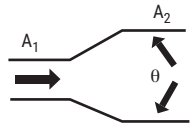
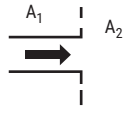
TUBING

Changes to airflow direction within a duct cause shock loss that can be expressed as an equivalent length of pipe. During contraction or expansion shock loss calculations, the smaller cross-sectional area is used to determine the airflow velocity. Figures 15.4 and 15.5 show loss coefficients for area changes and elbows in ducts. For example, in Figure 15.4, the first example of "gradual contraction at 45 degrees," the loss coefficient is 0.04; use the velocity at the small A2. Vent pipes and tubing that keep a constant cross-sectional area can use Table 15.11 to determine equivalent lengths of known shock losses. For example, four 25-m sections of lay-flat tubing with a single 90-degree bend (duct centerline radius = diameter of duct) and using three couplings would have an equivalent length of 4×25 + 46 + 3×2 = 152 m of straight tubing (not including entrance and exit losses). Equation 15.21 calculates the used head loss with the loss coefficient (from tables) and measured airflow velocity.

Type	Illustration	Conditions	Loss Coefficient
Gradual Contraction		θ 30° 45° 60°	C_2 0.02 0.04 0.07
Equal Area Transformation		$A_1 = A_2$ θ ≤ 14°	C 0.15
Flanged Entrance		A = ∞	C 0.34
Duct Entrance		A = ∞	C 0.85
Formed Entrance		A = ∞	C 0.03
Gradual Expansion		θ 5° 7° 10° 20° 30° 40°	C_1 0.17 0.22 0.28 0.45 0.59 0.73
Abrupt Exit		$A_2 = \infty$ $A_1/A_2 = 0.0$	1.00

"C" with a subscript indicates the cross-section at which velocity is calculated.

Courtesy of Schauenburg Flexadux Corporation

FIGURE 15.4 Loss coefficients for area changes in ducts

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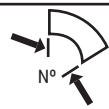
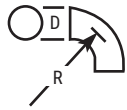
Type	Illustration	Conditions	Pressure Loss	
			Loss Coefficient	L/D Ratio
Nº		Rectangular or Round, with or Without Vanes	(N/90) Times Value for Similar 90° Elbow	
90° Round Section		Miter R/D = 0.5 0.75 1.0 1.5 2.0	1.30 0.90 0.45 0.33 0.24 0.19	65 No value given 23 17 12 10

L/D = length-to-diameter ratio
R/D = radius-to-diameter ratio

Courtesy of Schauenburg Flexadux Corporation

FIGURE 15.5 Loss coefficients for elbows in ducts

TABLE 15.11 Equivalent lengths for vent pipe and tubing

Source	Equivalent Length, m (ft)
Entrance loss*	10.7 (35)
Exit loss*	30.5 (100)
Bend (generic), 90°	10.7 (35)
Bend (generic), 60°	7.5 (24.5)
Bend (generic), 45°	5.3 (17.5)
Coupling, spiral-reinforced duct	2.4 (8)
Fiberglass duct	
90° round sharp† bend	7D (23D)
90° round smooth‡ bend	3.8D (12.5D)
90° oval sharp† vertical bend	6.1D (20D)
90° oval smooth‡ vertical bend	3.0D (10D)
90° oval sharp† horizontal bend	8.8D (29D)
90° oval smooth‡ horizontal bend	4.3D (14D)
45° bend	50% of 90° value
60° bend	70% of 90° value
Lay-flat tubing	
Coupling	2 (6)
90° bend§	46 (150)
45° bend	50% of 90° value
60° bend	70% of 90° value

* Used only if fan is not located at the point of loss.

Source: Hartman et al. 1997

† Sharp bend: duct centerline radius = 0.75D.

‡ Smooth bend: duct centerline radius = 1.25D.

§ Duct centerline radius = D.

Notes: (1) Most values derived from ABC Industries, Inc., laboratory measurements.
(2) The table variable D refers to duct diameter.

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$$H = C\left(\frac{V}{4{,}005}\right)^2 \quad \text{(EQ 15.21)}$$

where

H = head loss for fitting, in. water
C = loss coefficient (from Figures 15.4 and 15.5)
V = velocity, ft/min

VELOCITIES FOR MINERAL DUSTS AND GASES

Tables 15.12 and 15.13 show capture velocities for mineral dusts and conveying velocities of dusts and gases, respectively.

TABLE 15.12 Capture velocities for mineral dusts

Location	Toxicity	Capture Velocity	
		Meters per Second	Foot per Minute
Low-speed conveyor transfer	Low	0.5	100
	High	1.0	200
Welding	Low	0.5	100
	High	1.0	200
Conveyor loading	Low	1.0	200
	High	2.5	500
Grinding	Low	2.5	500
	High	10.0	2,000
Degreasing	Low	0.3	50
	High	0.5	100

Adapted from ACGIH 1995

TABLE 15.13 Conveying velocities of dusts and gases

	Meters per Second	Foot per Minute
Gases	5.0–10	1,000–2,000
Light dust	10–20	2,000–2,500
Mineral dust	18–20	3,500–4,000

Adapted from ACGIH 1995

UNDERGROUND SEALS AND BULKHEADS

The regulation for underground mine seals includes a requirement for seals designed to 345 kPa (50 psi) if the atmosphere behind the seal is monitored and inert, but requires seals to be designed to at least 830 kPa (120 psi) if the atmosphere is not monitored or is not inert (MSHA, n.d.). Sealed areas must be monitored, whether ingassing or outgassing (air flowing in or out), for methane and oxygen concentrations and the direction of leakage. Per MSHA (n.d.) regulations, atmospheres with seals of 830 kPa (120 psi) or greater must be sampled until the design strength is reached. Atmospheres with seals less than 830 kPa (120 psi) are monitored and maintained inert (MSHA, n.d.). The following is excerpted from 30 CFR § 75.335, "Seal Strengths, Design Applications, and Installation":

> (a) Seal strengths. Seals constructed on or after October 20, 2008 shall be designed, constructed, and maintained to withstand
> (1)(i) At least 50-psi overpressure when the atmosphere in the sealed area is monitored and maintained inert and designed using a pressure-time curve with an instantaneous

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overpressure of at least 50 psi. A minimum overpressure of at least 50 psi shall be maintained for at least four seconds then released instantaneously.

(ii) Seals constructed to separate the active longwall panel from the longwall panel previously mined shall be designed using a pressure-time curve with a rate of pressure rise of at least 50 psi in 0.1 second. A minimum overpressure of at least 50 psi shall be maintained; or

(2)(i) Overpressures of at least 120 psi if the atmosphere in the sealed area is not monitored, is not maintained inert, the conditions in paragraphs (a)(3)(i) through (iii) of this section are not present, and the seal is designed using a pressure-time curve with an instantaneous overpressure of at least 120 psi. A minimum overpressure of 120 psi shall be maintained for at least four seconds then released instantaneously.

(ii) Seals constructed to separate the active longwall panel from the longwall panel previously mined shall be designed using a pressure-time curve with a rate of pressure rise of 120 psi in 0.25 second. A minimum overpressure of 120 psi shall be maintained; or

(3) Overpressures greater than 120 psi if the atmosphere in the sealed area is not monitored and is not maintained inert, and

(i) The atmosphere in the sealed area is likely to contain homogeneous mixtures of methane between 4.5 percent and 17.0 percent and oxygen exceeding 17.0 percent throughout the entire area;

(ii) Pressure piling could result in overpressures greater than 120 psi in the area to be sealed; or

(iii) Other conditions are encountered, such as the likelihood of a detonation in the area to be sealed.

(iv) Where the conditions in paragraphs (a)(3)(i), (ii), or (iii) of this section are encountered, the mine operator shall revise the ventilation plan to address the potential hazards. The plan shall include seal strengths sufficient to address such conditions.

ACKNOWLEDGMENTS

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CHAPTER 16

Power: Electrical and Compressed Air

Daniel F. Kump, P.E.

ELECTRICAL POWER

Tables 16.1, 16.2, and 16.3, respectively, give ampacities, power factors, and electrical formulas. For more information on electricity, see Chapter 4, which covers physical science and engineering.

TABLE 16.1 Allowable ampacities*

Size	Temperature Rating of Conductor						Size
	60°C (140°F)	75°C (167°F)	90°C (194°F)	60°C (140°F)	75°C (167°F)	90°C (194°F)	
AWG or kcmil	Types† TW, UF	Types RHW, THHW, THW, THWN, XHHW, USE, ZW	Types TBS, SA, SIS, FEP, FEPB, MI, RHH, RHW-2, THHN, THHW, THW-2, THWN-2, USE-2, XHH, XHHW, XHHW-2, ZW-2	Types TW, UF	Types RHW, THHW, THW, THWN, XHHW, USE	Types TBS, SA, SIS, THHN, THHW, THW-2, THWN-2, RHH, RHW-2, USE-2, XHH, XHHW, XHHW-2, ZW-2	AWG or kcmil
	Copper			Aluminum or Copper-Clad Aluminum			
18	–	–	14	–	–	–	–
16	–	–	18	–	–	–	–
14	15	20	25	–	–	–	–
12	20	25	30	15	20	25	12
10	30	35	40	25	30	35	10
8	40	50	55	30	40	45	8
6	55	65	75	40	50	55	6
4	70	85	95	55	65	75	4
3	85	100	115	65	75	85	3
2	95	115	130	75	90	100	2
1	110	130	145	85	100	115	1
1/0	125	150	170	100	120	135	1/0
2/0	145	175	195	115	135	150	2/0
3/0	165	200	225	130	155	175	3/0
4/0	195	230	260	150	180	205	4/0
250	215	255	290	170	205	230	250
300	240	285	320	195	230	260	300
350	260	310	350	210	250	280	350
400	280	335	380	225	270	305	400
500	320	380	430	260	310	350	500
600	350	420	475	285	340	385	600
700	385	460	520	315	375	425	700
750	400	475	535	320	385	435	750
800	410	490	555	330	395	445	800
900	435	520	585	355	425	480	900
1,000	455	545	615	375	445	500	1,000
1,250	495	590	665	405	485	545	1,250
1,500	525	625	705	435	520	585	1,500
1,750	545	650	735	455	545	615	1,750
2,000	555	665	750	470	560	630	2,000

(continues)

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TABLE 16.1 Allowable ampacities* (continued)

Correction Factors							
Ambient Temperature, °C	For Ambient Temperature Other Than 30°C (86°F), Multiply the Allowable Ampacities Shown Above by the Appropriate Factor Shown Below						Ambient Temperature, °F
21–25	1.08	1.05	1.04	1.08	1.05	1.04	70–77
26–30	1.00	1.00	1.00	1.00	1.00	1.00	78–86
31–35	0.91	0.94	0.96	0.91	0.94	0.96	87–95
36–40	0.82	0.88	0.91	0.82	0.88	0.91	96–104
41–45	0.71	0.82	0.87	0.71	0.82	0.87	105–113
46–50	0.58	0.75	0.82	0.58	0.75	0.82	114–122
51–55	0.41	0.67	0.76	0.41	0.67	0.76	123–131
56–60	–	0.58	0.71	–	0.58	0.71	132–140
61–70	–	0.33	0.58	–	0.33	0.58	141–158
71–80	–	–	0.41	–	–	0.41	159–176

* Allowable ampacities of insulated conductors rated 0 through 2,000 V, 60°–90°C (140°–194°F) not more than three current-carrying conductors in raceway, cable, or earth (directly buried), based on ambient temperature of 30°C (86°F).

† The following is a list of wire abbreviations and descriptions (adapted from Multi Cable Corporation 2017–2019):

AWG = American wire gauge
FEP = fluorinated ethylene propylene insulation
FEPB = same as FEP with glass braid or asbestos-type outer covering
FEPW = same as FEP but water-resistant
MI = mineral-insulated, metal-shielded cable
RH = flame-retardant, moisture-resistant thermoset with nonmetallic covering
RHH = flame-retardant, moisture-resistant thermoset
RHW = 75°C rubber insulated, heat and moisture insulation with an outer cover of moisture-resistant, flame-retardant, nonmetallic covering
RHW-2 = continuous-temperature thermoset, for use in wet locations
SA = silicone rubber or silicone asbestos with glass or other braided covering
SIS = thermoplastic, flame-resistant switchboard wire
TBS = fire-retardant thermoplastic with fibrous outer braid
THHN = heat-resistant, flame-retardant thermoplastic with nylon or equivalent insulating jacket
THHW = flame-retardant, heat-resistant thermoplastic
THW = TW-wire with heavier flame-retardant, heat-resistant insulation
THW-2 = same as THW, but for use in wet locations
THWN = thermoplastic with flame-retardant, heat- and water-resistant outer nylon or equivalent insulation
THWN-2 = same as THWN, but for use in wet locations
TW = T-wire with water- and heat-resistant thermoplastic insulation
UF = waterproof underground feeder and branch circuit cable
USE = underground service entrance cable with heavy rubber water-resistant coating, not fire resistant
USE-2 = same as USE, but for use in wet locations
XHH = flame-retardant thermoset
XHHW = moisture-resistant, flame-retardant thermoset
XHHW-2 = same as XHHW, but for use in wet locations
ZW = modified ethylene tetrafluoroethylene
ZW-2 = same as ZW, but for use in wet locations

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TABLE 16.2 Power factor of typical alternating current loads

Load	Approximate Power Factor
Near-Unity Power Factor	
Incandescent lamps	1.0
Fluorescent lamps (with built-in capacitor)	0.95–0.97
Resistor heating apparatus	1.0
Synchronous motors (also built for leading power factor operation)	1.0
Rotary converters	1.0
Lagging Power Factor	
Induction motors (at rated load)	
Split-phase, below 1 hp	0.55–0.75
Split-phase, 1–10 hp	0.75–0.85
Polyphase, squirrel cage	
High-speed, 1–10 hp	0.75–0.90
High-speed, 10 hp and larger	0.85–0.92
Low-speed	0.70–0.85
Wound rotor	0.80–0.90
Groups of induction motors	0.50–0.85
Welders	
Motor-generator type	0.50–0.60
Transformer type	0.50–0.70
Arc furnaces	0.80–0.90
Induction furnaces	0.60–0.70
Leading Power Factor	
Synchronous motors	0.9, 0.8, 0.7, 0.6, etc. leading power factor depending on the rated leading power factor for which they are built
Synchronous condensers	Nearly zero leading power factor; output practically all leading reactive kilovolt-amperes
Capacitors	Zero leading power factor; output practically all leading reactive kilovolt-amperes

Source: Bise 2003

TABLE 16.3 Electrical formulas*

Required	Direct Current	Alternating Current: Single Phase	Alternating Current: Two Phase, Four Wire†	Alternating Current: Three Phase
Amperes when horsepower is known	$\frac{746(hp)}{(E)(eff)}$	$\frac{746(hp)}{(E)(eff)(pf)}$	$\frac{746(hp)}{2(E)(eff)(pf)}$	$\frac{746(hp)}{1.73(E)(eff)(pf)}$
Amperes when kilowatts are known	$\frac{1{,}000(kw)}{E}$	$\frac{1{,}000(kw)}{(E)(pf)}$	$\frac{1{,}000(kw)}{2(E)(pf)}$	$\frac{1{,}000(kw)}{1.73(E)(pf)}$
Amperes when kilovolt-ampere is known		$\frac{1{,}000(kva)}{E}$	$\frac{1{,}000(kva)}{2(E)}$	$\frac{1{,}000(kva)}{1.73(E)}$
Power kilowatts	$\frac{(I)(E)}{1{,}000}$	$\frac{(I)(E)(pf)}{1{,}000}$	$\frac{2(I)(E)(pf)}{1{,}000}$	$\frac{1.73(I)(E)(pf)}{1{,}000}$
Power kilovolt-ampere		$\frac{(I)(E)}{1{,}000}$	$\frac{2(I)(E)}{1{,}000}$	$\frac{1.73(I)(E)}{1{,}000}$
Power output horsepower	$\frac{(I)(E)(eff)}{746}$	$\frac{(I)(E)(pf)(eff)}{746}$	$\frac{2(I)(E)(pf)(eff)}{746}$	$\frac{1.73(I)(E)(pf)(eff)}{746}$

* If actual voltage is above or below the rated value, the correct value should be used in the table's formulas. E = volts; eff = efficiency (as a decimal); hp = horsepower; I = amperes; kva = kilovolt-amperes; kw = kilowatts; pf = power factor.

† In two-phase, three-wire circuits, the current in the common conductor is 1.41 times that in either of the other conductors.

Adapted from Gibbs 1971

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Motor Efficiency

- Synchronous motor efficiency: Depending on their rating, unity power factor synchronous motors have a full load efficiency of 93% to more than 97%. Motors with an 80% power factor have lower efficiencies (Gibbs 1971).
- To determine the efficiency of an induction motor, use the following values:
  - Use 90% for a 1,780-rpm motor (use as 0.90 in the formulas).
  - Use 85% for a 3,500-rpm motor (use as 0.85 in the formulas).
- When sizing a backup generator, allow 1 kW (kilowatt) per horsepower of load.
- When sizing a transformer, allow 1 kVA (kilovolt-ampere) per horsepower of load.

Useful Formulas

The following are useful formulas for balanced and near-balanced loads on four-wire systems:

- Single phase (one phase and neutral): $P = V \times I \times \cos \phi$
- Single phase (phase to phase): $P = U \times I \times \cos \phi$
- Three phase (three wires or three wires + neutral): $P = \sqrt{3} \times U \times I \times \cos \phi$
- Single phase (one phase and neutral): $Q = V \times I \times \sin \phi$
- Single phase (phase to phase): $Q = U \times I \times \sin \phi$
- Three phase (three wires or three wires + neutral): $Q = \sqrt{3} \times U \times I \times \sin \phi$
- Single phase (one phase and neutral): $S = V \times I$
- Single phase (phase to phase): $S = U \times I$
- Three phase (three wires or three wires + neutral): $S = \sqrt{3} \times U \times I$

where

P = active power, kW
V = voltage between phase and neutral
U = voltage between phases
I = line current
ϕ = phase angle between vectors V and I
Q = reactive power, kvar
S = apparent power, kVA

COMPRESSED AIR

See Chapter 4, which covers physical science and engineering, for more information on gases; see particularly the section on thermodynamics and heat transfer. See Tables 16.4 and 16.5 for air consumption multipliers. See Table 16.6 for capacity correcting factors for single-stage air compressors. Hose pressure loss is given in Table 16.7, and drilling machine air requirements are given in Table 16.8.

Use the following equation to determine the pipe diameter for compressed air lines:

$$D = \left(\frac{V^2 L}{2{,}000 (P_1^2 - P_2^2)} \right)^{0.2} \quad \textbf{(EQ 16.1)}$$

where

D = pipe diameter, in.
V = volume of free air, cfm
L = pipe length, ft
P_1 = absolute pressure at beginning of pipe, psig
P_2 = absolute pressure at end of pipe, psig

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TABLE 16.4 Air consumption multipliers for operation of rock drills*

Altitude, ft	0	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000	12,500	15,000
Multiplier	1.0	1.02	1.05	1.08	1.11	1.14	1.18	1.22	1.26	1.30	1.34	1.46	1.58

* Based on 80–100 psig air pressure. Source: Bise 2003

TABLE 16.5 Multiplier for air consumption by number of rock drills

Number of drills	1	2	3	4	5	6	7	8	9	10	12	15	20	30	40	50	70
Multiplier	1.0	2.0	3.0	4.0	5.0	6.0	6.8	7.5	8.2	9.0	10.5	12.6	16.0	23.5	31.0	38.0	52.5

Source: Bise 2003

TABLE 16.6 Factors for correcting capacity of single-stage compressors*

Altitude, ft	90 psig Factor	100 psig Factor
Sea level	1.000	1.000
1,000	0.988	0.987
2,000	0.972	0.972
3,000	0.959	0.957
4,000	0.944	0.942
5,000	0.931	0.925
6,000	0.917	0.908
7,000	0.902	0.890
8,000	0.886	0.873
9,000	0.868	0.857
10,000	0.853	0.840
11,000	0.837	–
12,000	0.818	–

* Based on 7% cylinder clearance. Source: Bise 2003

The following equation can be used to correct air pressure at different elevations as compressed air tools lose efficiency with less dense air:

$$\log P_2 = \log P_1 - \frac{\pm h}{122.4 \times (T_F + 461)} \quad \textbf{(EQ 16.2)}$$

where

P_2 = unknown absolute pressure, psig
P_1 = known absolute pressure, psig
h = difference in elevation (+h if increasing altitude, –h if decreasing altitude), ft
T_F = temperature in degrees Fahrenheit (usually 60°F)

See Chapter 15 on ventilation for more information; see Table 15.2 for barometric pressure, temperature, and air density at different altitudes.

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TABLE 16.7 Pressure loss in hose

Hose Length, inside diameter	Free Air, cfm	Line Pressure, psig						
		60	80	100	120	150	200	300
50 ft; ¾ in.	60	3.1	2.4	2.0	–	–	–	–
	80	5.3	4.2	3.5	2.9	2.4	1.8	1.2
	100	8.1	6.4	5.2	4.5	3.6	2.8	1.9
	120	–	9.0	7.4	6.3	5.1	3.9	2.7
	140	–	12.0	9.9	8.4	6.9	5.3	3.6
	160	–	–	12.7	10.8	8.9	6.8	4.6
	180	–	–	–	13.6	11.1	8.5	5.8
	200	–	–	–	16.6	13.5	10.4	7.1
	220	–	–	–	–	16.2	12.4	8.4
50 ft; 1 in.	120	2.7	2.1	–	–	–	–	–
	150	4.1	3.2	2.7	2.3	–	–	–
	180	5.8	4.6	3.8	3.2	2.6	2.0	1.3
	210	7.7	6.1	5.0	4.3	3.5	2.7	1.8
	240	–	7.9	6.5	5.5	4.5	3.4	2.3
	270	–	9.8	8.1	6.9	5.6	4.3	2.9
	300	–	12.0	9.9	8.4	6.9	5.3	3.6
	330	–	–	11.8	10.0	8.2	6.3	4.3
	360	–	–	13.9	11.9	9.7	7.4	5.0
	390	–	–	–	13.8	11.3	8.7	5.9
	420	–	–	–	15.9	13.0	10.0	6.8
	450	–	–	–	–	14.8	11.4	7.7
50 ft; 1¼ in.	200	2.4	–	–	–	–	–	–
	250	3.7	2.9	2.4	2.0	–	–	–
	300	5.2	4.1	3.4	2.9	2.3	1.8	1.2
	350	7.0	5.5	4.5	3.8	3.1	2.4	1.6
	400	8.9	7.0	5.8	4.9	4.0	3.1	2.1
	450	–	8.8	7.3	6.2	5.0	3.9	2.6
	500	–	10.8	8.9	7.6	6.2	4.7	3.2
	550	–	–	10.7	9.1	7.4	5.7	3.9
	600	–	–	12.6	10.7	8.7	6.7	4.6
	650	–	–	14.6	12.4	10.2	7.8	5.3
	700	–	–	–	14.3	11.7	9.0	6.1
	750	–	–	–	–	13.3	10.2	6.9
	800	–	–	–	–	15.0	11.5	7.8
50 ft; 1½ in.	300	2.1	–	–	–	–	–	–
	400	3.7	2.9	2.4	2.0	–	–	–
	500	5.6	4.4	3.7	3.1	2.5	1.9	1.3
	600	8.0	6.3	5.2	4.4	3.6	2.8	1.9
	700	–	8.5	7.0	5.9	4.9	3.7	2.5
	800	–	10.9	9.0	7.7	6.3	4.8	3.2
	900	–	–	11.2	9.5	7.8	6.0	4.1
	1,000	–	–	13.6	11.6	9.5	7.3	4.9
	1,100	–	–	–	14.0	11.4	8.8	6.0
	1,200	–	–	–	–	13.6	10.4	7.1
	1,300	–	–	–	–	15.8	12.1	8.3

Source: Bise 2003

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TABLE 16.8 Air requirements of representative drilling machines

Type of Machine	Hammer Diameter, in.	Free Air Required, cfm
Sinker	2⅜	70
Sinker	2½	95
Sinker	2⅝	110
Sinker	2¾	115
Stoper	2⁹⁄₁₆	140
Stoper	2¾	160
Drifter	2¾	130
Drifter	3	140
Drifter	3½	180
Drifter	4½	200

Source: Bise 2003

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CHAPTER

17

Pumping

W.R. Reed, P.E.

INTRODUCTION

Pumps are generally used to supply and remove water for mining and milling operations. The material presented here applies to centrifugal pumps, which are probably the most popular type of pump used in mining. Piston pumps (positive displacement pumps) are used in mine dewatering in high head applications.

LIQUID FLOW

Flow of a liquid through a pipe can be classified as *laminar* or *turbulent*. The flow type is determined using the Reynolds number, which is a dimensionless number.

$$\text{Re} = \frac{VD}{v} \quad \textbf{(EQ 17.1)}$$

where

Re = Reynolds number, dimensionless
V = average velocity, ft/s
D = average internal diameter, ft
v = kinematic viscosity of the fluid, ft^2/s

For water at 60°F, $v = 0.00001216\ ft^2/s$. Or the following equation can be used:

$$\text{Re} = \frac{3{,}160Q}{kD_I} \quad \textbf{(EQ 17.2)}$$

where

Re = Reynolds number, dimensionless
Q = rate of flow, gpm
D_I = internal diameter of pipe, in.
k = kinematic viscosity of the fluid, centistokes

If the value of Re is less than 2,000, then the flow is considered laminar. In this case, the flow particles follow separate distinct flow paths with no eddies or turbulence. If the value of Re is greater than 4,000, then the flow is considered turbulent. Re values between 2,000 and 4,000 are in a critical zone between laminar and turbulent. For pumping purposes, this critical zone is considered turbulent.

Table 17.1 shows the properties of water in customary units of measure. In this table, if $\mu \times 10^5 = 3.746$, then $\mu = 3.746 \times 10^{-5}$ lb-s/ft^2. For example, at 80°F, $\sigma \times 10^2 = 0.492$ or $\sigma = 0.00492$ lb/ft. $E_v \times 10^{-3} = 322$ or $E_v = 322{,}000$ psi.

PRESSURE

Absolute pressure is total pressure including atmospheric pressure. Gauge pressure is pressure above or below atmospheric pressure. The relationship between absolute and gauge pressure is shown in the following equation:

$$P_A = P_G + P_{ATM} \quad \textbf{(EQ 17.3)}$$

where

P_A = absolute pressure, lb/in.^{2}
P_G = gauge pressure, lb/in.^{2}
P_{ATM} = atmospheric pressure, lb/in.^{2} (generally 14.7 psi at sea level and 59°F)

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TABLE 17.1 Properties of water in English* units†

Temperature, °F	Specific Weight, lb/ft^3	Density, slugs/ft^3	Viscosity, lb-s/ft^2	Kinematic Viscosity, ft^2/s	Surface Tension, lb/ft	Vapor Pressure, psia	Vapor Pressure Head, ft	Bulk Modulus of Elasticity, psi
T	γ	ρ	$\mu \times 10^5$	$\nu \times 10^3$	$\sigma \times 10^2$	ρ_v	ρ_v/γ	$E_v \times 10^{-3}$
32	62.42	1.940	3.746	1.931	0.518	0.09	0.20	293
40	62.43	1.940	3.229	1.664	0.514	0.12	0.28	294
50	63.41	1.940	2.735	1.410	0.509	0.18	0.41	305
60	62.37	1.938	2.359	1.217	0.504	0.26	0.59	311
70	62.30	1.936	2.050	1.059	0.500	0.36	0.84	320
80	62.22	1.934	1.799	0.930	0.492	0.51	1.17	322
90	62.11	1.931	1.595	0.826	0.486	0.70	1.61	323
100	62.00	1.927	1.424	0.739	0.480	0.95	2.19	327
110	61.86	1.923	1.284	0.667	0.473	1.27	2.95	331
120	61.71	1.918	1.168	0.609	0.465	1.69	3.91	333
130	61.55	1.913	1.069	0.558	0.460	2.22	5.13	334
140	61.38	1.908	0.981	0.514	0.454	2.89	6.67	330
150	61.20	1.902	0.905	0.476	0.447	3.72	8.58	328
160	61.00	1.896	0.838	0.442	0.441	4.74	10.95	326
170	60.80	1.890	0.780	0.413	0.433	5.99	13.83	322
180	60.58	1.883	0.726	0.385	0.426	7.51	17.33	318
190	60.36	1.876	0.678	0.362	0.419	9.34	21.55	313
200	60.12	1.868	0.637	0.341	0.412	11	26.59	308
212	59.83	1.860	0.593	0.319	0.319	1452.70	33.90	300

* English or Imperial units.

Source: Daugherty et al. 1985

† In this table: If $\mu \times 105 = 3.746$, then $\mu = 3.746 \times 10^{-5}$ lb-s/ft^2. Example: at 800°F, $\sigma \times 102 = 0.492$ or $\sigma = 0.00492$ lb/ft. $E_v \times 10^{-3} = 322$ or $E_v = 322{,}000$ psi.

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To convert water pressure in units of pounds per square inch to water pressure head in units of feet, use the following equation:

$$P_{psi} = 0.433 \times P_{ft} \quad \text{(EQ 17.4)}$$

where

P_{psi} = water pressure, lb/in.^{2}
P_{ft} = water pressure head, ft

BERNOULLI EQUATION

The Bernoulli equation is the basis for calculating the head that a pumping system generates. Assuming no flow losses, the total head is the same for any point along a streamline.

$$\text{head} = H_{SP} + H_V + z \quad \text{(EQ 17.5)}$$

where

head = total head, ft
H_{SP} = static pressure head, ft
H_V = velocity pressure head, ft
z = elevation, ft

Calculating Static Pressure Head

Static head is the pressure that is induced normally to the flow. It is calculated using Equation 17.6:

$$H_{SP} = \frac{144P}{\gamma} \quad \text{(EQ 17.6)}$$

where

H_{SP} = static head, ft
P = pressure, lb/in.^{2}
γ = density, lb/ft^3

Equation 17.7 can also be used to calculate the static pressure head:

$$H_{SP} = \frac{P}{\gamma} \quad \text{(EQ 17.7)}$$

where

H_{SP} = static head, ft
P = pressure expressed as a height of a column of fluid, ft
γ = density, lb/ft^3

Calculating Velocity Head

Velocity head represents the kinetic energy of the moving liquid in the system. It considers the velocity of the liquid through the system. It is the basis for calculating entrance losses, losses through valves and fittings, and losses for sudden enlargements and exits. In pumping systems with high heads, velocity head is generally negligible and can be disregarded. However, for low head pumping systems, velocity head must be included.

$$H_V = \frac{V^2}{2g} \quad \text{(EQ 17.8)}$$

where

H_V = velocity head, ft
V = velocity of liquid, ft/s
g = gravitational constant = 32.174 ft/s^2

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Application of Bernoulli's Equation at Two Points

Because the total head is the same at any point along a streamline, the total heads at two different points will be equal minus any friction loss from point 1 to point 2. Therefore, going from point 1 to point 2 (Figure 17.1) is represented by Equation 17.9.

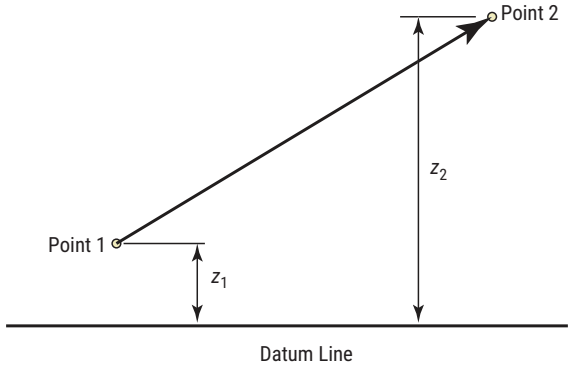

FIGURE 17.1 Elevation head in feet

$$H_{SP1} + H_{V1} + z_1 = H_{SP2} + H_{V2} + z_2 - H_f \quad \text{(EQ 17.9)}$$

where

H_{SP1} = static pressure head, ft at point 1
H_{V1} = velocity pressure head, ft at point 1
z_1 = elevation, ft at point 1
H_{SP2} = static pressure head, ft at point 2
H_{V2} = velocity pressure head, ft at point 2
z_2 = elevation, ft
H_f = friction loss caused by fluid movement from point 1 to point 2

Alternatively, Equation 17.10 can be used:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - H_f \quad \text{(EQ 17.10)}$$

where

P_1 = pressure expressed as a height of a column of fluid, ft at point 1
γ = density, lb/ft^3
V_1 = velocity of liquid, ft/s at point 1
g = gravitational constant = 32.174 ft/s^2
z_1 = elevation, ft at point 1
P_2 = pressure expressed as a height of a column of fluid, ft at point 2
V_2 = velocity of liquid, ft/s at point 2
z_2 = elevation, ft
H_f = friction loss due to fluid movement from point 1 to point 2

CALCULATING DYNAMIC HEAD

To select a pump, determine the flow rate in liters per second or gallons per minute and the total dynamic head in meters or feet. Because the dynamic head capability of the pump depends on the flow rate, this is an iterative process. Generally, only the elevation and friction loss components are needed to calculate head. The elevation head is the difference in elevation of the pump's discharge and the delivery point. The friction head is the amount of head used in overcoming friction in the pump's discharge pipeline.

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$$\text{head} = H_{el} + H_f \quad \text{(EQ 17.11)}$$

where

head = total dynamic head, m (ft)
H_{el} = elevation head, m (ft)
H_f = friction head, m (ft)

In most cases, head to create velocity of flow is negligible and thus is often ignored (Bise 2003). Properly designed pipelines limit friction loss. The Darcy–Weisbach equation is provided (Equation 17.12) and is generally used in conjunction with the Moody diagram (Figure 17.2). The Hazen–Williams equation (Equation 17.13) is commonly used to calculate pipeline friction losses (Warner 1992).

$$H_f = f\frac{LV^2}{D_I 2g} \quad \text{(EQ 17.12)}$$

where

H_f = friction loss, ft
f = friction factor, dimensionless, but dependent on pipe surface roughness and the Reynolds number
L = pipeline length, ft
V = flow velocity, ft/s
g = gravitational acceleration, 32.2 ft/s^2
D_I = inside pipe diameter, ft

Also, the following can be used:

$$H_{f100} = \frac{K\left(\frac{Q}{C}\right)^{1.852}}{D^{4.87}} \quad \text{(EQ 17.13)}$$

where

H_{f100} = friction loss per 100 m (100 ft) of pipe
K = constant = 1,045 for customary units or 1.22×10^{12} for SI units
Q = flow rate, L/s (gpm)
C = retardation coefficient = 120 for coated steel or 150 for polyvinyl chloride (PVC)
D = inside pipe diameter, mm (in.)

For laminar flow, that is, Re < 2,000, use the following equation:

$$f = \frac{64}{\text{Re}} \quad \text{(EQ 17.14)}$$

where

f = friction factor, dimensionless, but dependent on pipe surface roughness and the Reynolds number
Re = Reynolds number, dimensionless

For turbulent flow, that is, Re > 4,000, either the Moody diagram (Figure 17.2) or the Colebrook formula (Equation 17.15) can be used to calculate the friction factor (White 2011).

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D_I} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \text{(EQ 17.15)}$$

where

f = friction factor, dimensionless, but dependent on pipe surface roughness and the Reynolds number
Re = Reynolds number, dimensionless
ε = absolute roughness, ft
D_I = inside pipe diameter, ft

Table 17.2 lists roughness values for commercial duct, Table 17.3 contains pipe friction loss data, and Table 17.4 gives equivalent lengths of pipe for some common fittings and valves. Generally, friction losses caused by fittings and valves are negligible in long pipelines.

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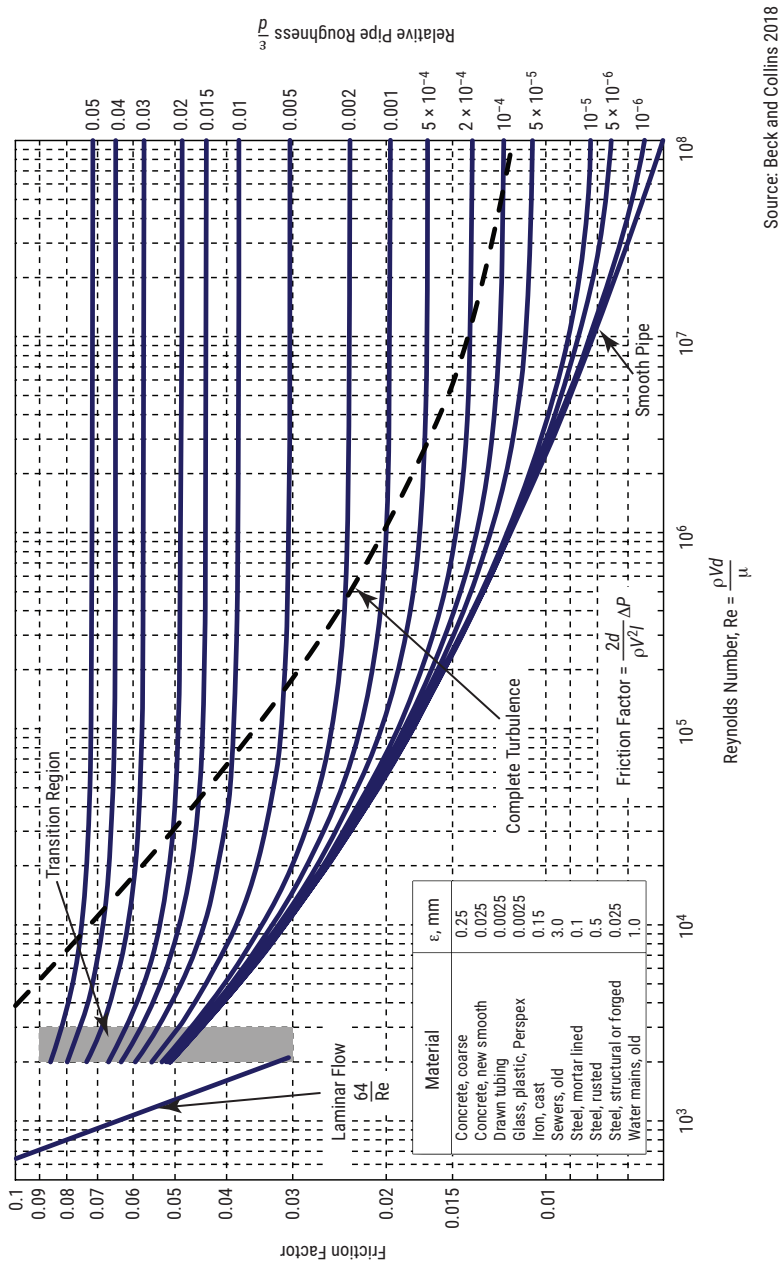

Source: Beck and Collins 2018

FIGURE 17.2 Moody diagram

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TABLE 17.2 Roughness of pipes

Material	Condition	Value, mm
Aluminum, drawn/pressed	New	0.0013–0.0015
Aluminum, drawn/pressed	Used	to 0.03
Asbestos-cement	New, smooth	0.03–0.1
Brass, drawn/pressed	New	0.0013–0.0014
Brass, drawn/pressed	Used	to 0.03
Cast iron	Average city sewerage	1.2
Cast iron	Incrusted	to 3.0
Cast iron	New, bituminized	0.10–0.13
Cast iron	New, with skin	0.2–0.6
Cast iron	Operating several years, cleaned	1.5
Cast iron	Slightly rusty	1.0–1.5
Clay	New, clay tile	9.0
Clay, drainage-pipe	New, calcined	0.7
Concrete	New, medium rough	1.0–2.0
Concrete	New, rough	2.0–3.0
Concrete	New, smooth	0.3–0.8
Concrete	Operating several years	0.2–0.3
Concrete, centrifugal	New, smooth plastered	0.1–0.15
Concrete, centrifugal	New, without plaster	0.2–0.8
Concrete, steel	New, smooth	0.1–0.15
Copper, drawn/pressed	New	0.0013–0.0015
Copper, drawn/pressed	Used	to 0.03
Glass, drawn/pressed	New	0.0013–0.0015
Glass, drawn/pressed	Used	to 0.03
Plastic, drawn/pressed	New	0.0013–0.0015
Plastic, drawn/pressed	Used	to 0.03
Rubber	New, smooth	0.0016
Steel	After long operation cleaned	0.15–0.20
Steel	Homogeneous corrosion pits	0.15
Steel	Intensely incrusted	2.0–4.0
Steel	Slightly rusty and incrusted	0.15–0.40
Steel, longitudinal welded	New, bituminized	0.01–0.05
Steel, longitudinal welded	New, galvanized	0.008
Steel, longitudinal welded	New, rolling skin	0.04–0.1
Steel, weldless	New, commercial size galvanized	0.10–0.16
Steel, weldless	New, neatly galvanized	0.07–0.10
Steel, weldless	New, pickled	0.03–0.04
Steel, weldless	New, rolling skin	0.02–0.06
Steel, weldless	New, unpickled	0.03–0.06
Stoneware	—	0.25
Wood	After long operating	0.1
Wood	New	0.2–1.0

Courtesy of Spraying Systems Co.

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TABLE 17.3 Pipe friction loss

Flow Rate, gpm	Friction Loss, ft/100 ft per inside diameter measurement							
	HDPE,* 2 in.	Steel, 2 in.	HDPE, 4 in.	Steel, 4 in.	HDPE, 6 in.	Steel, 6 in.	HDPE, 8 in.	Steel, 8 in.
25	1.3	2.0	0.0	0.1	–	–	–	–
50	4.8	7.1	0.2	0.2	–	–	–	–
75	10.1	15.1	0.3	0.5	0.0	0.1	–	–
100	17.3	25.8	0.6	0.9	0.1	0.1	–	–
125	26.1	39.1	0.9	1.3	0.1	0.2	–	–
150	36.7	55.0	1.2	1.9	0.2	0.3	0.0	0.1
175	48.9	73.2	1.6	2.5	0.2	0.3	0.1	0.1
200	62.6	93.8	2.1	3.2	0.3	0.4	0.1	0.1
300	–	–	4.5	6.7	0.6	0.9	0.2	0.2
400	–	–	7.7	11.5	1.1	1.6	0.3	0.4
500	–	–	11.6	17.4	1.6	2.4	0.4	0.6
750	–	–	24.7	36.9	3.4	5.1	0.8	1.2
1,000	–	–	42.1	63.1	5.8	8.7	1.4	2.1
1,500	–	–	–	–	12.3	18.5	3.0	4.5
2,000	–	–	–	–	21.0	31.5	5.1	7.7
2,500	–	–	–	–	31.8	47.7	7.8	11.7
3,000	–	–	–	–	44.7	67.0	10.9	16.4
4,000	–	–	–	–	76.3	–	18.7	28.0
5,000	–	–	–	–	–	–	28.3	42.4

* HDPE = high-density polyethylene.

TABLE 17.4 Equivalent feet of straight pipe for fittings

Fitting	Inside Diameter of Fitting			
	2 in.	4 in.	6 in.	8 in.
90° elbow	5.0	10	15	20
45° elbow	2.5	5.0	8.0	10
Tee straight	3.5	7.0	10	14
Tee side	10	20	30	40
Gate valve, open	1.5	2.6	4.0	5.0
Swing check valve, open	9.0	17	25	35

Adapted from Crane Company 2018

NET POSITIVE SUCTION HEAD

Pumps do not pull fluid up a suction pipe; atmospheric or external pressure pushes it up the suction pipe of a pump (Bise 2003). Atmospheric pressure provides most of a pump's net positive suction head (NPSH). At sea level, atmospheric pressure is 14.7 psi (34 ft of water head; some consider 23 ft to be the practical limit of suction lift; Warner 1992). Table 17.5 contains equivalent feet of suction head at various elevations at perfect vacuum.

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TABLE 17.5 Equivalent feet of suction head available at perfect vacuum*

Elevation, ft	Elevation, m	Head of water, ft
0	0	34.0
2,000	609.6	31.6
4,000	1,219.2	29.4
6,000	1,028.8	27.3
8,000	2,438.4	25.2
10,000	3,048.0	23.4
15,000	4,572.0	19.1

* Water at 75°F.

Adapted from Heald 2018

PUMP CHARACTERISTIC CURVES

Pump manufacturers publish pump characteristic curves (pump curves) that show the relationship between pump discharge rate in gallons per minute (liters per second) and head in feet (meters). See Figure 17.3 for a typical pump curve. Frequently, manufacturers also show pump efficiency and horsepower on pump curves.

Series Pumps

By connecting pumps in series, the pump combination's head is increased. To plot the curve for a series pump arrangement, add the pumps' heads at a given flow rate.

Parallel Pumps

By connecting pumps in parallel, the pump combination's flow is increased. To plot the curve for a parallel pump arrangement, add the pumps' flow rates at a given head.

Best Efficiency Point

A pump is designed to operate at its best efficiency point (BEP), which is the flow head point on its curve at which the pump operates at its greatest efficiency. In Figure 17.3, the best efficiency is approximately 85% with the pump operating at 650 gpm and approximately 78–79 ft of head. To operate at this point, brake horsepower is approximately 17.5 bhp (brake horsepower).

In Figure 17.3, a system curve is plotted on this diagram (dashed line). The system curve shows how much head is required for a given flow rate through a pipe network. This network includes all piping, fittings, and devices from the free liquid surface level on the intake side of the pump to the discharge of the pipe network. The pump will operate at the intersection of the pump curve and the system curve.

Allowable Operating Range

Although a pump is designed to operate at the BEP, pump manufacturers often provide greater flexibility by specifying an allowable working range for the pump. For example, Figure 17.4 displays the performance curve of a typical centrifugal pump. This curve shows the working range of the pump. In Figure 17.4, changes in rotations per minute to this pump will either change head, flow, or a combination thereof.

System Head Curve

A system head curve is a plot of the expected head at which one plans to run a pump. The intersection of the system head curve and the pump curve is the operating point at which the pump is forecast to run. Figure 17.5 shows the system head curve versus flow rate. This figure shows the amount of friction head (H_f) required for the pump to overcome in order to provide flow rate (Q). The other curves labeled C_w (concentration solids by weight) are the H_f curves for slurries at the corresponding percent concentration solids by weight. The limiting settling velocity of the mixtures is shown by V_L.

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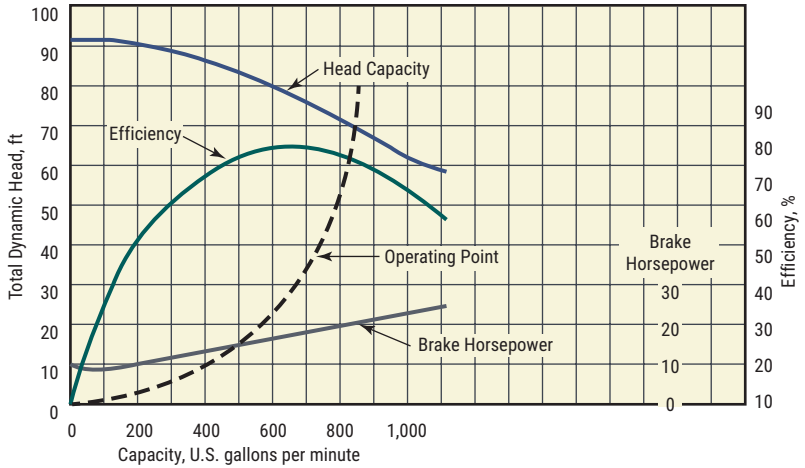
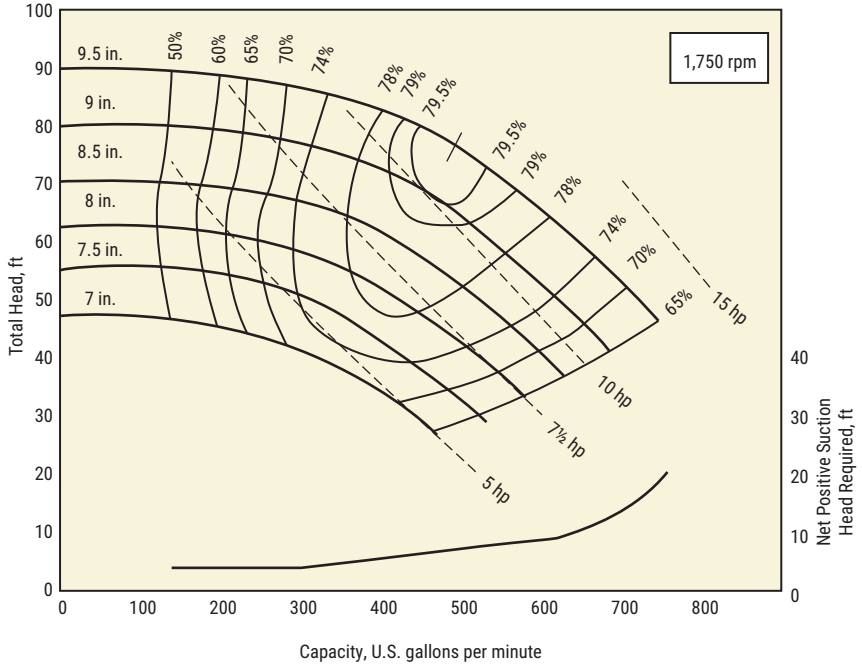


Source: Weir Slurry Group 2009

FIGURE 17.3 Typical pump characteristic curve



Adapted from Weir Slurry Group 2009

FIGURE 17.4 Typical pump performance curve

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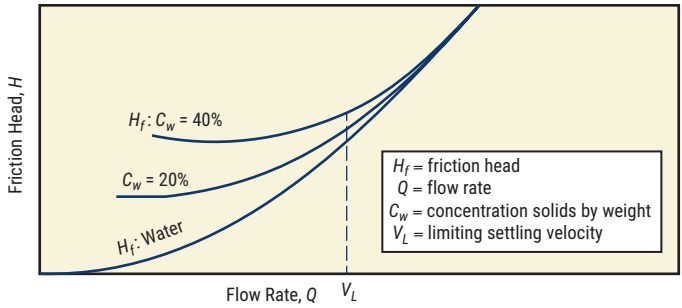

Source: Weir Slurry Group 2009

FIGURE 17.5 Typical system head curve

CENTRIFUGAL PUMPS

A single-stage (single-impeller) pump is normally appropriate for high volume and low to moderate heads. For heads greater than about 250 ft (76.2 m), multistage pumps are generally used. A multistage pump has two or more stages and is essentially for high head pumping (Warner 1992). It is also common to place multiple single-stage pumps in series either directly one after another or along a pipeline.

PUMPING POWER FORMULAS

$$\text{bhp} = \frac{QH}{3{,}960E_p} \qquad \textbf{(EQ 17.16; Warner 1992)}$$

where

bhp = brake horsepower
Q = flow rate (gpm) at specific gravity 1.0
H = head, ft
E_p = pump efficiency (decimal)

Equation 17.16 can be rearranged to solve directly for Q and H

$$\text{kW input} = \frac{0.746\,\text{bhp}}{E_m} \qquad \textbf{(EQ 17.17)}$$

where

kW input = kilowatt input to the motor, kW
bhp = brake horsepower
E_m = motor efficiency

Efficiency Values

Both pump and motor values are available from the manufacturers. The following efficiency values are generally acceptable:

- Eff_p = pump efficiency = 70% (use as 0.70 in the preceding formulas)
- Eff_m = motor efficiency = 90% for a 1,780-rpm motor (use as 0.90 in the formulas)
- Eff_m = motor efficiency = 85% for a 3,500-rpm motor (use as 0.85 in the formulas)

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FLOW RATE FORMULA AND OTHER FACTS

See Chapter 4 on physical science and engineering for more information.

$$Q = V \times A \quad \text{(EQ 17.18)}$$

where

Q = flow rate, ft^3/s
V = velocity, ft/s
A = area, ft^2 per 1 cfs = 448.83 gpm

1 psi = 2.31 ft of water
1 ft^3 = 7.48 gal

AFFINITY LAWS

When the speed of a centrifugal pump is changed, a pump's operation changes according to three fundamental laws (Bise 2003):

1. Flow rate (Q) varies directly as speed.
2. Head (H) varies as the square of speed.
3. Brake horsepower varies as the cube of speed.

For pumps of similar geometry and operating on the same dimensionless point on a pump curve,

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_3}\right)^3 \quad \text{(EQ 17.19)}$$

$$\frac{H_2}{H_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_3}\right)^2 \quad \text{(EQ 17.20)}$$

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_3}\right)^5 \quad \text{(EQ 17.21)}$$

where

Q = flow rate, gpm
n = speed, rpm
D = diameter of impeller, ft
H = head, ft
P = power
ρ = density of liquid

Subscripts 1 and 2 refer to two non-cavitating operating conditions.

WATER HAMMER

Pressure caused by an abrupt decrease in water velocity because of valve closure can be shown with the following equation:

$$T_r = 2\frac{L}{c_p} \quad \text{(EQ 17.22)}$$

where

T_r = time of round trip pressure wave from point of closure to water inlet into system, s
L = length of pipe, ft
c_p = celerity = velocity of pressure wave in pipe, fps

For instantaneous or rapid valve closure $\left(T_r < 2\frac{L}{c_p}\right)$,

$$p_h = \rho c_p V \quad \text{(EQ 17.23)}$$

where

p_h = water hammer pressure, lb/ft^2
ρ = density of water, $slug/ft^3$
c_p = celerity = velocity of pressure wave in pipe, fps
V = velocity of water, fps

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$$p_{tot} = p_h + p \quad \text{(EQ 17.24)}$$

where

p_{tot} = total pressure at valve immediately after closure, lb/ft^2
p_h = water hammer pressure, lb/ft^2
p = pressure in pipe upstream of valve prior to valve closure, lb/ft^2

For slow valve closure $\left(T_r > 2\frac{L}{c_p}\right)$,

$$p_h = \frac{2LV\rho}{t_c} \quad \text{(EQ 17.25)}$$

where

p_h = water hammer pressure, s
ρ = density of water, slug/ft^3
t_c = time of valve closure, s
L = length of pipe, ft
V = velocity of water, fps

POSITIVE DISPLACEMENT PUMPS

Positive displacement pumps differ from centrifugal pumps as they are generally piston and/or diaphragm pumps with inlet and outlet poppet valves. They are employed in pumping through very long pipelines (2–50 km) because of their ability to generate high pressures, which are well in excess of multistaged centrifugal pumps. They are also used as chemical injection pumps to inject chemicals into high-pressure streams. However, their design flow rate range is limited from 50 to 1,000 L/s, mainly because of the pump's large physical size. They are most suitable for transporting slurries with high concentrations of fine particles. The maximum particle size is dictated by the poppet valves, which can jam in semi-open position by large particles (Grzina et al. 2002).

SLURRIES: SPECIFIC GRAVITY AND CONCENTRATION EQUATIONS

Equations 17.26 through 17.31 are used to determine specific gravity and concentration in slurries.

$$Sw = \frac{S(S_m C_w - S_m)}{(C_w - S)} = \frac{(SC_v - S_m)}{(C_v - 1)} = \frac{S[C_v(C_w - 1)]}{[C_w(C_v - 1)]} \quad \text{(EQ 17.26)}$$

$$S = \frac{S_w C_w (C_v - 1)}{[C_v(C_w - 1)]} = \frac{S + (S_m - S_w)}{(C_v)} = \frac{S_w C_w}{\left[C_w - 1 + \frac{S_w}{S_m}\right]} \quad \text{(EQ 17.27)}$$

$$S_m = \frac{S_w}{\left[1 - C_w\left(1 - \frac{S_w}{S}\right)\right]} = S_w + C_v(S - S_w) = \frac{S_w(C_v - 1)}{[C_w - 1]} \quad \text{(EQ 17.28)}$$

$$C_w = \frac{S(S_m - S_w)}{[S_m(S - S_w)]} = \frac{SC_v}{[S_w + C_v(S - S_w)]} = \frac{1 + S_w(C_v - 1)}{S_m} \quad \text{(EQ 17.29)}$$

$$C_v = \frac{(S_m - S_w)}{(S - S_w)} = \frac{S_w}{\left[S_w - S + \frac{S}{C_w}\right]} = \frac{1 + S_m(C_w - 1)}{S_w} \quad \text{(EQ 17.30)}$$

$$\frac{C_w}{C_v} = \frac{S}{S_m} \quad \text{(EQ 17.31)}$$

where

S_w = specific gravity of liquid
S = specific gravity of solid
S_m = specific gravity of mixture
C_w = % concentration of solids by weight
C_v = % concentration of solids by volume

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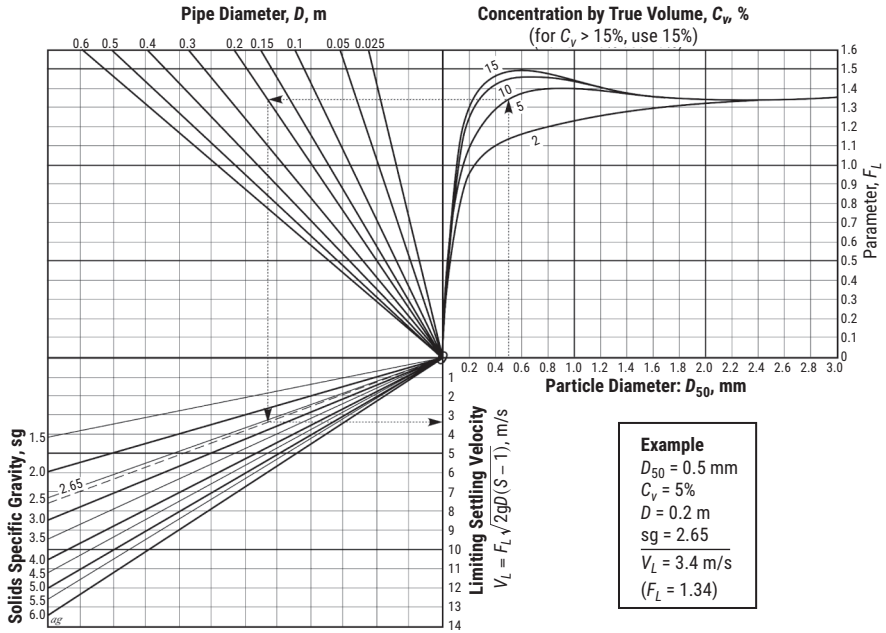
PUMPING SLURRIES

When pumping slurries, the limiting velocity (V_L) is important to calculate. This is usually accomplished using either Durand's (Figure 17.6) or Wilson's methods (Figure 17.7). Generally, slurries having high concentrations (C_w) require small flow rates (Q) and small pipe diameters (D). For low concentrations (C_w), the opposite is true (Grzina et al. 2002).

Durand's Method

1. For a fixed solids mass flow (M), select a value of C_v and calculate the corresponding flow rate (Q).
2. Select a standard inner pipe diameter (D).
3. Obtain from the Durand diagram the corresponding limiting velocity (V_L) and multiply it by the pipe cross-sectional area (A) to obtain the limiting flow rate (Q_L).
4. If the flow rate (Q) is 10%–15% larger than the limiting flow rate (Q_L), the pipe diameter is correct and the solids will not settle during pumping.
5. If the flow rate (Q) is equal or smaller than the limiting flow rate (Q_L), then select the next smaller pipe diameter (D) and restart at step 1.
6. If no reasonable pipe diameter satisfies the requirement, then select another value of C_V and start over.



Source: Grzina et al. 2002

FIGURE 17.6 Durand's method of calculating the limited settling velocity of a slurry

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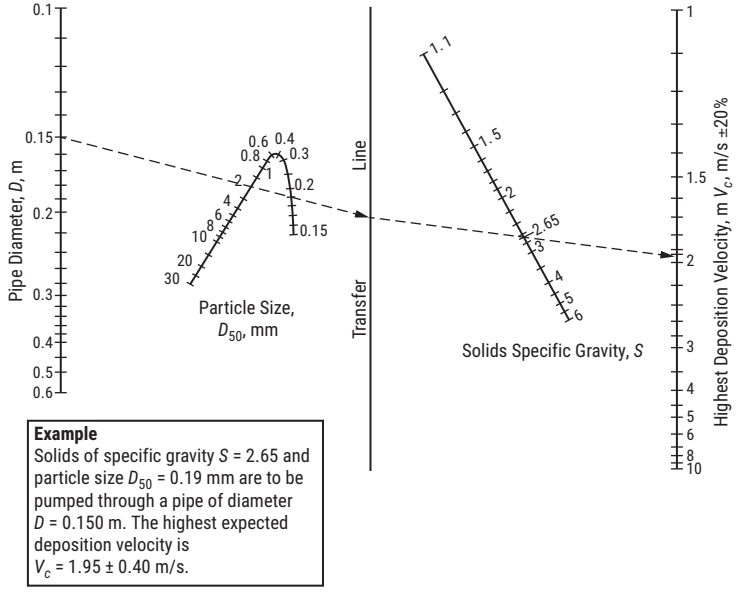

Source: Grzina et al. 2002

FIGURE 17.7 Wilson's method of calculating the limited settling velocity of a slurry

DREDGING

During the dredging process, the particle size distribution continuously varies. The dredging operator can move the suction head of the dredge to ensure that the solids do not settle out and that the dredge pipeline does not become blocked.

Because of the variation in particle size of the solids, complicated methods for calculating friction head losses are not necessary. A simple method for predicting slurry heads based on friction losses caused by water has been developed by the Colorado School of Mines (1963). The method requires calculating the head loss of the entire system then applying the correct factor from Table 17.6.

The head losses caused by water are multiplied by the correct factor from the table to determine the head losses caused by the slurry.

TABLE 17.6 Colorado School of Mines factors for friction losses in dredging

Worst Type of Material Dredged	Factor
Light silt, mud or silt, but no sand	1.10
Mud, fine sand, or soft clay	1.15
Medium sand, mud and clay mixed	1.20
Tough clay, coarse sand, and/or gravel	1.30
Coral or shell	1.40
Coarse gravel and boulders without clay	1.50

Source: Colorado School of Mines Research Foundation 1963

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SOLIDS SETTLING

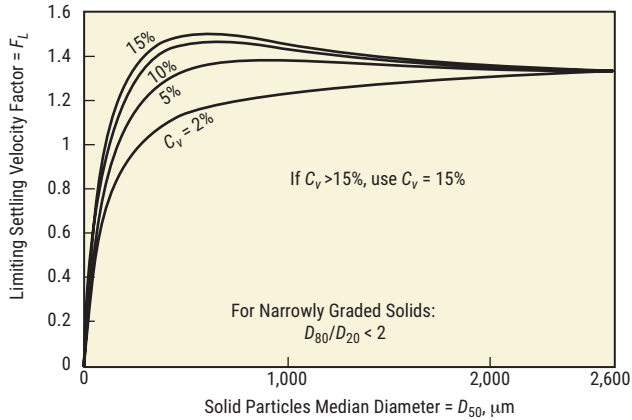
To calculate the velocity of liquid that will allow the slurry to flow without the solids settling, the following equation developed by Durand and Condolios (Miedema 2017) can be used:

$$V_L = F_L \sqrt{[2gD(S-1)]} \quad \text{(EQ 17.32)}$$

where

V_L = Durand's limiting solids settling velocity in pipe, m/s
F_L = Durand's parameter for limiting settling velocity in a pipe, dimensionless
g = gravitational acceleration, 9.81 m/s^2
D = inside diameter of pipe, m
S = specific gravity of dry solids

Figure 17.8 shows Durand's graph of limiting settling velocity.



Source: Grzina et al. 2002

FIGURE 17.8 Durand's limiting settling velocity

Slurry Friction Loss Data

Slurries can be classified into six different categories (Weir Slurry Group 2009):

1. Homogenous slurry
2. Heterogeneous slurry
   – Category A
   – Category B
   – Category C
   – Category D

Homogenous Slurry

In homogenous slurry, all particles measure less than 50 μm and have low C_w or concentration of solids by weight (<20%). For these slurries, friction loss (H_f) is similar to water or can use the empirical method shown for category A slurries. The yield of homogeneous slurries is also critical (e.g., pumping something that shears easily will be almost impossible to pump).

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Heterogeneous Slurry

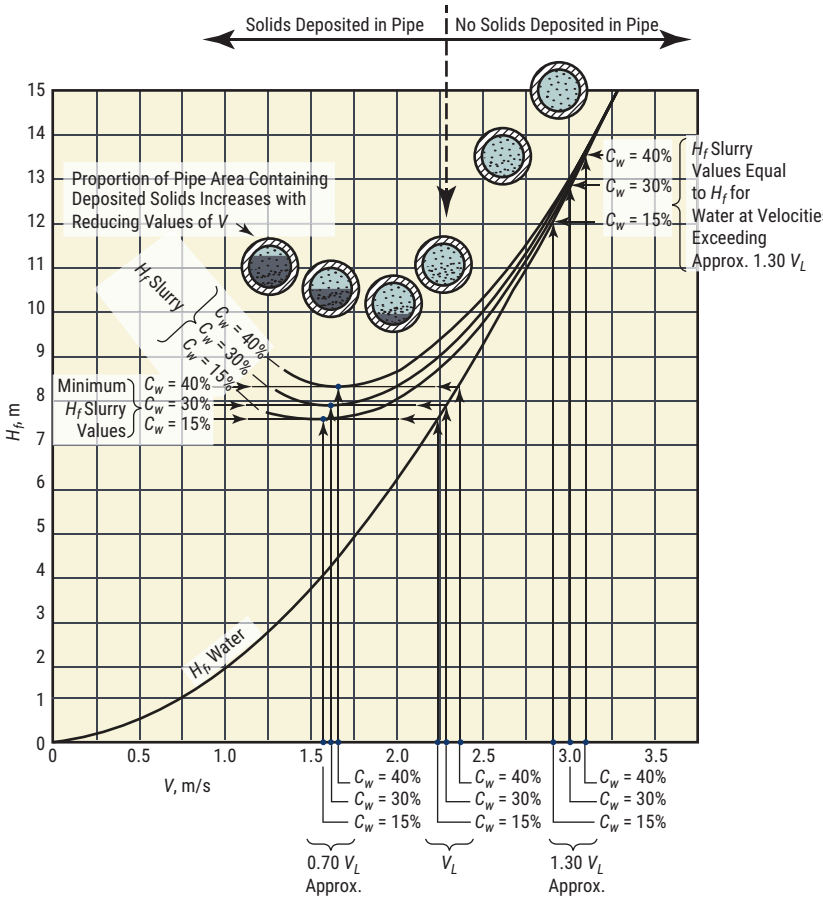
In heterogenous slurry, there are four different subcategories.

Category A

- Particles are coarser than 50 μm and finer than 300 μm. C_w measures from 0% to 40%. Figure 17.9 is used to calculate H_f. H_f calculations are based on V_L or limiting settling velocity.
- When flow velocity is at 1.3 V_L, then slurry H_f is equivalent to water H_f.
- When flow velocity is at 0.0 V_L, then slurry H_f is at its minimum value, and slurry H_f is equivalent to H_f of water at V_L.

Category B

- Particles are coarser than 50 μm and finer than 300 μm, but C_w is greater than 40%.
- H_f values will vary with the range beginning at values calculated using Figure 17.9 with C_w measuring to 40%. Maximum H_f values can be double or more than these lowest values. The lowest values are often calculated and then adjusted using field observations.



Source: Weir Slurry Group 2009

FIGURE 17.9 Typical H_f curve for category A slurries

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Category C

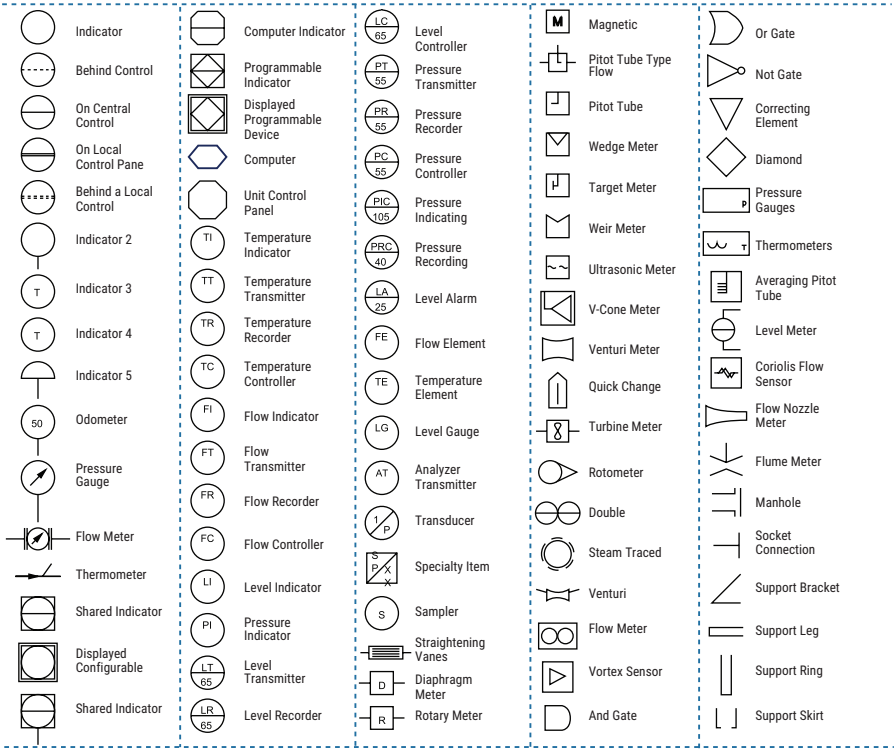
- Particles are coarser than 300 μm and C_w ranges from 0% to 20%.
- These slurries are generally slurries from dredging activities where C_w is less than 20% because of large particles of gravel and coarse sand.
- For design, the minimum flow average flow velocity is not less than V_L when F_L is 1.40, where F_L is the limiting settling velocity factor. Therefore, for velocities of V_L and greater, H_f of the slurry is equal to 1.10 H_f for water.

Category D

- Particles are coarser than 300 μm and C_w is greater than 20%.
- H_f values vary with the range beginning at values calculated using Figure 17.9. However, maximum H_f values can be three times or more than these lowest values, especially with velocities greater than V_L.

Piping and Instrumentation Diagrams

Piping and instrumentation (P&I) diagrams show the process flows using symbols for various operations. Figures 17.10 through 17.22 display standard symbols used in P&I diagrams (EdrawSoft 2014–2019). Symbols are connected using instrument lines. Figure 17.23 depicts symbols for the different connecting lines used in P&I diagrams (EdrawSoft 2014–2019).



Source: EdrawSoft 2014–2019

Figure 17.10 Instrument symbols

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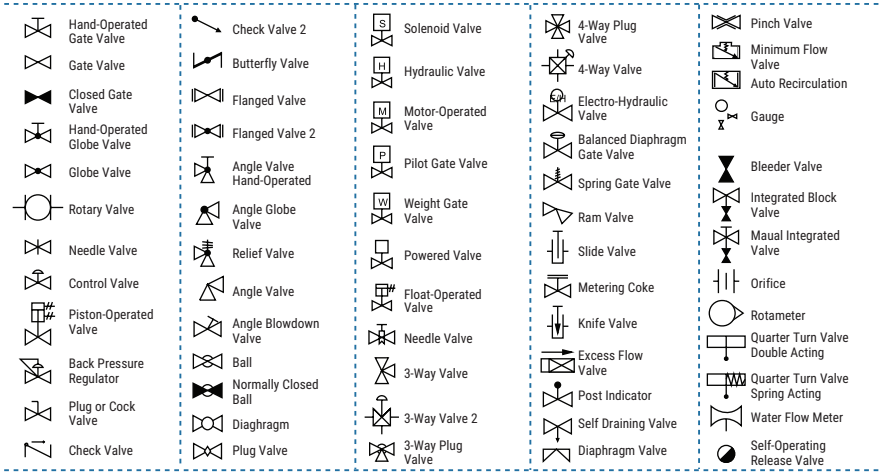
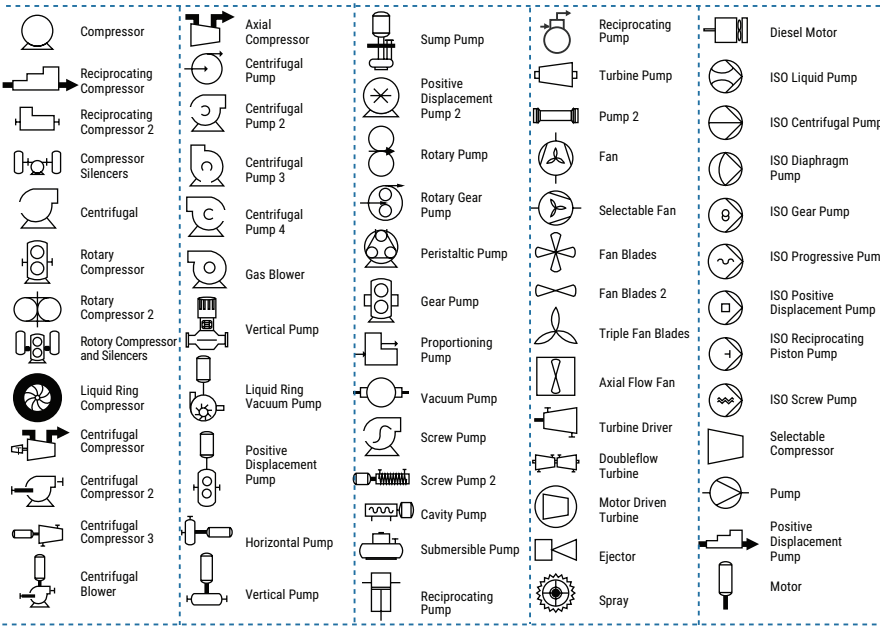


Source: EdrawSoft 2014–2019

Figure 17.11 Valve symbols



Source: EdrawSoft 2014–2019

Figure 17.12 Pump symbols

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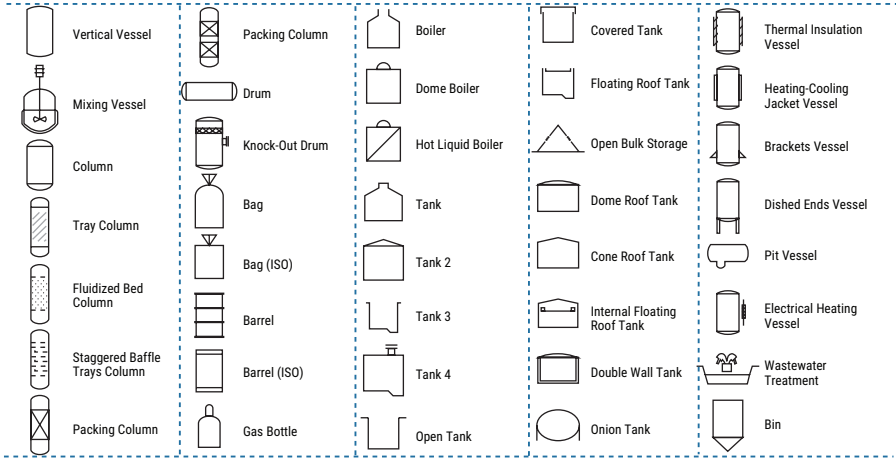
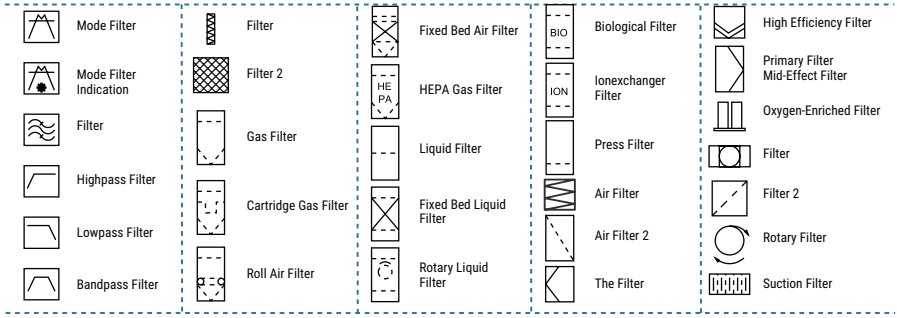


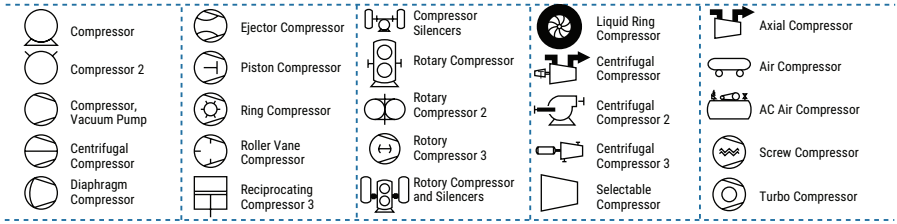
Source: EdrawSoft 2014–2019

Figure 17.13 Vessel symbols



Source: EdrawSoft 2014–2019

Figure 17.14 Filter symbols



Source: EdrawSoft 2014–2019

Figure 17.15 Compressor symbols

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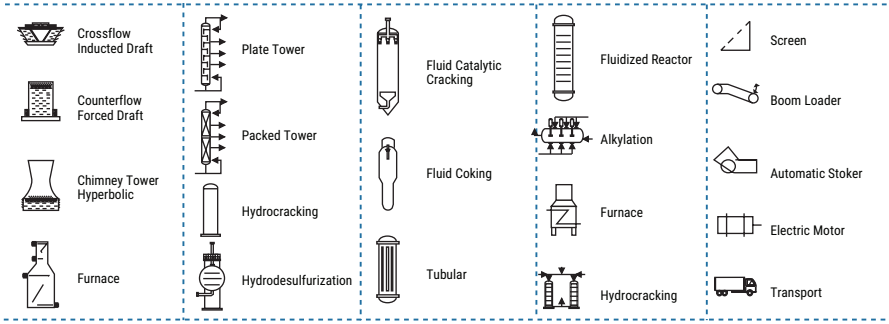
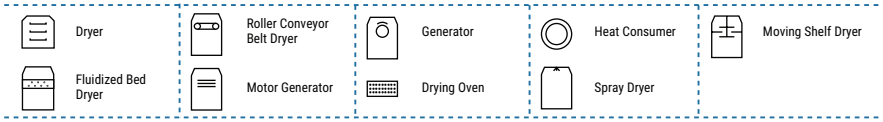


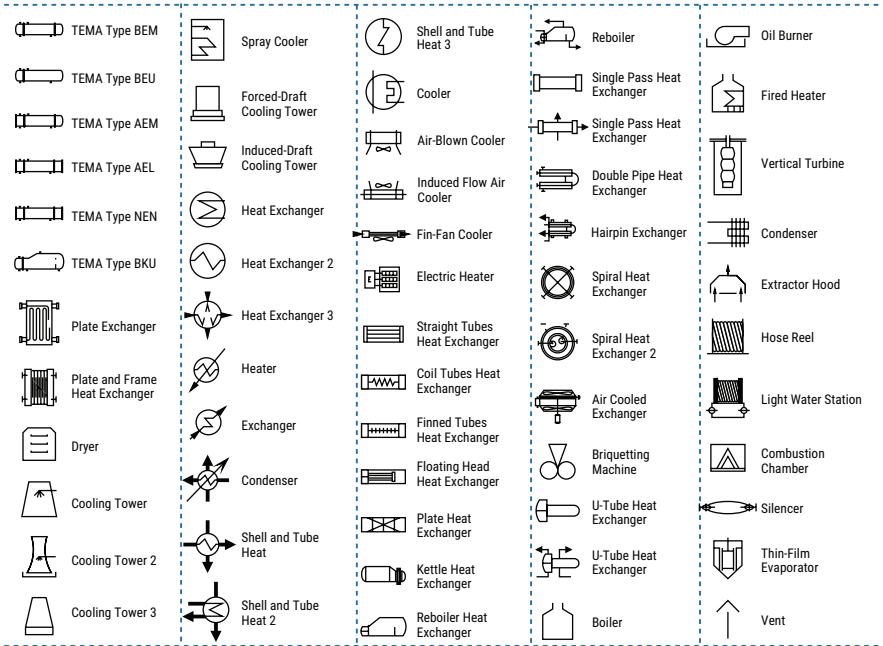
Source: EdrawSoft 2014–2019

Figure 17.16 General symbols



Source: EdrawSoft 2014–2019

Figure 17.17 Dryer symbols



Source: EdrawSoft 2014–2019

Figure 17.18 Heat exchanger symbols

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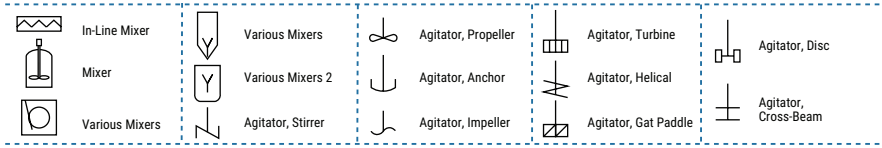
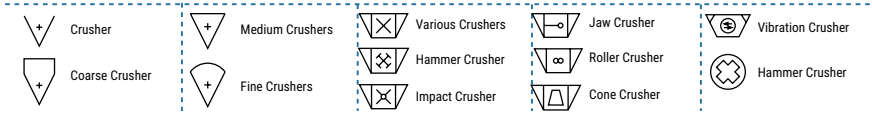


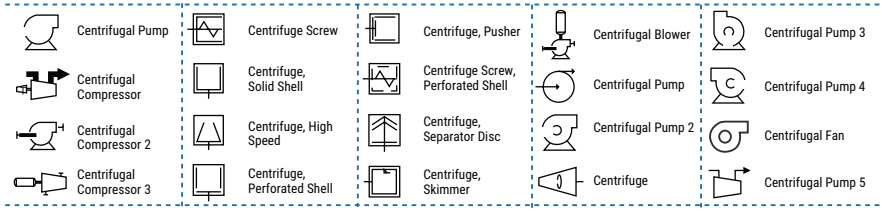
Source: EdrawSoft 2014–2019

Figure 17.19 Mixer symbols



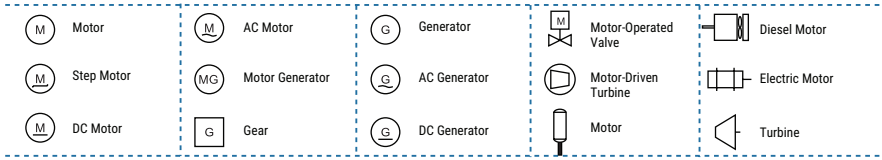
Source: EdrawSoft 2014–2019

Figure 17.20 Crusher symbols



Source: EdrawSoft 2014–2019

Figure 17.21 Centrifuge symbols



Source: EdrawSoft 2014–2019

Figure 17.22 Motor symbols

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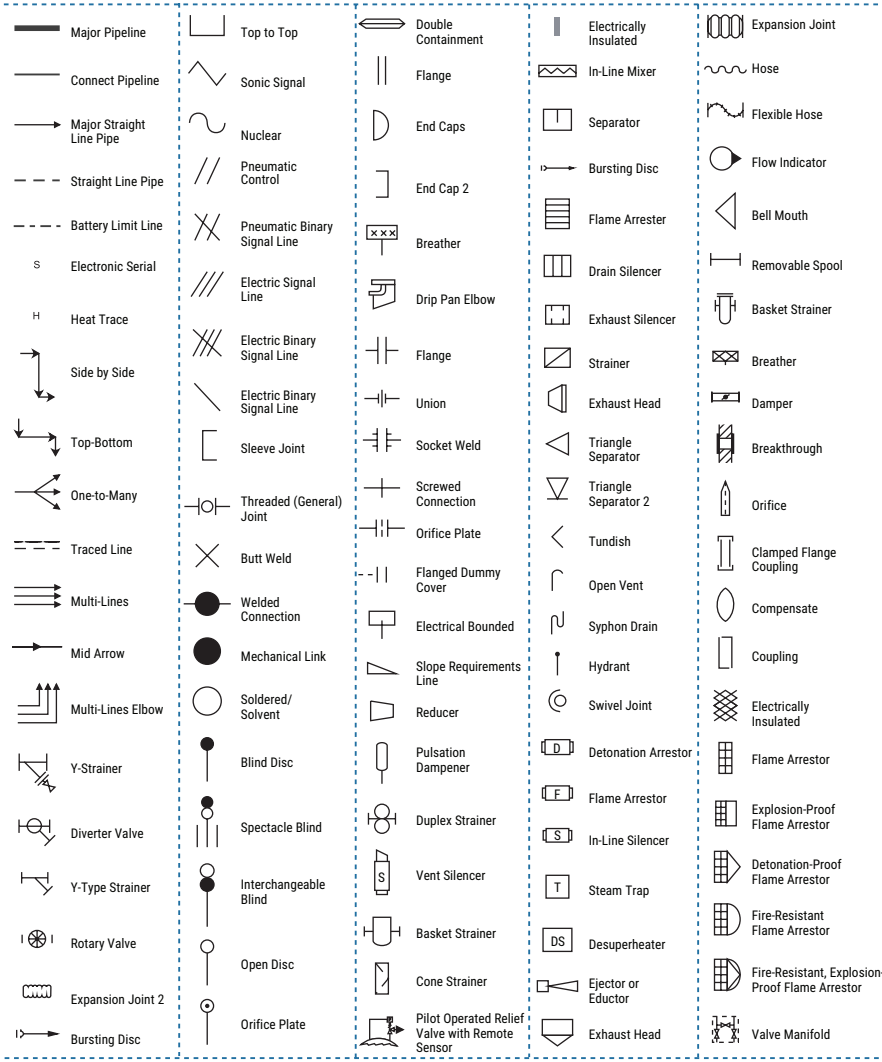

Source: EdrawSoft 2014–2019

Figure 17.23 Piping symbols and connecting lines

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ACKNOWLEDGMENTS

The author acknowledges Daniel F. Kump, P.E., who wrote the "Pumping" chapter in the previous edition of this handbook.

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CHAPTER

18

Placer Mining

Louis W. Cope, P.E. and Richard Jolk, P.E.

INTRODUCTION

Placer ore bodies are alluvial deposits that can contain the economic metals or minerals of gold, tin, magnetite, titanium, tungsten, zircon, garnet, diamonds, and semiprecious gemstones, to name the most common valuable constituents. These deposits are usually formed by deposition in rivers, on shorelines, or colluvially by in-place weathering.

Mining of the deposits can vary from small operations ranging from 1–5 yd^3/h by hand or mechanized digging with small machines. Operations ranging from 5–200 yd^3/h can be land-based, usually skid-mounted, or barge-mounted units with separate digging machines. These latter items can be bulldozers, front-end loaders, backhoe excavators, draglines, cutterheads, or suction machines. When the digging and recovery machinery are on the same barge with the recovery section, such as on a 200–2,000 yd^3/h floating dredge, the digging is either by a bucket ladder or bucket-wheel excavator.

Ores are usually treated in two steps. First the muck is slurried before it is screened in a rotating trommel. Recovery of target metals or minerals is normally by a combination of sluices, jigs, centrifugal concentrators, vibrating tables, or all four.

GEOLOGY

Placer deposits are formed through flowing and repetitive motion alluvial deposition of sediments (referring to stream/river placers and beach placers, respectively). Concentration of valuable metals and/or minerals in a placer deposit is attributable to the differences in specific gravity between the heavy (and usually the valuable) constituents and the host rock. Aeolian placer deposits have also formed but have only been known to contain constituents with a lower specific gravity (such as magnetite or hematite). Deposits of gold, silver (and combinations thereof), gemstones, chromite, garnet, zircon, and native copper (known as *exotic copper*) have formed as placers in alluvium.

Three common-sense exploration methods were presented by Black and Youngson (1999):

1. Location of previous workings. These are getting more difficult to find considering the relocating and rework of historic deposits having already occurred.
2. Placer–source relationships. As multiple sedimentary cycles and/or tectonic events alter the original depositional characteristics, this technique becomes more difficult.
3. Geologic similarities between prospective deposits and known placers. This method uses knowledge of the geology, but there are potential major differences in ore grade between known deposits versus prospective findings.

EXPLORATION

Exploration techniques used to find placer deposits generally rely on drilling at locations identified though a sound understanding of the regional and local geology. The relatively high specific gravity of metal values in placers requires extra care be taken when drilling and sampling. When practical, sonic drilling is commonly the best technique for taking composite samples of heavy mineral placers.

Details regarding exploration and drilling methods applicable to placer deposits can be found in the pertinent geology and exploration sections of this reference handbook.

MINE GEOLOGY

Efficient mining of placers requires a sound understanding of their size, shape, orientation, material types, rock strengths, and grade of values contained. Unfortunately, these parameters are often very difficult to quantify. Placers can be very large as in the Alaskan Yukon or very small as found in ephemeral streambeds.

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The shape of placers can be envisioned to some extent considering the characteristics of alluvial deposition. Placer deposits silhouette the channels of streams and rivers, the heavy minerals being deposited where the velocity of the fluid has dropped below the carrying velocity at which all the material is being conveyed.

Modern placers retain a near horizontal orientation whereas ancient placers may be tilted, faulted, overthrust, or rolled into anticlinal or synclinal structures, substantially tilted from horizontal.

Sediments forming a placer deposit may be loose like sand and gravel. Alternatively, the sediments may be compacted and/or cemented. Host rock associated with a placer can likewise be alluvially deposited, also being composed of loose, compacted, or cemented sediments. The base host rock in which the alluvial flowed can be made up of any rock type.

Drill recovery and proper assay of heavy mineral content is often poor, even using modern tools and techniques. Geostatistical analysis results in very short unpredictable ranges because of the nugget effect.

Ultimately, the delineation of placer deposits through extensive drilling and sampling programs is often more costly than delineation through active mining and processing of the excavated material.

It is most appropriate for the mining engineer (and metallurgist) to expect detailed ore volume, location, and grade information from the geologist. It should be fully understood to project management that the mining and processing of the materials excavated may not meet production expectations if such information is unavailable from the geologist or if the information is of less than sterling quality.

Management may decide to pursue mining as a lower cost alternative to the expense of additional detailed drilling, sampling, and assaying. This frequently happens considering how difficult it is to properly sample and evaluate a placer deposit. When this does occur, however, it must be understood that the mining engineer has been tasked with excavating material that may or may not be of ore grade.

PLACER MINING

The primary factors that affect placer mining are (1) the characteristics of the material being worked (deposit size, shape, orientation, material types, rock strengths, and grade of values contained); (2) the availability of power, water, equipment, materials, labor, infrastructure, and logistic support; and (3) project location and climate.

Placer mines, like all other modern mines, are designed to maximize return to the investor while minimizing risk. A reasonable mine life should be determined and a mine plan developed based on the best alternative mining method to be used.

Placer deposits are mined like all other sedimentary deposits. Consideration should be given to the continuity of placer structures and the associated ramifications to the mine plan. Placer mining is commonly thought of as a surface mining operation, either wet or dry, during which material is excavated and processed using common mining equipment. Dredging is also still extensively used in wet pits that produce sand and aggregate in addition to placer values. *Heat blankets* are employed at times to retain heat and ultimately increase minability in cold or frozen ground.

Conventional underground mining methods may be used when deposits of sufficient value and characterization are located well below the surface. Mining methods used in such instances likely would include some forms of cut-and-fill stoping or square setting, both methods reflecting the poor geotechnical characteristics of the sedimentary or conglomerate rock type.

Details regarding the mining methods applicable to specific placer deposits may be found in the pertinent mining sections of this reference handbook.

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PLACER PROCESSING

Gravity separation is used almost exclusively for the recovery of values from placer mining. The use of automated optical air sorters for specific minerals may be applicable to placers in some situations.

Details regarding processing methods applicable to placer deposits may be found in the pertinent mineral processing sections of this reference handbook.

PLACER ORE BODIES

Tables 18.1 through 18.3 show the grain size of the material, commonly found heavy minerals, and the approximate swell of earth and gravel after disturbance in placer deposits.

TABLE 18.1 Grain size of material in placer deposits

Names of Particles	Average Diameter, mm (in.)
Boulders	Greater than 256 mm (10 in.)
Cobbles	64 to 256 mm (2½ to 10 in.)
Pebbles	4 to 64 mm ($^{3}/_{16}$ to 2½ in.)
Gravel	Greater than 2 mm
Sand	2 to $^{1}/_{16}$ mm
Silt	$^{1}/_{16}$ to $^{1}/_{256}$ mm
Clay	Less than $^{1}/_{256}$ mm

Source: Wells 1969

TABLE 18.2 Common heavy minerals in placers

Mineral	Specific Gravity	
	From	To
Magnetite	–	5.2
Ilmenite	5.6	5.7
Garnet	3.2	4.3
Zircon	4.2	4.7
Hematite	4.9	5.3
Chromite	4.3	4.6
Epidote	3.2	3.5
Olivine	–	3.3
Limonite	3.6	4.0
Rutile	–	4.2
Pyroxene	–	3.3
Monazite	4.9	5.3
Platinum group metals	14	19

Note: This is a partial list, arranged in approximate order of frequency. Source: Wells 1969

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TABLE 18.3 Approximate disturbed swell of materials

Material	Loose		Bank	
	kg/m³	lb/yd³	kg/m³	lb/yd³
Basalt	1,960	3,300	2,970	5,000
Bauxite, kaolin	1,420	2,400	1,900	3,200
Caliche	1,250	2,100	2,260	3,800
Carnotite, uranium ore	1,630	2,750	2,200	3,700
Cinders	560	950	860	1,450
Clay, natural bed	1,660	2,800	2,020	3,400
Dry	1,480	2,500	1,840	3,100
Wet	1,660	2,800	2,080	3,500
Clay and gravel, dry	1,420	2,400	1,660	2,800
Wet	1,540	2,600	1,840	3,100
Coal, anthracite, raw	1,190	2,000	1,600	2,700
Washed	1,100	1,850	–	–
Coal, ash, bituminous	530–650	900–1,100	590–890	1,000–1,500
Bituminous, raw	950	1,600	1,280	2,150
Washed	830	1,400	–	–
Decomposed rock	–	–	–	–
75% Rock, 25% earth	1,960	3,300	2,790	4,700
50% Rock, 50% earth	1,720	2,900	2,280	3,850
25% Rock, 75% earth	1,570	2,650	1,960	3,300
Earth, dry packed	1,510	2,550	1,900	3,200
Wet excavated	1,600	2,700	2,020	3,400
Loam	1,250	2,100	1,540	2,600
Granite, broken	1,660	2,800	2,730	4,600
Gravel, pitrun	1,930	3,250	2,170	3,650
Dry	1,510	2,550	1,690	2,850
Dry, 6–50 mm (¼–2 in.)	1,690	2,850	1,900	3,200
Wet, 6–50 mm (¼–2 in.)	2,020	3,400	2,260	3,800
Gypsum, broken	1,810	3,050	3,170	5,350
Crushed	1,600	2,700	2,790	4,700
Hematite, iron ore, high grade	1,810–2,450	4,000–5,400	2,130–2,900	4,700–6,400
Limestone, broken	1,540	2,600	2,610	4,400
Crushed	1,540	2,600	–	–
Magnetite, iron ore	2,790	4,700	3,260	5,500
Pyrite, iron ore	2,580	4,350	3,030	5,100
Sand, dry, loose	1,420	2,400	1,600	2,700
Damp	1,690	2,850	1,900	3,200
Wet	1,840	3,100	2,080	3,500
Sand and clay, loose	1,600	2,700	2,020	3,400
Compacted	2,400	4,050	–	–
Sand and gravel, dry	1,720	2,900	1,930	3,250
Wet	2,020	3,400	2,230	3,750
Sandstone	1,510	2,550	2,520	4,250
Shale	1,250	2,100	1,660	2,800
Slag, broken	1,750	2,950	2,940	4,950
Snow, dry	130	220	–	–
Wet	520	860	–	–
Stone, crushed	1,600	2,700	2,670	4,500
Taconite	1,630–1,900	3,600–4,200	2,360–2,700	5,200–6,100
Top soil	950	1,600	1,370	2,300
Taprock, broken	1,750	2,950	2,610	4,400

Adapted from Caterpillar 2014

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PLACER EVALUATION

Tables 18.4 through 18.8 describe prospecting and sampling methods, as well as evaluation and effects of gold particles. Fluvial (water laid) placer deposits are anisotropic (highly elongated, ellipsoid). Drill hole spacing should reflect the shape. Optimum spacing can be estimated from a semivariogram. As shown in Table 18.5, downslope from the original ore deposits are the hillside or colluvial deposits, and below them are the alluvial deposits of redeposited minerals. Gold placers may extend for 40 to 50 miles along a river system (Bullock et al. 2011).

TABLE 18.4 Sampling methods

Metallic Minerals—Au, Sn, Pt
Deposits other than beach sand, thawed ground:
A. Depth to about 15 ft
1. Dry: Hand-dug pits
Power digger, square or cylindrical bucket
Occasionally placer churn drill, cased hole
2. Wet: Hand-dug pits with caisson or lagged lining
Placer churn drill, cased hole
B. Depth 15–200 ft
1. Dry or wet: Placer churn drill, cased hole (rarely open hole)
Becker percussion drill
Deposits other than beach sand, frozen ground:
A. Depths to 200 ft or more
1. Placer churn drill, open hole
2. Hand-dug shafts, usually little or no lagging
3. Becker percussion drill
Beach sand deposits (at or above present sea level):
A. Depth to about 15 ft—greater depth rarely encountered
1. Above water level: Hand auger
Occasionally placer churn drill
2. To below water level: Hand auger with pipe casing
Placer churn drill

Source: Daily 1973

TABLE 18.5 Character of gold versus distance from source

5 miles	Rough nuggety
8 miles	Small nuggety, waterworn
11 miles	Fine granular
25 miles	Fine scaly

Source: Wells 1969

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TABLE 18.6 Effect of a single gold particle on a sample

Size of Drill Hole or Channel, in.	Size of Gold Particle and Effect on Sample, $/yd^3		
	20 mesh (6.57 mg)	40 mesh (0.91 mg)	60 mesh (0.27 mg)
7½ diameter	0.58	0.08	0.025
5¼ diameter	1.18	0.16	0.05
3 diameter	3.60	0.50	0.14
3 × 6	1.42	0.02	0.06
6 × 6	0.71	0.10	0.03
6 × 12	0.35	0.05	0.015
12 × 12	0.18	0.025	0.0075
16-in. pan*	1.18	0.16	0.05

*180 pans/yd^3.

Note: Values shown are those that would result from one gold particle in a 1-ft sample increment or drive, and are based on gold weights determined by the author, with gold at $35/oz. Source: Wells 1973

TABLE 18.7 Classification of gold particles

- Coarse gold (nuggets), which remains on a 10-mesh screen (openings 1/16 in.)
- Medium gold (small nuggets), which passes 10-mesh and remains on a 20-mesh screen (openings 1/32 in.); value about 1 cent apiece
- Fine gold, which passes 20-mesh and remains on a 40-mesh screen (openings 1/64 in.); value about 1/5 cent per color
- Very fine gold, which passes a 40-mesh screen

Adapted from Boericke 1933

TABLE 18.8 Sample methods: advantages and disadvantages

Bulk Sampling

- **Advantages.** The advantages of bulk sampling include a good view of the gravel in place and knowledge of the amount of force required to excavate it. Some of the same digging machines used in bulk sampling are used in production. The large samples obtained minimize *nugget effect* and errors.
- **Disadvantages.** Because pits or trenches may not reach bedrock in all cases, bulk sampling is a serious disadvantage. It is often difficult to accurately measure a pit because of curves in the excavation, if by a backhoe, and sloughing from the sides. If groundwater is encountered, sample integrity is lost. Larger samples require larger transport and processing equipment, and the situation becomes more complicated. As unit costs of each sample are high, the exploration cost to delineate reserves is very expensive.

Drill Sampling

- **Advantages.** Historically, drilling has been the sample method of choice and has been proven by operation in most cases. Drilling gives greater coverage in developing gravel grade and reserves at less unit cost, the profile of bedrock, and indications of enriched and barren areas.
- **Disadvantages.** Drilling gives a small sample that aggravates the nugget effect. At times, boulders will necessitate abandonment of uncompleted drill holes. Vibration from driving casing or from a reverse-circulation hammer drill can cause gold to migrate downward into the gravel. Drill rigs require at least minimal access roads and drill pads.

Source: Cope 1992a

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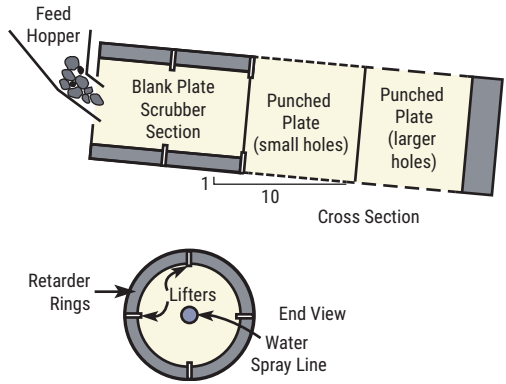
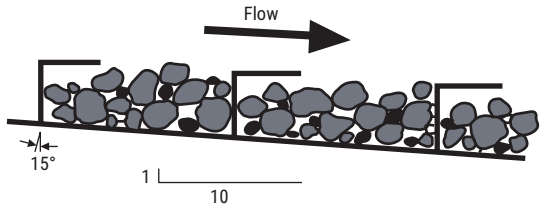
PLACER OPERATIONS

Placer ore is unconsolidated material, usually found on the surface. It is excavated by many means including pick and shovel, dragline, front-end loader, backhoe, bulldozer, bucket ladder, bucket wheel, hydraulicking, and suction. After it is excavated, the target valuable minerals are recovered using various processing methods (Figures 18.1–18.6). Tables 18.9 through 18.12 show concentration and treatment methods, as well as water requirements in placer operations.



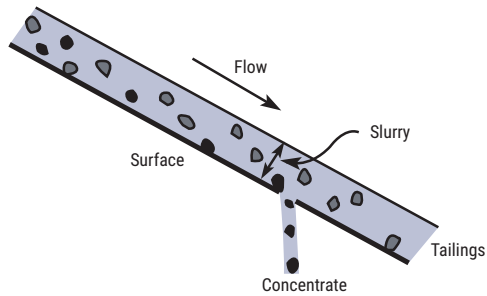
Source: Cope 1992b

FIGURE 18.1 Elements of a scrubbing/screening trommel



Source: Cope 1992b

FIGURE 18.2 Cross section of a sluice with Hungarian riffles



Source: Cope 1992b

FIGURE 18.3 How spirals and cones work

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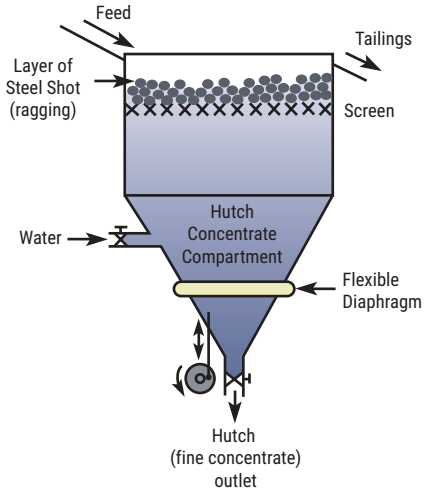
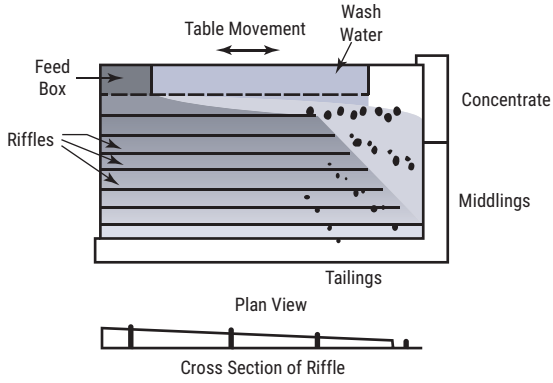


Source: Cope 1992b

FIGURE 18.4 Components of a jig



Source: Cope 1992b

FIGURE 18.5 How a vibrating table separates minerals

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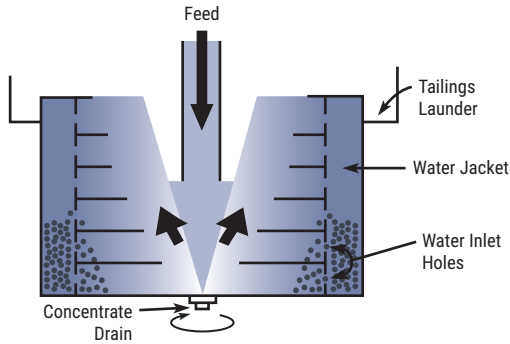

Source: Cope 1992b

FIGURE 18.6 Cross section of a centrifugal separator

TABLE 18.9 Methods of placer deposit concentration

	Methods of Concentration	
	Wet Processing	**Dry Processing**
Tin, phosphate, precious metals	Primary	–
Heavy mineral sand deposits such as ilmenite, rutile, zircon	–	Primary
Sizing	Unlike most mineral processing in which comminution is required to achieve a specific liberation size, particles generally have been already liberated by the time placer deposits are being processed.	
Sizing	Cyclones to de-slime and remove clays. Attrition scrubbers to remove clay coatings on particles.	Trommel screens to remove large particles
Sizing	Hydraulic classifiers, wet vibratory screens	Vibrating screens
Gravity separation	Spiral classifiers, hydraulic classifiers, centrifugal jigs, and wet shaking tables	–
Magnetic separation	Wet magnets	Dry magnets
Electrostatic separation	–	High-tension rolls, electrostatic plates
Flotation	Used to separate silica from phosphate	–

Adapted from Richards 2011

TABLE 18.10 Water requirements

Mining Method	Water Requirements
Rocker boxes	4–5 gpm, 50–100 gal/yd^3
Small-scale hand mining	170–225 gpm for steep 12-in.-wide sluice
Ground sluicing	22,000–162,000 gal/yd^3
Hydraulicking	2,000–32,000 gal/yd^3
Stationary washing plants	650–2,000 gal/yd^3
Land-based mobile plants	480–3,200 gal/yd^3
Floating dragline-fed dredges	570–2,500 gal/yd^3
Bucket-line dredges	3,500–10,000 gal/yd^3

Adapted from Wells 1969

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TABLE 18.11 Water requirements, feed rates of a standard sluice box

Gold Particle Size	Water, L/s/m of width	Material feed, $m^3/h/m$ of width
Less than 1 mm	40	0.75
Greater than 1 mm	80	0.75

Adapted from Dunne et al. 2018

TABLE 18.12 Treatment methods: advantages and disadvantages

Method	Advantages	Disadvantages
Sluice	Simple Inexpensive Construction materials available everywhere No power required	Reduced gold recovery Time-consuming to clean up Low capacity
Fabric	Simple Inexpensive No power required Improved fine gold recovery Best used in conjunction with a normal sluice	Low capacity
Amalgamation	Simple Inexpensive No power required Recovers very fine gold Best used for final cleanup	Low capacity Cost of makeup of lost mercury Legal and moral constraints Not all gold will amalgamate
Thin film	No moving parts in separating machine Recovers fine gold	Requires screening to fine size High headroom required Pump and piping wear
Jig	Feed-size forgiving Feed-rate forgiving Requires little operator attention Recovers fine gold	Complicated machine Several adjustments
Vibrating table	Can produce high-grade gold concentrate Takes jig concentrate as produced Best for cleanup Separation clearly visible	Requires screening to fine size Requires considerable operator attention
Bowl	Recovers fine gold Requires little operator attention Long history of use in cleanup Longer run gives better concentrate	Requires screening to relatively fine size Has to be stopped to be cleaned up Requires absolutely clean water (for water-injection models)
Pan	Minimum equipment required Simple Inexpensive Often used in last stage of cleanup Best used for prospecting	Very low capacity Hard work

Source: Cope 1992b

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COSTS

An example for heavy mineral extraction cost is given in Table 18.13. And other costs are shown in Table 18.14.

TABLE 18.13 Mineral extraction costs*

Area	Ore, US$/ton	Heavy Mineral Concentrate, US$/ton†
Mining (by dredge)	0.60	5.10
Wet concentrator	1.40	13.45
Mineral separation plant	0.60	6.50
Product handling and delivery	0.95	10.95
General and administrative	0.35	3.75
Concurrent reclamation	0.10	1.05
Total operating cost	4.00	40.80

* Feed rate = 1,100 tons per hour to the wet concentrator.

† Ore heavy mineral grade 12.5%.

Courtesy of Outotec

TABLE 18.14 Bucketline dredge capital costs*

Bucket Capacity		Cost US$ (millions)
m^3	ft^3	
0.283	10	9
0.425	15	16
0.566	20	23
0.850	30	50

*Not including transportation, insurance, or duties.

Source: McLean 1992

GRAVEL

See Table 18.15 for general gravel information.

TABLE 18.15 Useful gravel facts

- One gold pan typically has a diameter of 10–17 in.
- One gold pan can accommodate 0.01–0.10 ft^3 of gravel.
- One cubic yard of gravel weighs about 2,600 lb.
- One person can shovel up to 16 tons of coal or 10 tons of gravel per day.

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CHAPTER 19

In Situ Leaching

Paul D. Chamberlin, P.E., Ray V. Huff, Bibhuti B. Panda, P.E., and Joseph R. Stano, P.E.

In situ leaching (ISL) generally means leaching undisturbed ore rather than in-place rubble. It does not imply heap or dump leaching. Commercial operations have been used for uranium, copper, boron, trona, nahcolite, and potash. Potential operations are possible for precious metals, manganese, and some industrial minerals. Sulfur is not leached in situ; it is melted.

Ore and Deposit Characteristics

Many deposits are formed by precipitation of values from either percolating meteoric or upwelling fluids. Thus, the deposit has or had both porosity and permeability (Tables 19.1 and 19.2).

Shear zones have higher values than fractured crystalline ores. Permeability is a property of the porous rock. Hydraulic conductivity is a property of the rock pores and viscosity of the fluid as a system. Porosity is the ratio of volume of void to total volume of the rock.

TABLE 19.1 Approximate ranges of porosity and permeability

Permeability	Pervious				Semipervious					Impervious			
Unconsolidated sand and gravel	Well-sorted gravel		Well-sorted sand or sand and gravel		Very fine sand, silt, loess, loam					–			
Unconsolidated clay and organic	–				Peat		Layered clay			Unweathered clay			
Consolidated rocks	Highly fractured rocks				Oil reservoir rocks			Fresh sandstone		Fresh limestone, dolomite		Fresh granite	
k (cm)^2	0.001	0.0001	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}	10^{-13}	10^{-14}	10^{-15}
k (millidarcy)	10^{+8}	10^{+7}	10^{+6}	10^{+5}	10,000	1,000	100	10	1	0.1	0.01	0.001	0.0001

Adapted from Bear 1988

TABLE 19.2 Rock type porosity (typical)

Rock Type	Porosity or Void Volume, %
Unconsolidated deposits	
Gravel	25–40
Sand	25–50
Silt	35–50
Clay	40–70
Rocks	
Fractured basalt	5–50
Karst limestone	5–50
Sandstone	5–30
Limestone, dolomite	0–20
Shale	0–10
Fractured crystalline rock	0–10
Dense crystalline rock	0–5

Adapted from Freeze and Cherry 1979

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The permeability values of the deposit must be in flow channels (not encapsulated). Ideally, the deposit should have an ore zone beneath the water table, uniformly high permeability, low porosity, be bounded by low permeability formation(s), and have values lining the flow channels.

Permeability Assumptions

Permeability

- Varies with depth,
- Depends on porosity,
- Depends on the connectivity of the flow paths in the rock,
- Depends on the fracture geometry parameters (length, spacing, orientation, and aperture), and
- Is a directional quantity that can be affected by heterogeneous or directional properties of the pore geometry.

In general, ISL is suitable to sedimentary hosted deposits that meet the following well-defined conditions:

- Confined aquifer hosting the deposit with no critical hydrologic connectivity to adjacent formations
- Effective (free-fluid) ore porosity and corresponding permeability
- Appropriate mineralogy and geochemistry, enabling the effective leaching of metals of interest with regard to both thermodynamic conditions and kinetic rates

Lixiviants

Gangue minerals may dictate the nature and concentration of lixiviant.

- Uranium—carbonate/bicarbonate with an oxidant (O_2 or peroxide)
- Oxide copper—dilute sulfuric acid
- Sulfide copper—sulfuric acid with oxidant, often ferric sulfate, $Fe_2(SO_4)_3$, or iron chloride, $FeCl_3$
- Boron—hydrochloric acid
- Precious metals—complexing agent (CN^- or Cl^-) and oxidant (dissolved O_2 or OCl^-)
- Manganese—reducing acid (water with sulfur dioxide [SO_2])

Wells and Well Fields

Hydrogeological conditions for efficient ISL programs include the following:

- Sufficient hydraulic head above the extractor screens (in the order of 10–100 m)
- Adequate thickness (up to 10 m) of the mineralized zone (too low thickness impedes targeting; too high thickness requires staged screening of wells)
- Moderate depth of deposit from the surface (costs for drilling and well construction)
- No or minor interference by impermeable intersections (e.g., shale, clayey layers, cemented sediments) that could disturb the hydraulic connection between injection and production wells

Factors affecting the ISL operation include the following:

- Deposit morphology
- Optimization of well field design
- Deposit heterogeneity with respect to hydraulic permeability and ore morphology

The following conditions are required for successful ISL mining, especially for uranium (U) mining:

- The ore body must be located below the water table.
- The ore deposit geometry must be of sufficient size and shape to allow circulation of lixiviant through injection and extraction wells.

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- The host rock must be sufficiently permeable to allow the lixiviant to pass through the formation.
- The formation must be confined so that lixiviant flow can be contained and directed by injection and extraction wells.
- The mineralogy must be such that U can be extracted from the ore material by the relatively benign oxidants and complexants used in ISL mining.

Regional-Scale Hydrogeological Model

The setup of regional-scale hydrogeological models with embedded well fields requires the specific data input and procedures (calibrations) for reliable simulations. The appropriate representation of the regional geological stratigraphy is fundamental. Additionally, possible hydraulic connectivity between strata and along tectonic faults or thinning of confining layers should be considered. Input data can be derived from geophysical surveys (in particular, seismic) and borehole data (logging data, core assays). The permeability and porosity of all relevant formations (strata) including the mineralized ore zone are deduced from

- Geophysical borehole logging,
- Laboratory assays of available core samples,
- Fracture data from core logs, and
- Aquifer pumping tests.

The initial hydraulic head data (potentiometric surfaces) are used for the calibration of the hydrological model and for simulating natural groundwater flow (baseline conditions). The anisotropy of permeability (lateral vs. vertical) is quite significant and should be determined. All relevant boundary conditions need to be appropriately considered, in particular, sink and source terms that contribute to the groundwater flow regime such as recharge/discharge, precipitation/evaporation, pumping/injection, and surface water boundaries.

Wells and Well Fields

Shallow wells (<300 m) can be substantially less costly when low-cost materials of construction can be used. An inverted five-spot well pattern, consisting of one injection well and four production wells, is common. This provides wells at the perimeter of the well field, which maximizes the recovery of pregnant solution, although the solution grade is somewhat diluted.

Other well patterns are line drive and inverted four-spot, seven-spot, and nine-spot. Well spacing is optimized by iterative designs, considering the cost of wells and predicted values recovered per five-spot unit.

The approximate flow rate in the five-spot pattern is

$$q = \frac{1.035 \times 10^{-4} kh\Delta p}{\mu\left[\ln\left(\frac{d}{rw}\right) - 0.619\right]} \qquad \text{(EQ 19.1)}$$

where

q = flow rate, gpm
k = permeability, mD (millidarcy)
h = vertical ore interval open to flow, ft
Δp = applied differential pressure, psig
μ = viscosity of the fluid, cP (centipoise)
d = spacing between injection and production wells, ft
rw = well bore radius, ft

Materials of construction are fiber-reinforced plastic (FRP) for high-pressure and corrosive environments, other plastic materials for lower pressure and less corrosive environments, and stainless steels or other metallics as required for pumps or valves.

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Operating Considerations

- Annual plant production rate depends on the grade of pregnant solution and the number of producing wells.
- Well flow rate mostly depends on the permeability of the ore zone, the differential pressure between injection and production wells, the spacing of the wells, and the thickness of the ore interval open to flow.
- Injection pressure is limited by rock parting pressure.
- Δp is related to injection pressure and the drawdown of fluid in production wells.
- Equations describing flow from various well patterns can be found in water flood manuals published by the Society of Petroleum Engineers.

Troubleshooting

- Look for reduced flow rates from production wells over time.
- Look for increasing injection pressures over time.
- Look for unexpected decreases in pregnant solution grade over time.
- Data collection includes fluid injection rates and pressures for each well, fluid production rates from each well, and pregnant solution grades from each well.
- Diagnosing most troubles often requires down-hole wireline measurements.

IN SITU LEACHING OF URANIUM

Following are the specifics of in situ leaching of uranium, including environmental concerns.

Deposit Characteristics

- Roll-type sandstone deposits (redox fronts) are typical.
- Ore beds usually are unconsolidated or loosely cemented with calcium carbonate.
- Typical depths are from 45 to 137 m with the maximum at 304–609 m.
- Typical ore grades are between 0.04% and 0.2% triuranium octoxide (U_3O_8).
- Predominant ore minerals are uraninite, coffinite, and sometimes carnotite; radioactive disequilibrium is relatively common.
- Common accessory minerals are quartz, feldspars, clays, calcite, pyrite, and carbonaceous material.
- Less common minerals are jordisite; the vanadates, selenates, arsenates, and humates; and hydrogen sulfide, and methane.
- Typical hydrology is in confined aquifers, with the leach zone preferably below the water table.
- Typically, 10%–30% open porosity with permeability of 50–1,000 millidarcies is seen.
- Typical reserves are 10,000,000–30,000,000 lb U_3O_8 with isolated pods less than 1,000,000 lb.

Wells and Well Fields

- Modified water well designs are used. Usually, they are rotary-drilled and constructed with plastic casings (polyvinyl chloride [PVC] and FRP) cemented back to the surface with single or multiple completions. The most common casing diameter is nominal 4 in.; 6 in. is sometimes used in production wells and for deep completions.
- Well completions—injection and production wells are perforated in the selected ore zone(s) and usually lined with stainless well screens.
- Typical well field spacings are 15.2–61.0 m (50–200 ft).
- Predominant well field configurations are five-spots and line drives.
- Monitor wells are completed in upper and sometimes lower aquifers with provisions for water level measurements and water sampling.
- Typical single well yields are 19–38 L/min (5–10 gpm).

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In Situ Leaching Process

Most operations are continuous with injection of fortified barren leach solution. The flow through the ore is controlled by induced pressure gradients and well spacings. Uranium dissolution is accomplished; and submersible pumps are used to withdraw the pregnant liquor from the recovery wells. On completion of leaching, groundwater goes through chemical restoration.

The typical concentration of uranium in the pregnant leach solution is 35–200 ppm U_3O_8, with peaks as high as 1,000 ppm. Injected leach solutions have low concentrations of oxidant (50–500 ppm) and lixiviant (500–2000 ppm).

Oxidation of the uranium minerals is the rate-controlling step. Dissolved oxygen is the most common oxidant; hydrogen peroxide is used in shallow deposits (<76 m) and to accelerate the leach.

The primary lixiviant is a variable mixture of sodium carbonate and bicarbonate adjusted with carbon dioxide to regulate the pH between 7 and 9. Ammonium carbonate was used in early projects. Leaching has been conducted successfully with pH levels as high as 10. Lower pH levels tend to dissolve more calcium carbonate mineral, which is present in most roll-front deposits.

Sulfide minerals (such as pyrite and jordisite) in the ore tend to react with the oxidant to produce acid and lower the pH of the pregnant leach solution. Oxidation of sulfide minerals can mobilize molybdenum, arsenic, and polythionates. Radium is generally mobilized. Radon gas is entrained in leach solutions and provisions are made for in-plant ventilation.

Hydrogen sulfide is sometimes present in leach liquor. Principal leaching reactions are

$$2UO_2 + O_2 = 2UO_3$$

$$UO_3 + 2HCO_3^- = UO_2(CO_3)_2^{-2} + H_2O$$

$$UO_3 + CO_3^{-2} + 2HCO_3^- = UO2(CO_3)_3^{-4} + H_2O$$

$$2CaCO_3 + CO_2 + H_2O = 2Ca(HCO_3)_2$$

$$FeS_2 + O_2 = Fe^{+2} + SO_4^{-2}$$

Plant Recovery Process

- Pregnant solution is pumped from the well field(s) to a plant feed tank and clarified with sand filters and cartridge filters.
- Uranium is extracted with anion exchange resins (capacity 16–80 kg U_3O_8 per cubic meter).
- Ion exchange (IX) equipment can be either fixed bed or moving bed systems.
- Resins are stripped with alkaline or acidic chloride brines, which concentrates the U_3O_8 5–20 times.
- Eluate is neutralized with acid.
- Uranium is precipitated with ammonia, sodium hydroxide, or hydrogen peroxide to produce a slurry of ammonium diuranate, sodium diuranate, or uranyl peroxide.
- Uranium precipitate is dewatered by thickening and centrifugation (or alternately filtration) to produce a moist yellow cake.
- Cake is dried or calcined to a yellow cake product that is actually yellow, gray, or black, depending on calcining temperature. The product contains from 71% to 95% U_3O_8.
- Barren leach solution is replenished with leach chemicals and reinjected.

The principal recovery reactions are

Na diuranate: $8Na^+ + 2UO_2(CO_3)_3^{-4} + 6NaOH = Na_2U_2O_{7(solid)} + 6Na_2CO_3 + 3H_2O$

NH_4 diuranate: $2UO_2^{+2} + 6NH_4OH = (NH_4)_2U_2O_{7(solid)} + 4(NH_4)^+ + 3H_2O$

Uranyl peroxide: $UO2^{+2} + H_2O_2 + 2H_2O = UO_42H_2O_{(solid)} + 2H^+$

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Environmental Concerns

- Dispose of excess leach solution by solar evaporation or reverse osmosis.
- Dispose of (1) scale residues and plugged filter elements (both radioactive); (2) by-product precipitates, such as molybdenum sludges; (3) surplus IX brines; and (4) groundwater restoration residues.
- Restore well fields by groundwater flushing with concurrent aboveground separation of dissolved solids.
- Accomplish aboveground radium removal with selective IX resins or barium chloride precipitation.
- Impound separated brines.
- Fix uranium and molybdenum underground.

Operating Problems and Remedies

- Plugged wells are usually caused by scale buildup on well screens and relieved by acidification with hydrochloric acid. Bacterial plugging can be relieved by injecting hypochlorite. Severe cases may also require stimulation by surging and jetting.
- Induced formation fracturing generally is self-healing after pump pressure is relieved.
- For IX resin fouling, perform the following steps:
  1. Perform an acid wash to remove calcium scale on resin beads.
  2. Oxidize hydrogen sulfide, polythionates, and thiomolybdates in the pregnant leach solution so they will not poison the resin; beware that too much oxidant can ruin resins.
  3. Pre-extract selenium in a sacrificial resin column.
  4. Condition the resins with chemicals.
  5. Replace any irreversibly fouled resins (Mo can poison resin).
- For premature declines and abrupt drops in preg grade
  1. Check assays.
  2. Check injection chemical concentrations.
  3. Check injection pH.
  4. Check for plugged wells.
- For gas blockage, temporarily curtail or reduce oxidant input.
- To protect well field surface piping from freezing, install fast drainage devices and blow out with compressed carbon dioxide.

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CHAPTER

20

Mineral Processing

Paul D. Chamberlin, P.E.

COMMINUTION by D. Erik Spiller

Mineral processing is the liberation and separation of minerals. It extends from metal-bearing to industrial to energy and fertilizer minerals, and finally to modern industrial and household recycling. *Comminution* is the umbrella term for *liberation*, and especially when used in reference to brittle rocks as normally encountered in the realm of metal-bearing ores. Comminution is comprised of a series of unit operations wherein mined ore is processed into finer sizes resulting in an eventual effective economic liberation of the contained values. Many avenues lead to liberation as shown in Figure 20.1.

Mined ore is first delivered to primary crushing from which it is reduced in size following various commercially accepted methodologies. For example, primary crushing most commonly done in gyratory or jaw crusher machines, followed by grinding in a semiautogenous grinding (SAG) mill, followed by further grinding in a ball mill produces liberated ore minerals. However, recent advances in high-pressure grinding roll (HPGR) technology and stirred media grinding are providing new options for comminution and liberation.

PARTICLE CHARACTERIZATION by James D. Newman, P.E.

Particles are characterized by physical properties (size, shape, density, surface area, hardness, conductivity, magnetism, permeability, porosity, color, tenacity, and luster), chemical properties (chemical composition, solubility, reactivity, vapor pressure), and mineralogical properties (mineral species present and proportions of mineral phases).

Blasting, material transfer, crushing, and grinding cause particle size reduction. The decrease in particle size causes increased liberation and particle surface area. Two methods in determining size distribution are Schuhmann (1940) and Rosin–Rammler (Rosin and Rammler 1933). Schumann is best used for coarser product sizes:

$$W_p = 100\left(\frac{x}{k}\right)^m \% \qquad \text{(EQ 20.1)}$$

where

W_p = weight percent passing
x = size
k = size modulus, size at W_p = 100% passing
m = distribution modulus

Rosin–Rammler was originally developed for coal:

$$W_r = 100e^{\left[-\left(\frac{x}{a}\right)^b\right]} \% \qquad \text{(EQ 20.2)}$$

where

W_r = weight percent retained
x = size
a = size modulus, size at W_r = 36.8% particles retained
b = slope constant from $\ln(100/W_r)$ versus $\ln x$

Throughout the process, there is a constant need for metallurgical accounting to track and determine the feed, product, and recirculation. To balance the volume of slurry, the following equation can be used:

$$\frac{M_{\text{slurry}}}{\rho_{\text{slurry}}} = \frac{M_{\text{solid}}}{\rho_{\text{solid}}} + \frac{M_{\text{liquid}}}{\rho_{\text{liquid}}} \qquad \text{(EQ 20.3)}$$

where

M = mass
ρ = density

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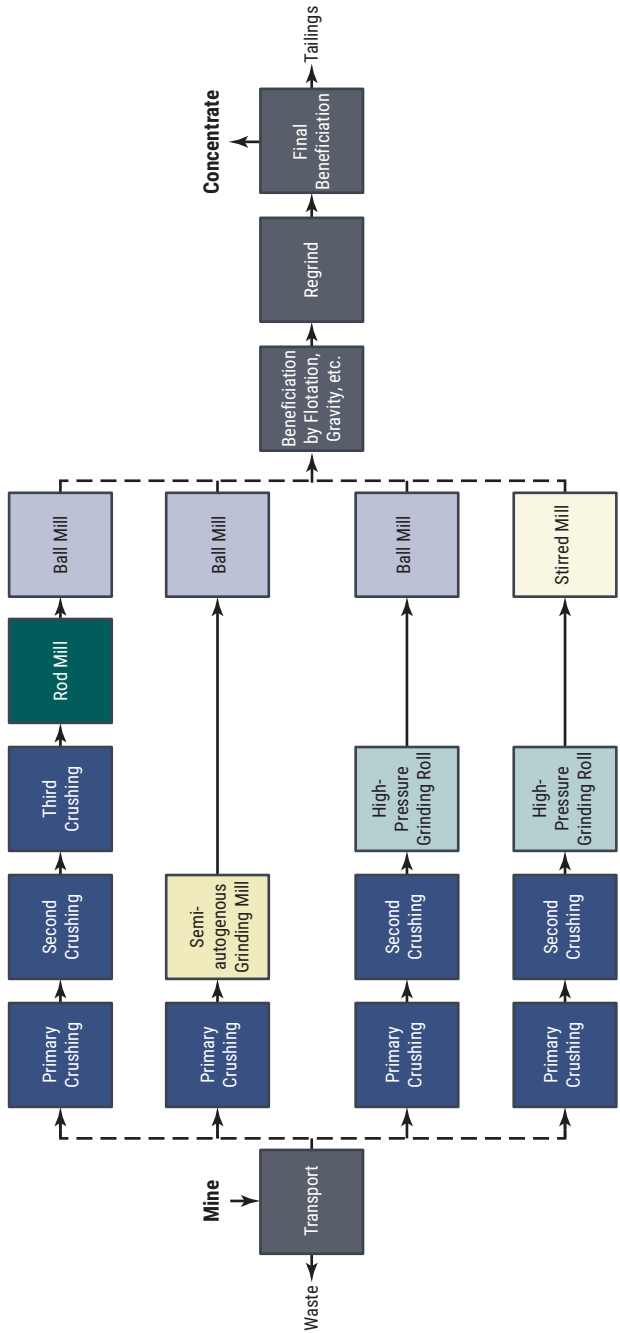

FIGURE 20.1 Common pathways in mineral comminution and separation

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CRUSHING by Paul D. Chamberlin, P.E.

Crusher Feed

Crushers work best with a constant feed rate, and, being choke fed, surge capacity may be needed between crushing stages. In addition, all crushers work best when fines are removed, including product size and smaller. Screens can be used ahead of crushers to remove fines to improve crusher performance. Changing the shape of the crushing chamber in jaw and cone crushers to suit the size distribution of the ore offers better wear and energy usage rates.

$$\text{circulating load (CL) of original feed (\%)} = \frac{100}{\left(\frac{e}{r}\right) - 1} \quad \textbf{(EQ 20.4)}$$

where

e = screen efficiency, %
r = oversize in crusher product, %
CL = (total feed to crusher) – (original crusher feed), t

When operating a crusher in a closed loop with a scalping screen, the circulating load ratio is calculated as

$$\text{CLR} = \frac{U}{O} = \frac{(f - o)}{(u - f)} \quad \textbf{(EQ 20.5)}$$

where

CLR = circulating load ratio
U = screen underflow, stph (short tons per hour)
O = screen overflow, stph
f = feed weight percent, %
o = overflow weight percent, %
u = underflow weight percent, %

This formula is more accurate for +12 mm crushing and less accurate for –1.7 mm crushing. Rod mill feed should be finer than F_{80} = 33 mm. Primary ball mill feed should be finer than F_{80} = 12.7 mm. Crusher discharge is typically 100% passing two times the closed side setting (CSS); for example, if CSS = 12.7 mm, then the discharge will be 100% –25.4 mm.

Types of Crushers

The following discusses the usual types of crushers used in mining: primary, secondary, tertiary, and quaternary. The types apply to the size reduction brought about by crushing. A mining operation commonly requires several crushing steps to reduce particle size to the targeted size for input into downstream processing. Table 20.1 shows the maximum feed sizes and reduction ratios for primary through quaternary crushing.

$$\text{reduction ratio} = \frac{F_{80}}{P_{80}} \quad \textbf{(EQ 20.6)}$$

TABLE 20.1 Crusher types

Type	Primary	Secondary	Tertiary	Quaternary
Maximum feed size, in.	60	25	10	3
Recommended minimum product, in.	6–4	4–¾	1–⅛	½–20 mesh
Reduction ratio	4:1–6:1	4:1–6:1	4:1–6:1	—

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Jaw

Jaw crushers can be used for low-tonnage applications. A general estimate for jaw crusher capacity is

$$Q = 1.5F \quad \text{(EQ 20.7)}$$

where

Q = quantity, m^3/h
F = discharge area at open setting, cm^2

The typical reduction ratio is 3:1 to 4:1, but it can be 6:1 to 8:1. If the abrasion index is >0.7, then a double-toggle style crusher should be used. The maximum feed size is approximately 90% of the feed opening for single-toggle jaw crushers and approximately 80% for double-toggle jaw crushers.

Cone and Gyratory

The gyratory reduction ratio is usually 3:1 or 4:1, but it can be 8:1 or 10:1. The largest rock in the feed should be 80%–90% of the open side setting (OSS) of the feed opening. Fifty percent of the feed passing the midpoint setting of the crusher chamber is ideal. Problems occur when >80% of the feed passes the midpoint. The midpoint opening is about 0.5 times the closed side receiving opening plus CSS. The midpoint opening should be choke fed for lowest steel consumption, greatest reduction ratio, and lowest kilowatt hours per ton. The circulating load is reduced by using a CSS slightly smaller than the screen's opening. Normally, 60% of the crusher discharge is less than that of the CSS.

Bearing damage in some cone crushers can be avoided by dissipating the bearing heat through idling the crusher without feed for 10–15 minutes before shutdown. On cold start-up, the crusher can be idled without feed for 10 minutes, and then the clamping pressure can be slowly stepped up over a period of 20 minutes. Motors should have reduced voltage starters (65% of voltage and 42% of amperes); otherwise, inrush amperes may be 5.6 times the rated amperes.

Liner consumption for cone crushers is given by the following equation (A_i values are given in Table 20.2). The type of cone crusher is not a factor.

$$\text{liner consumption } \left(\frac{\text{lb}}{\text{ton}}\right) = 0.043\, Ai^{1.255}\, h_d^{0.254} \quad \text{(EQ 20.8)}$$

where

lb = total weight of the liners including unused waste portion at the end of liner life
h_d = head diameter of the cone crusher, in.

TABLE 20.2 Work index and abrasion indices (averages)

Material	W_i	A_i	Material	W_i	A_i
Amphibolite	16 ± 3	0.2–0.45	Hornfels	18 ± 3	0.7 ± 0.2
Andesite	16 ± 2	0.5 ± 0.1	Limestone	12 ± 3	0.001–0.03
Basalt	20 ± 4	0.2 ± 0.1	Magnetite ore	8 ± 3	0.2 ± 0.1
Diabase	19 ± 4	0.3 ± 0.1	Marble	12 ± 3	0.001–0.03
Diorite	19 ± 4	0.4	Porphyry	18 ± 3	0.1–0.9
Dolomite	12 ± 3	0.01–0.005	Pyrite ore	10 ± 3	0.6 ± 0.2
Ferrosilicon	11 ± 2	0.25 ± 0.7	Quartzite	16 ± 3	0.75 ± 0.1
Gabbro	20 ± 3	0.4 ± 0.1	Sandstone	10 ± 3	0.1–0.9
Granite	16 ± 6	0.55 ± 0.1	Syenite	19 ± 4	0.4 ± 0.1
Hematite ore	11 ± 3	0.5 ± 0.3			

Source: Ottergren and Steer 1996

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Vertical Shaft Impactor

For a vertical shaft impactor (VSI), the feed should be <5 cm to avoid high wear on the rotor tips. The minimum product size is typically <3.3–1.7 mm. VSIs should operate in a closed circuit with very efficient screening to remove as much of the fines and moisture as possible from the crusher feed. Fines make spongy rock beds, reduce crushing efficiency, and increase power consumption; and clay lumps and wood plug the crusher. To control dusting, water can be added. VSIs need a constant feed rate. The Bond work index is not applicable to VSIs. A larger-diameter rotor creates more fines (attrition) and increases power draw; extra throughput can be obtained by staging reduced portions of the feed. Typically, circulating loads are approximately 300%. Wear steel costs from about $0.04 to $0.80 per ton. The abrasion index should be <0.15.

Horizontal Shaft Impactors

Horizontal shaft impactors, also called *hammer mills*, are used for low abrasive or friable ore. Some machines have internal rock pockets, so crushing is mostly autogenous; others depend on anvils (rock on steel). Reduction ratios are high (up to 35). Hammer mills work on wet, sticky ores.

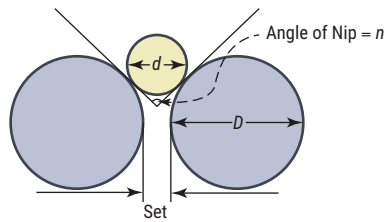
Rolls

Roll crushers produce breakage mostly by compression and some by shear. Single, dual, or quad rolls are available. Ore should be <172,000 kN/m^2 (<25,000 psi) compressive strength. They can be toothed or smooth. Figure 20.2 shows the angle of nip and the following equation relates the size of particle, roll, angle of nip, and the set size.

$$\cos\frac{n}{2} = \frac{D + \text{set}}{D + d} \qquad \textbf{(EQ 20.9)}$$

where

n = angle of nip (as shown on the figure)
D = roll diameter
d = diameter of particle



Source: Blake and Komarek 1985

FIGURE 20.2 Angle of nip

Other Considerations

- The coarse ore storage capacity should be about 2 days.
- The feed pocket ahead of the primary crusher should have a capacity of at least two times that of the ore truck.

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GRINDING by Paul D. Chamberlin, P.E.

Blasting, followed by crushing, is the most energy efficient size-reduction technique. Grinding is the least effective method for breaking rock, but it is generally needed to achieve the liberation size required in mineral processing. When grinding is necessary, the following should be done:

- Remove fine material from a grinding machine as quickly as possible.
- Consider screens instead of cyclones on mill discharges.
- Replace constant-speed cyclone feed pump motors with variable frequency drives (VFDs).
- Incline cyclones to reduce pumping head.
- Gradually return collected spills to the circuit to minimize process upsets.
- Consider VFDs on autogenous and semiautogenous grinding mills to accommodate variations in feed properties such as hardness, feed rate, and particle size distribution.
- Be aware that chemicals such as surfactants and dispersants may reduce grinding power.

Types of Grinding Mills

Grinding mills for mineral processing are of two basic types: steel rods that tumble and size rock in a rotating drum, and ball mills that tumble and size rock in a rotating drum. Characteristics are discussed in the following subsections.

Rod and Ball Mills

The following formulas dictate the power required to reduce material in rod and ball mills.

$$W = \frac{10W_i}{\sqrt{P_{80}}} - \frac{10W_i}{\sqrt{F_{80}}} \qquad \textbf{(EQ 20.10)}$$

where

W = kW·h per short ton
W_i = work index (kW·h/st), reduce from ∞ to 80% passing 100 µm
P_{80} = 80% passing size of product, µm
F_{80} = 80% passing size of feed, µm

For the Bond ball mill work index (W_i), the following equation (McIvor 2015) is suggested:

$$W_{iBM} = \frac{49.05}{P_{100}^{0.23} GpR^{0.82} \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right)} \qquad \textbf{(EQ 20.11)}$$

where

W_{iBM} = Bond ball mill work index, kW·h/t
GpR = net grams per revolution of product
P_{80} = 80% passing size of product, µm
F_{80} = 80% passing size of circuit feed
P_{100} = 100% passing size of circuit feed

$$\text{actual grinding mill horsepower} = 1.341\ W \times t \times EF_t \qquad \textbf{(EQ 20.12)}$$

where

1.341 = hp/kW·h
t = short tons per hour
EF_t = product of all efficiency factors
EF_1 = dry grinding factor (rod or ball mills) = 1.3; for wet grinding, EF_1 = 1
EF_2 = open circuit factor (ball mill only) = 1.04–1.7 (average 1.2); for closed circuit, EF_2 =1
EF_3 = mill diameter (ft) factor: 3 ft = 1.217, 4 ft = 1.149, 5 ft = 1.099, 6 ft = 1.059, 7 ft = 1.027, 8 ft = 1.000, 9 ft = 0.977, 10 ft = 0.956, 11 ft = 0.938, 12 ft = 0.922, 13+ ft = 0.907
EF_4 = oversize feed factor (rod or ball mills) = 1 except when W_i is >14.0 and/or F is >16,000 µm (rod) or 4,000 µm (ball); call the factory in these cases. Alternately,

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$$0.5\,EF_4 = \frac{R_r + (W_i - 7)\left(\frac{F - F_0}{F_0}\right)}{R_r} \quad \text{where } F_0 = 16{,}000 \times \sqrt{\frac{13}{W_i}} \text{ for rod mills}$$

$$\text{and } F_0 = 4{,}000 \times \sqrt{\frac{13}{W_i}} \text{ for ball mills, } R_r = \text{ratio of reduction} = \frac{F_{80}}{P_{80}}$$

EF_5 = fineness factor (ball mill only when P_{80} is <70 μm) = $[P_{80} + 10.3]/1.145\,P_{80}$
EF_6 = reduction ratio (rod mill) = 1.0 when R_r is 10–20; otherwise, call the factory
EF_7 = reduction ratio (ball mill) = 1.0 when R_r is <5.0; otherwise, call the factory
EF_8 = feed preparation factor = 1.4 if only a rod mill and open circuit crushing; 1.2 for rod + ball mill circuit if open circuit crushing; 1.2 if only a rod mill and closed circuit crushing; 1.0 for rod + ball mill circuits if closed circuit crushing

To approximate plant operating work indices, multiply laboratory work indices for rod and SAG mills by 1.4. Wet grinding power is about 25% lower than that required for equivalent dry grinding, but media wear is five to seven times as much as dry grinding. For dry grinding, media consumption is much less; moisture should not exceed 1% unless special handling equipment is installed. And, the rod mill feed should be <19 mm, and the ball mill feed should be <19 mm for soft ores and <7 mm for hard ores.

Critical speed is the speed at which the grinding media will not stick to the shell of the mill because of centrifugal force as shown in the following equation:

$$C_S = \frac{42.305}{\sqrt{D}} \qquad \textbf{(EQ 20.13)}$$

where

D = diameter of the inside liners, m
C_s = critical speed, rpm (Table 20.3)

Mill power is nearly proportional to revolutions per minute. New liner designs allow for greater mill revolutions per minute; for example, a 20-ft-diameter mill at 77% of C_s gives more throughput. General rules for older critical-speed percentages are shown in Table 20.3.

Overall availability for large ball mills can be as high as 98%; however, 95% is more typical for rod mills and ball mills, and 85%–90% is common for SAG mills.

$$\text{charge volume (\% filled with rods/balls)} = 113 - 126\left(\frac{H}{D}\right) \qquad \textbf{(EQ 20.14)}$$

where

H = distance from top of inside liners to top of charge, ft
D = diameter inside the liners, ft

The charge volume is usually 45% for wet grinding and 40% for overflow ball mills. It ranges between 28% and 35% for dry grinding. The void space in a rod charge is 30%, and in a ball charge, it is 42%. The weight of rods is approximately 5.5 t/m^3; balls weigh about 4.6 t/m^3.

Rowland and Kjos (1978) define the largest diameter of rods or balls that should be added as makeup media in the following equations.

TABLE 20.3 Critical-speed rules

Inside Diameter of Liners		Average Percent of Critical Speed	
Meters	Feet	Rod Mills	Ball Mills
0.91–1.83	3–6	76–73	80–78
1.83–2.74	6–9	73–70	78–75
2.74–3.66	9–12	70–67	75–72
3.66–4.57	12–15	67–64	72–69
4.57–5.49	15–18	–	69–66

Source: Rowland and Kjos 1978

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Rods

$$R = 25.4\left(\frac{F^{0.75}}{160}\right)\sqrt{\frac{(W_i \times \text{sg})}{\left(\sqrt{\frac{D}{3.281}} \times \%C_s\right)}} \quad \text{(EQ 20.15)}$$

where

R = rod diameter, mm
F = 80% passing feed size, μm
W_i = work index, kW·h/t
sg = specific gravity of ore
C_s = % of critical speed
D = diameter inside the liners, m

Balls

$$B = 25.4\left(\sqrt{\frac{F}{k}}\left[\sqrt[3]{\frac{\text{sg} \times W_i}{\sqrt{\frac{D}{3.281}} \times \%C_s}}\right]\right) \quad \text{(EQ 20.16)}$$

where

B = ball diameter, mm
k = 350 for wet overflow mills, 330 for wet diaphragm mills, and 335 for dry diaphragm mills (all other terms are the same as for rod diameter)

The largest ball diameter generally used in ball mills and SAG mills is 125 mm (5 in.). If there is no pebble crusher, the SAG grate openings are generally 25 mm, and 125-mm balls can peen them shut, causing an issue with throughput.

Another formula to determine makeup ball size is

$$B = 6 \times \log d_k \times \sqrt{d} \quad \text{(EQ 20.17)}$$

where

d_k = largest particle in the product, μm
d = largest particle in the feed, mm

Bond Work Index

The Bond work index is a useful determination in a laboratory Bond mill to determine material (ore) hardness.

$$BW_i = \frac{44.5}{(P_1^{0.23})(G_{bp})^{0.82}\left(\frac{10}{\sqrt{\text{P}_{80}}} \times \frac{10}{\sqrt{\text{F}_{80}}}\right)} \quad \text{(EQ 20.18)}$$

where

P_1 = 100% passing product size
P_{80} = 80% passing product size
F_{80} = 80% passing feed size
G_{bp} = grams per revolution
BW_i = Bond ball mill work index, kW·h/t

Ball and Liner Wear

The following guide is based on forged steel (Rowland and Kjos 1978):

- Wet rod mill rod liner wear = 0.175 (A_i – 0.020)0.2 (kg/kW·h)
- Wet rod mill liners liner wear = 0.175 (A_i – 0.015)0.3 (kg/kW·h)
- Wet ball mill balls liner wear = 0.175 (A_i – 0.015)0.333 (kg/kW·h)
- Wet ball mill liners liner wear = 0.013 (A_i – 0.015)0.3 (kg/kW·h)
- Approximate wear = 1.0 kg of balls per 15 kW·h of power to the mill

The typical wear rate of steel balls in wet ball mills is 0.4 kg/t (range 0.15–0.75). For high-chrome balls, it is 0.1 kg/t (range 0.05–0.3); 12% chrome balls are probably the most economic; slugs may be an economic

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alternative. For grinding mill liners, 12% chrome is probably the most economic metal liner, but rubber is probably the most economical for secondary grinding and regrinding ball mills. Because the wear rate of rubber in primary grinding mills can be high, consider steel-capped rubber liners.

Semiautogenous Grinding Mills

- Conduct tests on each ore type in the deposit: Testing for SAG mill design can take many different approaches, including the JK drop weight test and closely related SMC Test; MacPherson grindability test; Bond impact, rod, and ball mill work index tests; SAG power index (SPI) test; and pilot plant–scale testing. Obtaining representative samples is critical.
- To obtain the SAG mill kilowatt estimate, ±5%, use the same formula as for a ball mill but substitute the autogenous work index for the ball mill work index (MacPherson 1989; Austin 1990; Scott and Barratt 1987).
- Maintain overall volume in the 30%–35% range. Charge volume can range from 5% to 18% depending on ore hardness.
- Assay and test SAG mill cobbles by asking if they are worth crushing in a SAG mill, ball mill, and pebble crusher (SABC) circuit.
- Use vibrating screens or a rubber-lined trommel rather than cyclones on mill discharge.
- To prevent mill overload, use bearing pressure as a predictive tool:
  - Rising power at a fixed mill load suggests lowering the volume (bearing pressure) set point to get more impact grinding; it may also mean that the ore feed does not have enough large media and that a coarser blend of ore is needed.
  - When the mill load does not decrease at maximum water input and the power draw is high, stop the mill feed for about 30 minutes. Continue running the mill to remove solids from the mill.
  - Install an expert control system that monitors the different mill operating parameters.
- If the initial set of mill liners are troublesome, the initial grates may need to be changed to include pebble ports. In SABC circuits, set the crusher CSS as tight as possible to prevent oversize from reporting to the mill.
- Install magnets and a metal detector to reject steel from the pebble crusher feed.
- Install a variable-speed pump with a short acceleration time on SAG mill discharge.
- The operating availability for a well-run SAG mill circuit should be 90%–92% or more.
- With a compression-type bolt sealer, retighten liner bolts after 1–2 hours of full load operation, again after 6–8 hours, and again after 5–7 days.
- The mill bearing lubrication system should be closely monitored for bearing temperatures, lube temperature, pinion vibration, and temperatures.
- The shutdown sequence is as follows:
  1. Shut off the feed.
  2. Shut off the water.
  3. Allow the mill to turn five revolutions to pump out the slurry.
  4. Shut off the power.
  5. Hose and bar down overhead muck and balls. Do not grind out the mill to inspect or maintain the liners or to determine ball charge volume (only complete this when absolutely necessary as it can damage the liners and grates). Unless maintenance must be done on the lubrication system, it should be left running.
- Wash muck from the pebbles before conveying them to the SABC cone crusher.

Table 20.4 shows the recommended torque for bolts in a SAG mill.

Dry Grinding Systems

- Run the mill at average 24% charge load.
- Do not try to grind finer than 1.7 mm.
- Limit velocities to 1,500 m/min in classifier ductwork.
- Use clean air fans, not dirty air fans. Ceramic line all dirty air fans.

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TABLE 20.4 Torque

Bolt Diameter, mm	Torque, kg·m		
	Grade 2	Grade 5	Grade 8
32	117	201	293
38	180	252	401
44	281	—	638
51	423	—	961

Source: Kjos 1986

High-Pressure Grinding Rolls

HPGR machines are essentially a modern roll crusher using two counter-rotating rolls to grind ore. Very high pressure between the rolls is provided by hydraulic cylinders used to push one floating roll against a fixed roll. The pressure causes interparticle breakage as ore passes between the rolls. These units cannot tolerate tramp iron (non-crushable materials), which can damage the roll tires. Typical tire configurations include studs and protective cheek plate systems to constrain the ore within the roll area. High steel wear is overcome by segmented roll faces that are partially covered with studs of very hard metal between which ore gets jammed; that is, much of the roll face is rock, and grinding is partially autogenous. Wear drive maintenance of the roll surface, cheek plates, and feed chutes are the primary operating costs.

HPGRs must be choke fed from an overhead hopper, and generally require <3% moisture in the ore. Less than full feed rate causes excessive and uneven wear on the rolls. HPGRs generate many microfractures that make subsequent grinding easier and provide higher extractions in subsequent leaching operations.

Stirred Media Mills

Stirred media mills, including vertical and tower mills, are used for fine wet or dry grinding and for regrinding.

- For F_{80} <0.15 mm, a stirred media mill's energy usage is approximately 25% less than a ball mill's energy usage.
- Because Bond energy equations show 20%–50% more power than is actually needed when grinding in a 10-µm range, do not apply the equations directly.

For stirred media mills

- The circulating load is approximately 300%–1,000%.
  - Percent solids in the mill are approximately 10%–70%.
  - Percent solids in the overflow are approximately 2%–45%.
  - Feed size is 5 mm or less.
- Makeup balls are generally 25 mm or less in diameter; maximum size is 32 mm. Total metal wear is less than for ball mills in the same application.
- The availability of scheduled operating time is approximately 95%–98%.

PUMP SIZING by Shubham Verma, P.E.

Use the following equation to estimate the power requirement for a slurry pump:

$$\text{power (hp)} = \frac{\text{flow rate (gpm)} \times \text{total dynamic head (ft)} \times \text{specific gravity (sg)}}{3{,}960 \times \text{pump efficiency}} \quad \textbf{(EQ 20.19)}$$

The total dynamic head (TDH) is a sum of static lift or height, friction losses in the pipe, pressure losses across equipment (e.g., cyclone), and the velocity head. For example, see the application shown in Figure 20.3.

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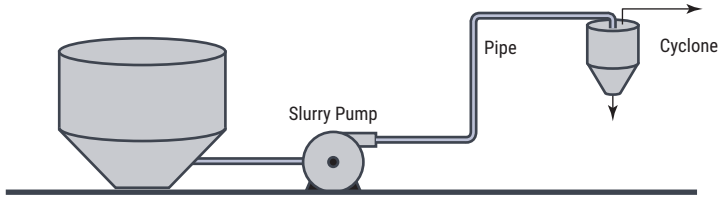

FIGURE 20.3 Slurry pump, pipe, and cyclone schematic

$$TDH = \frac{0.78\,V_S^2}{2g} + \frac{V_i^2}{2g} + \frac{2.31\Delta P}{\rho_P} + f_s L_s + f_d L_d + h_d - h_s \qquad \text{(EQ 20.20)}$$

where

V_s = slurry velocity at pump suction line, ft/s
V_i = slurry velocity at cyclone inlet, ft/s
ΔP = pressure drop across cyclone, psi
ρ_P = relative density of slurry
f_s = friction loss rate in suction line, ft/1,000 ft of pipe
L_s = equivalent length of suction line, ft
f_d = friction loss rate in discharge line from pump centerline to cyclone inlet, ft/1,000 ft of pipe
L_d = equivalent length of discharge line, ft
h_d = vertical height from pump centerline to cyclone overflow, ft
h_s = vertical height from pump centerline to slurry level in the sump, ft
g = 32.2 ft/s^2

PIPELINE SIZING by Shubham Verma, P.E.

For shorter pipe runs, the velocities shown in Table 20.5 are considered good parameters. By using these velocities and a known slurry flow rate, the pipe size can be closely estimated.

TABLE 20.5 Velocity rules

Stream	Example	Velocity, ft/s
Coarse-particle applications	Semiautogenous grinding mill discharge	12–16
Medium-particle sizing	Leach/flotation feed	8–12
Fine-particle sizing	Concentrates	4–8

The volumetric flow rate of the slurry must also take froth into consideration if applicable. Presence of froth increases the overall slurry volume and hence affects the line sizing. Froth factors are usually specific to the application but can range from 1.1 to 1.5 for brittle froth and from 1.5 to 2.0 for tenacious froth. Following are some best practices for plant piping layout:

- Keep pipe routing as straight as possible. Every bend is a potential wear point.
- Pipe routing should allow for easy personnel access to different areas and pieces of equipment in the plant.
- A reasonable number of flush and drain connections should be provided with easy access in the piping.
- Shutoff valves, if needed, should be as close as possible to the flange on the tank or a T in the pipe.
- Piping should be well supported on both suction and discharge of pumps to minimize stresses on the pump flanges. In applications where high vibrations are common, consider using expansion joints.
- Long radius bends (three to five times the pipe diameter size) help in minimizing the line wear.

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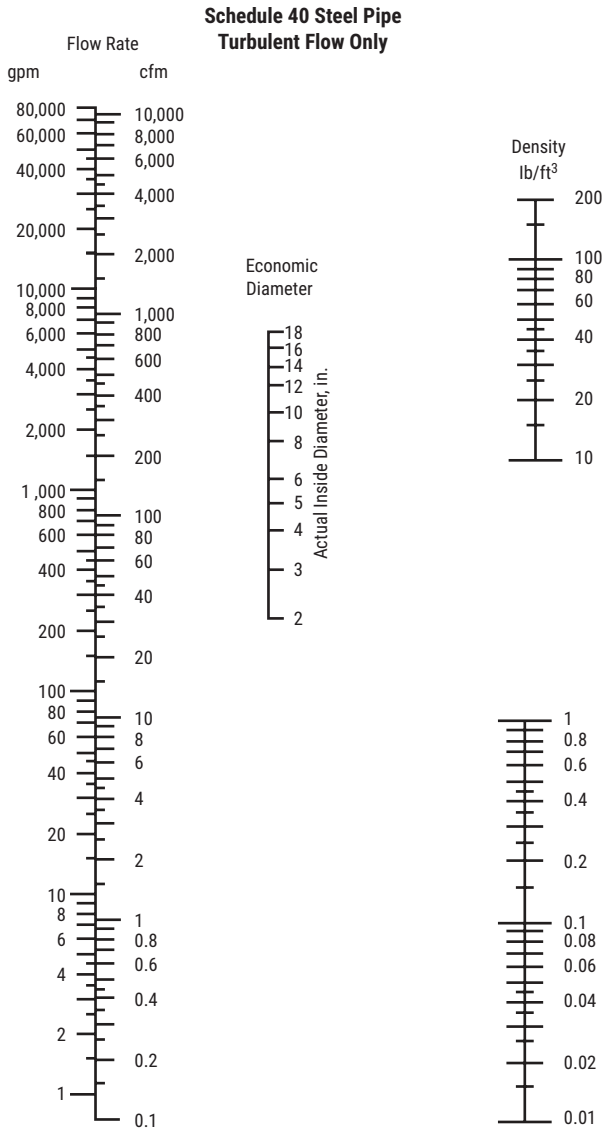
- The horizontal pipes should have some slope (generally 2–5 degrees depending on material) to drain the line during shutdown.
- Pump suction spools should be as short as possible to avoid sanding.
- Horizontal or vertical piping is best. Avoid angled piping as it produces preferential settling and potential to sand out.
- To obtain economic pipe diameters, connect the values of flow rate and density (Figure 20.4).



Courtesy of E.I. du Pont de Nemours and Company, as appears in Perry et al. 1969

FIGURE 20.4 Economic pipe diameter

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CLASSIFICATION by Paul D. Chamberlin, P.E.

The main controls for most classifiers are (1) varying the percentage of solids, (2) varying the input and output volumes, and (3) varying the viscosity of slurry systems (Table 20.6). For air separators and classification below 250 μm, a unit with a rotor is preferable, and variable-speed drives are needed. Circulating loads in high-efficiency air separators are 50%–400%; in traditional units, they are 200%–1,000% (cubic feet per minute [cfm] of air to tons per hour [tph] of load). In the pool area of spiral, rake, or drag classifiers, minimize turbulence.

Two stages of classification in series invariably give cleaner separations than a single stage. Use intermittent full-flow discharge of solids from settling cones rather than continuous slow flows.

TABLE 20.6 Classifiers

Classifier	Maximum Feed Size, mm	Feed Rate, t/h	Volume Percent Solids Feed/ Overflow/Underflow	Power, kW	Notes
Hydrocyclones	1.4 mm–45 μm	To 20 m^3/min	4–35/2–15/30–50	35–400 kN/m^2 pressure head	Relatively efficient separations
Air separators	2 mm–38 μm	To 2,100	—	4–500	—
Spiral, rake, drag	25	5–850	Variable/2–20/45–65	0.5–110	Gives relatively clean sands; may eliminate pump in grinding circuit
Log washer	100	40–450	—	10–60	Breaks agglomerates
Hydraulic bowl	12	5–225	Variable/2–15/50–65	5–25	Gives very clean sands
Rake clarifier	25	1–150	Variable/0.4–15/20–35	1–11	Gives very clean sands
Cone	6	2–100	Variable/5–30/35–60	0	Simple
Elutriator	7.5	4–120	15–35/0.4–5.0/20–35	1	Simple

Adapted from Kelly and Spottiswood 1982

Air Classifier Performance Measurements

The following measurements of performance, including yield, efficiency, and selectivity, for air classifiers are from Klumpar (1987a and 1987b). Yield, or recovery, is the amount of product per unit of feed.

$$Y_f = 100\frac{C(A - B)}{A(C - B)} \quad \textbf{(EQ 20.21)}$$

where

Y_f = yield of fines
A = cumulative mass fraction of feed passing screen x
B = cumulative mass fraction of coarse product passing screen x
C = cumulative mass fraction of fines product passing screen x

Efficiency E is the difference between fine Y_f and coarse Y_c product yields.

$$E = Y_f - Y_c = 100\frac{(C - A)(A - B)}{A(1 - A)(C - B)} \quad \textbf{(EQ 20.22)}$$

Selectivity S is the weight percentage of all particles of a given diameter in the feed D that go to the coarse product. The particles of a given diameter are defined as the narrow fraction passing one screen (e.g., 200 mesh, 74 μm) but retained on the next smaller screen (250 mesh, 63 μm). The average diameter of that fraction is the arithmetic mean or 68.5 μm. Selectivity for particles of average diameter D is

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$$S = \frac{b(c - a)}{a(c - b)} \quad \text{(EQ 20.23)}$$

where

a = mass fraction of feed passing screen x and retained on next smaller screen
b = mass fraction of coarse product passing screen x and retained on next smaller screen
c = mass fraction of fines product passing screen x and retained on next smaller screen

Performance is best characterized by the plot of selectivity versus average particle size D. Plot ($c - a$) versus ($c - b$) for each D: Draw a straight line through the points and measure the line's slope s. Typically, the ordinate is selectivity on a probability scale; average particle size D is the abscissa on a logarithmic scale. Ideal classification results in a vertical line intersecting the curve at 50% selectivity.

$$s = \frac{(c - a)}{(c - b)} \quad \text{(EQ 20.24)}$$

SCREENING by Paul D. Chamberlin, P.E.

Many materials are used in screen construction. Polyurethane and rubber screens have much less open area than woven wire screens, but typically have lower wear.

- Woven wire is widely used.
- Polyurethanes (many varieties) are best for most abrasive situations. Good for wet screening, they are available in modular deck configurations that are interchangeable with woven wire panels. Be sure to install thicker panels or solid panels for deck sections consistently impacted. Protect the screens when welding nearby.
- Rubber (many varieties) is best in dry and high-impact areas.
- Perforated plate can be metal or covered with rubber or polyurethanes.
- Wedge wire, bar decks, and grizzly bars can be custom designed for specific needs.

Types of Screens

Screens are available in many configurations (Table 20.7). Typical screen availability is 92%–96%. Following is a list of different screen types:

- Vibrating—horizontal or inclined—with circular, elliptical, or linear strokes
- Stationary and stationary sieve bend
- Circular, also known as trommel
- Wet or dry
- Mogensen Divergator
- Grizzly
- Dewatering
- Scalping
- Washing
- Banana
- Interstage carbon
- Rotating probability screens for wet screening fine materials
- Electrically heated screens to reduce plugging in winter (These do not work well with >2%–3% moisture.)
- Liwell (Alternate flexing and tensioning of the synthetic deck prevents plugging.)
- Bradford breakers for coal and oilsand
- Multiple deck

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TABLE 20.7 International standard and U.S. sieve sizes

Standard	U.S. Sieve	Standard	U.S. Sieve	Standard	U.S. Sieve
125.0 mm	5.000 in.	11.2 mm	0.438 in.	600 μm	No. 30
106.0 mm	4.240 in.	9.5 mm	0.375 in.	500 μm	No. 35
100.0 mm	4.000 in.	8.0 mm	0.313 in.	425 μm	No. 40
90.0 mm	3.500 in.	6.7 mm	0.265 in.	355 μm	No. 45
75.0 mm	3.000 in.	6.3 mm	0.250 in.	300 μm	No. 50
63.0 mm	2.500 in.	5.6 mm	No. 3.5	250 μm	No. 60
53.0 mm	2.120 in.	4.75 mm	No. 4	212 μm	No. 70
50.0 mm	2.000 in.	4.00 mm	No. 5	180 μm	No. 80
45.0 mm	1.750 in.	3.35 mm	No. 6	150 μm	No. 100
37.5 mm	1.500 in.	2.80 mm	No. 7	125 μm	No. 120
31.5 mm	1.250 in.	2.36 mm	No. 8	106 μm	No. 140
26.5 mm	1.060 in.	2.00 mm	No. 10	90 μm	No. 170
25.0 mm	1.000 in.	1.70 mm	No. 12	75 μm	No. 200
22.4 mm	0.875 in.	1.40 mm	No. 14	63 μm	No. 230
19.0 mm	0.750 in.	1.18 mm	No. 16	53 μm	No. 270
16.0 mm	0.625 in.	1.00 mm	No. 18	45 μm	No. 325
13.2 mm	0.530 in.	850 μm	No. 20	38 μm	No. 400
12.5 mm	0.500 in.	710 μm	No. 25	25 μm	No. 500
				20 μm	No. 625

Source: Matthews 1985

Troubleshooting

- Distribute feed across the full width of the screen so that the entire screen is used. Poor feed distribution is a major cause of poor screening.
- Present the feed to the screen in the direction of its centerline; feed that approaches at an angle will segregate by size, with coarse particles migrating to the far side and fines remaining in the center. Without coarse particles to scrub the deck, damp sticky fines tend to blind openings.
- To reduce blinding, make screens more efficient, and increase screen capacity, use woven wire screen with a large percentage of open area, that is, smaller wire diameters rather than larger diameters. Use backing wire.
- Prevent blinding by having the openings in the sizing screen slightly smaller than the openings in the backing underneath.
- Worn-out screens should be replaced as soon as possible to prevent short-circuiting.
- If screen cloth tears down the support ribs, it is undertensioned.
- Ripping screen cloth along the hook strip is caused by tightening end-draw bolts first.
- Ensure that the resonant frequency of the screen support structure is at least three times the operating frequency of the screen.
- Replace broken steel coil springs as a complete set rather than individually.
- Replace screen bearings in sets rather than individually.
- Install access around the screen so maintenance can be easily accomplished.
- All coil springs should be of the same height and should not sag.

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- To minimize vibration to the main structure, the vibrating screen should be located on its own columns and supports.
- Provide approximately 6 cm of clearance between the vibrating unit and any stationary structure. Start and stop the unit with and without load; note clearances, especially when stopping.
- The motor must be properly aligned and on a pivoting motor base. Sheaves should be monitored for wear and cleanliness. Determine the vibrator speed and stroke and compare with specifications.
- For overheated or noisy screens, check for proper lubrication fluids. Check for failed bearings.
- If the screen has erratic motion, check that the screen is level; that coil springs are not broken, weak, or jammed with rock; and that feed is evenly presented to screen.
- Watch screen discharge and how clean it is (establish a baseline); if discharge depth increases or if fines piggyback on the coarse side, there is a problem.
- Install plug chute switches on both undersize and oversize screen discharge chutes. Inspect chutes for wear.

Check Screen Sizing

The formula and factors presented in Table 20.8 are only guides. A separate calculation is required for each deck of a multiple-deck screen. Screening area is measured in square feet. *U* is the total of short tons per hour of material in the feed to the screen that is smaller than the screen opening size (i.e., tons per hour of undersize in the feed). Factor A is the total of short tons per hour that can pass through one square foot of screen deck when the feed contains 25% oversize and 40% halfsize. G is the factor that applies when the open area of screening surface is less than the open area shown in the Factor A chart. The basic operating conditions are as follows:

- Feed to the screening deck contains 25% oversize and 40% halfsize.
- Feed is granular free-flowing material.
- Material weighs 100 lb/ft^3.
- The operating slope of the screen is 18–20 degrees with flow rotation for an inclined screen and 0 degrees for a horizontal screen.
- The objective screening efficiency is 95%.

The screen efficiency is determined by the percent of undersize in the feed that actually passes divided by the percent of undersize in the feed. The bed depth at the discharge end of a screen should be less than four times the size of the screen opening for 1,605 kg/m^3 material and less than three times the size for 802 kg/m^3 material. The following formula can be used:

$$DBD = \frac{(O \times C)}{(5 \times T \times W)} \quad \textbf{(EQ 20.25)}$$

where

DBD = discharge-end bed depth, in.
O = oversize, stph
C = bulk density, ft^3/st (short ton)
T = rate of travel, fpm (nominal 75 fpm for inclined screen at slope of 18–20 degrees with flow rotation and nominal 45 fpm for horizontal screen)
W = width of screening area, ft

First determine the width of the screen deck based on the bed depth calculation, then select the length of the screen. For wet screening, apply water at a rate of 19–27 L/min per cubic meter of ore; use some of the water to prepare the slurry in the feed box for presentation to the screen. Very little screening of fines takes place between spray bars where no water is washing over the material.

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TABLE 20.8 Factors for calculating screen area

Formula: screening area = $\frac{U}{A \times B \times C \times D \times E \times F \times G \times H \times J}$

FACTOR A		
Surface Square Opening, in.	Open Area, %	Passing a Square Foot, stph
4	75	7.69
3½	77	7.03
3	74	6.17
2¾	74	5.85
2½	72	5.52
2	71	4.90
1¾	68	4.51
1½	69	4.20
1¼	66	3.89
1	64	3.56
⅞	63	3.38
¾	61	3.08
⅝	59	2.82
½	54	2.47
⅜	51	2.08
¼	46	1.60
3/16	45	1.27
⅛	40	0.95
3/32	45	0.76
1/16	37	0.58
1/32	41	0.39

FACTOR B (percent of oversize in feed deck)							
Percent oversize	5	10	15	20	25	30	35
Factor B	1.21	1.13	1.08	1.02	1.00	0.96	0.92
Percent oversize	40	45	50	55	60	65	70
Factor B	0.88	0.84	0.79	0.75	0.70	0.66	0.62
Percent oversize	75	80	85	90	95		
Factor B	0.58	0.53	0.50	0.46	0.33		

FACTOR C (percent of halfsize in feed deck)							
Percent halfsize	0	5	10	15	20	25	30
Factor C	0.40	0.45	0.50	0.55	0.60	0.70	0.80
Percent halfsize	35	40	45	50	55	60	65
Factor C	0.90	1.00	1.10	1.20	1.30	1.40	1.55
Percent halfsize	70	75	80	85	90		
Factor C	1.70	1.85	2.00	2.20	2.40		

FACTOR D (deck location)			
Deck	Top	Second	Third
Factor ID	1.00	0.90	0.80

(continues)

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TABLE 20.8 Factors for calculating screen area (continued)

FACTOR E (wet screening)										
Opening, in.		1/32	1/16	1/8	3/16	1/4	3/8	1/2	3/4	1
Factor E		1.00	1.25	2.00	2.50	2.00	1.75	1.40	1.30	1.25
FACTOR F (material weight)										
lb/ft^3	150	125	100	90	80	75	70	60	50	30
Factor F	1.50	1.25	1.00	0.90	0.80	0.75	0.70	0.60	0.50	0.30

FACTOR G (screen surface open area)

$$\text{Factor G} = \frac{\text{percent open area of surface being used}}{\text{percent open area indicated in capacity}}$$

FACTOR H (shape of surface opening)	
Square	1.00
Short slot (3–4× width)	1.15
Long slot (>4× width)	1.20

FACTOR J (efficiency)	
95%	1.00
90%	1.15
85%	1.35
80%	1.50
75%	1.70
70%	1.90

Source: VSMA 1998

SORTING by Douglas K. Maxwell, P.E.

These general rules are useful for all sorting techniques (see Table 20.9 for commercial sorting systems):

- Lighting is critical; the greatest differences are seen when particles are wet.
- Wash slimes from particles to expose all surfaces.
- Sorts are best when feed is closely sized; for example, large-to-small should be 2:1 or 3:1.
- Typically, only one surface per particle is evaluated; thus, liberation is critical to sort quality. Expect some misplacement of particles.
- Sorting rate depends on particle size; bigger particles give more tons per hour.

Hand Sorting

- Feed preparation—little preparation is needed for tramp material or oversize removal.
- Feed presentation—particles should cover <20% of the sorting surface.
- Sorting rates
  - Rate is approximately constant in particles per worker hour. Tonnage depends on particle size.
  - Gem sorting takes more evaluation time per particle.

Laser Reflectance (Photometric)

- Feed preparation
  - Maximum particle size is 120 mm; minimum particle size is technically 10 mm, but is approximately 20 mm for most economic operations.
  - One sorter can handle different size ranges at different times (i.e., campaigning).

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TABLE 20.9 Commercial sorting systems

Property	Sensor	Separation	Particle Size, mm	Sorting Rate	Rate Units	Application	Typical Use
Visual appearance	Human eye	Hand sorting	3–450	0.05–10	Metric tons per worker shift	Gems, industrial minerals, coal, metal ores, tramp material oversize	Rougher, cleaner, scavenger
Laser reflectance	Photomultiplier	Trajectory deflection	10–120	10–100	Metric tons per hour	Magnesite, limestone, wollastonite, feldspar, quartz, spodumene, gold ore, silver ore	Rougher, cleaner
Radioactive grade	Scintillometers and camera	Trajectory deflection	10–120	10–100	Metric tons per hour	Uranium ore, gold ore	Rougher, scavenger
X-ray fluorescence	Photomultiplier	Trajectory deflection	3–75	3–40	Metric tons per hour	Diamonds	Cleaner

- Feed presentation
  - Dust control is essential for the sensing system.
  - Separate the particles; particles should cover <20% of the sorting surface.
- Separation
  - Horizontal belt speed should be about 4 m/s.
  - Reflectance measurement is done after particles leave the belt and start free trajectory.
  - Air jets are down firing and are controlled by an electronic processor.
- Sorting rate depends on particle size. The general rule is metric tons per hour equal the average particle size in millimeters (i.e., 60 t/h for 40- to 80-mm particles).
- Sort characteristics
  - Sort is based on reflectance of the scanned laser spot. Red (helium-neon) laser is used most; blue (argon) lasers are occasionally used. Any visible light laser can be adapted for use.
  - Expect 10%–20% misplacement of particles.

Radioactive

- Feed preparation
  - Maximum particle size is 120 mm.
  - Each sorter is designed to handle only one limited size range.
- Feed presentation is with feed directed to channels in which particles are spread apart for separate evaluation.
- Separation
  - Belt moves at 4 m/s.
  - Scintillometers measure radiation of each particle in each channel and videocameras measure each particle's length and width (i.e., size).
  - Down-firing air jets are controlled by the processor.
- Sorting rate approximates particle size (i.e., 60 t/h for 40- to 80-mm particles).
- Sort characteristics
  - Particle grade equals scintillation count divided by the camera's measurement of size.
  - Expect 10%–20% misplacement of particles.

X-Ray Fluorescence Diamond Sorters

- Feed presentation is with a monolayer of material on an inclined chute in total darkness.
- Separation
  - Diamonds fluoresce with visible blue light when exposed to X-rays. Glow is detected by a bank of photomultipliers that trigger a down-firing air jet as material drops off the end of the chute.
  - Sorters are mostly used to clean gravity concentrates; however, in Russia, they are also used for roughing.
- The sorting rate of gravity concentrates is typically 2–5 t/h and up to 40 t/h for roughing of feed measuring –20 mm.
- Sort characteristics
  - Every diamond must be collected, so concentrates are low grade. Low grade is also caused by other minerals that fluoresce and are collected.
  - Diamond losses may be caused by diamonds that do not fluoresce brightly enough or quickly enough to be detected.
  - Sorters are enclosed machines and they offer a high degree of security.

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GRAVITY CONCENTRATION by D. Erik Spiller

The key to successful gravity concentration of minerals is careful attention to both liberation of the components to be separated and presentation of the feed material to the separating machine. Many mechanical devices are used to separate minerals based on particle-specific gravity differences. Effective commercial separations can be made on materials as coarse as 1,200 mm (5 in.) to as fine as 0.01 mm (10 μm); the upper size is limited only by the size of the separating machine, and of course, liberation. The following guidelines cover gravity separation via the classical system definitions of stratification, flowing film, and shaking (Kelly and Spottiswood 1982); density systems (i.e., dense medium separations [DMS]), are not included (Table 20.10).

Concentration becomes more expensive and machine capacity lower as particle size decreases. Gravity concentration is usually difficult below 74 μm (200 mesh) and very difficult below 38 μm (400 mesh). Flowing film separators, such as sluices, Reichert cones, and spirals, tend to work best on –1.0 mm (16 mesh) to +100 μm (about 150 mesh) liberated particles. Numerous applications offer satisfactory recovery efficiencies that extend to finer sizes. Jigs perform well on coarser materials up to 75 mm, and in practice, their recoveries drop off significantly on particles finer than about 150 μm (100 mesh), although there are exceptions, especially when applied to high specific gravity (sg) differentials found with gold, tin, and so forth. Shaking tables and their variations can be effective down to 20 μm when treating narrowly sized feed materials.

Virtually all gravity-concentrating machines tend, at times, to confuse gravity concentration with size classification. It is the nature of the system. Narrower feed size distributions produce better gravity concentration results. Gravity-concentrating machines are most effective at producing either a concentrate or tailings. It takes multiple-unit operations, cleaners, and scavengers to do both.

More than about 10 wt % slimes (roughly –10 μm particles) significantly reduces performance, except for machines designed for beneficiation of slimes (Mozley multi-gravity separator [MGS] and centrifugal-based machines). The reason for this is the increased viscosity of the fluid/pulp.

TABLE 20.10 Commercial characteristics of gravity concentration machines*

Type	Machine	Operating Size Range, mm	Water Requirements†	Capacity‡
Stratification	Jigs—conventional	0.10–100	High	Medium
	Jigs—circular	0.05–100	High	High
	Jigs—centrifugal	0.03–2.0	High	Medium
Flowing film	Sluice box	0.15–10.0	High	Medium
	Reichert cone	0.05–1.5	Low	High
	Pinched sluice	0.05–1.5	Low	Medium
	Strake	0.15–2.0	High	Low
	Spiral	0.03–2.0	Medium	Medium
Shaking	Shaking table	0.02–2.0	Medium	Medium
	Orbital	0.01– 0.07	High	Low
	Centrifugal	0.01–0.15	High	Low
	Crossbelt	0.01–0.03	High	Low
Centrifugal	Spinning bowls	0.01–1.7	Very high	High
Air dry	Pneumatic jig	0.5–25	None	Medium
	Air table	0.15–6	None	Low

* Generalized—specific machine operating on site-specific materials may or may not perform to these characteristics.

† Relative.

‡ Relative per unit—multiple units always equate to high capacity.

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A consistent feed rate and feed pulp density are essential for shaking tables (all types), spiral concentrators, and Reichert cone concentrators. Sluices are a little less sensitive to feed rate. Jigs and centrifugal-based devices are relatively insensitive to feed rate and feed pulp density up to the point of overloading, and then performance dramatically drops.

The concentration criterion *CC* suggests if gravity concentration is practical (other than dense medium, heavy liquid, etc.).

$$CC = \frac{(D_h - D_f)}{(D_l - D_f)} \quad \textbf{(EQ 20.26)}$$

where

D_h = the density of the heavy particles
D_l = the density of the light particles
D_f = the density of the fluid (usually water at 1.0)

Gravity concentration will usually be successful if $CC > 2.5$. When $CC < 1.25$, gravity concentration is virtually impossible. When $2.5 > CC > 1.25$, gravity concentration is very difficult, but may be possible to some extent with narrow feed size classification and slow and careful feed presentation.

Complete component liberation is not necessary before introducing feed to gravity separation. However, there must be *effective liberation*, which is the size at which a separation can be made that will have a positive economic impact on downstream processing.

Feeding dry material to a wet machine is discouraged because certain minerals, including metallic gold, are difficult to wet and they can float off and be lost to tailings. It is best to provide a feed mixing/wetting tank in the circuit prior to a gravity separation device. This same tank will provide surge protection to feed-rate sensitive gravity separators.

Dry gravity separation can be effective, although it is never as efficient as wet separation. A word of caution on dry separators: *Dry deposits* are often not really dry and, therefore, feed dispersion problems can impede performance at commercial scale.

For maximum sluice performance, barren coarse rock (+25 mm, or even +6 mm, if possible) should be removed from the feed. A constant feed rate at a constant feed pulp density (water addition) improves performance. Site-specific factors, including mineral associations, particle size, and relative abundance of heavies, influence sluice performance; testing is always necessary. Each deposit or operation should determine cleanup intervals based on riffle loading and tonnage processed; shorter is often better. Finally, for all practical purposes, a strake performs the same as a sluice with respect to mineral recovery.

Installing free-gold gravity recovery equipment on the circulating load in a grind circuit (i.e., cyclone underflow) improves gold recovery. Treating as little as 10 wt % of the circulating load can be effective, although up to 30% may be optimal. Free-gold recovery can improve the amount of payable gold from processing by moving the free gold to a different saleable product. Capturing the gravity-recoverable free gold increases security issues, but also makes mill metallurgical accounting easier and more accurate. Collected spills should be gradually returned to the circuit.

FLOTATION by Deepak Malhotra

Three main groups of variables for flotation are (1) ore and mineral properties, (2) reagent combinations and flotation environment, and (3) flotation machine characteristics.

The objective of rougher flotation is to maximize recovery at as coarse a grind as possible while maintaining grade at an acceptable level. The objective of cleaner flotation is to produce a high grade at acceptable recovery. Liberated particles float fast; middlings float slowly; thus, the need to regrind middlings. Very fine grinding does not increase recovery. Efficiency drops off below 15–25 μm.

Recovery and grade have an inverse relationship. Increased recovery may not mean increased profit. For example, a finer grind costs money but may give higher recovery, and pulling froth harder may give higher recovery but result in a low-grade unsalable concentrate. Economic recovery occurs when revenue from incremental recovery equals or exceeds the incremental cost of getting it.

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Operating Controls

The following are variables that can be controlled:

- Soluble impurities in process water
- Alkalinity or acidity of process water
- Type of grinding mill
- Type of grinding media
- Oxidation during grinding
- Pulp density during grinding
- Grinding time
- Chemicals added during grinding
- Grinding temperature
- Particle size
- Particle size in flotation
- Pulp density in flotation
- Temperature in flotation
- pH in flotation
- Circulating load in flotation
- Flotation time
- Impeller speed
- Froth height
- Degree and type of aeration
- Feed flow rate
- Pulp level control

And noncontrollable variables are

- Mineral and gangue nature
- Soluble constituents in ore
- Degree of oxidation of ore
- Grain size and mineral/gangue associations
- Hardness of minerals and gangue
- Relative time for different minerals in grinding circuit

Plants treating more than 2,000 stpd (short tons per day) can justify expensive instrumentation such as onstream analyzers for tailings; even smaller plants must have at least automatic samplers. For smooth operation with all flotation cell types, maintain a constant feed volume and density.

The order of reagent addition can be critical for industrial minerals or two-product systems such as Pb–Zn. Proper mixing and dilution of reagents is important for efficient operations. Xanthates in sulfide flotation are the cheapest collectors; they are also nonselective. Dithiophosphates and carbamates are more selective and less pH sensitive than xanthates.

Reagents must be added at the proper points in the circuit for best performance. Water-soluble collectors are usually added after grinding to the conditioner or pump sump before flotation. Non-water-soluble collectors should be added to the grinding mill.

Reagents affect the froth in addition to recovery and grade. Too much oil can kill froth; too much frother can create overfrothing conditions. Froth level should be deep in roughers and shallow in scavengers. The last cell should be used as the control cell. It should have little or no desirable mineralization in the froth to ensure maximum recovery.

Following is a list of additional operating controls to consider:

- Sulfidization often increases recovery of oxide minerals; determine the percentage of oxidized mineral first.
- Pulp level in mechanical cells should be controlled only on the last cell of the bank by weirs and dart valves. Pulp level in column cells is regulated by tailings removal rate.
- The higher the flotation air, the lower the retention time; the higher the feed rate, the lower the retention time.
- To avoid short-circuiting, a minimum of four cells in a row are needed in roughers. It can also be controlled by proper agitation and aeration. Sample rougher overflows (tailings) to determine if flotation time is sufficient. Short-circuiting of material is common in hog trough–type cells.
- Rougher flotation usually floats <10% of feed weight; hence, a quick method of checking recovery is to check tailings and feed assays, that is, $R = 100(1.0 - t/f)$. Cleaner flotation usually recovers more than

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50% of cleaner feed mass. Check the upgrading ratio of concentration (i.e., *c*/*f*, for flotation efficiency; a ratio <1:1 indicates poor selectivity).

- Pulp density affects residence time and, hence, recovery. Roughers are typically 30%– 40% solids and cleaners typically <20% solids. Increase pulp density by increasing the number of cleaner cells.
- Adjust column cell air supply pressure to the supplier's recommendation.
- Recycling of streams can help in some situations and hinder in others.
- Process water in industrial minerals can be acidic in one part of the plant and basic in another; keep them separate to avoid problems. Process water separation may also be needed because of the presence of certain chemicals in them. For example, water used in a moly circuit should not be used in a copper circuit. Moly process water typically has sodium hydrosulfide (NaHS), which is a copper depressant.
- Flotation circuits can be open circuit (easiest to operate), cocurrent, or countercurrent (hardest to operate even with small upsets).
- Slimes create problems of poor selectivity, high reagent consumption, and low concentrate grade. Add dispersant such as sodium silicate to the grinding circuit; however, overdosing causes problems in thickening, filtration, and settling in the tailings pond.
- If cells sand up, the cause is probably worn impellers or diffusors.
- To check airflow rate in the cells, place an inverted 2-L (liter) cylinder filled with water in the cell's pulp and time the emptying of the cylinder.
- Staged flotation reactors (SFRs) have begun to be used in flotation plants in both roughing and cleaning stages. Their design uses significantly less air, which results in a cleaner flotation product in fewer stages. Also, washing can be done on all stages.

Testing

- Conduct a recovery versus grind P_{80} series.
- If flotation is poor and recovery low, test a feed sample in the laboratory without additional reagents; follow up tests with additional collector dosages.
- Investigate other reagents.
- Perform tests at various agitation speeds and compare recovery and grade. If higher agitation gives higher recovery with the same grade, increase airflow to cells.
- Calculations (Taggart 1956):

$$Ff = Cc + Tt \quad \textbf{(EQ 20.27)}$$

$$\text{ratio of concentration} = K = F/C = \frac{(c - t)}{(f - t)} \quad \textbf{(EQ 20.28)}$$

$$R = \%\ \text{recovery} = 100\frac{c(f - t)}{f(c - t)} \quad \textbf{(EQ 20.29)}$$

$$\text{enrichment ratio} = \frac{c}{f} \quad \textbf{(EQ 20.30)}$$

$$\text{pulp density and percent solids} = x = \frac{100s(D - l)}{D(s - l)} \quad \textbf{(EQ 20.31)}$$

where

F, C, T = weight of feed, concentrate, and tails
f, c, t = assays for feed, concentrate, tails
x = % solids by weight
s = density of solid
l = density of liquid

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SOLID–LIQUID SEPARATIONS by David S. Davies, P.E.

In solid–liquid (S–L) separations, the most important variables are the size distribution of the solids and the density of the solids. Solids with relatively uniform particle size will normally separate well, even fine sizes. The separation of solids with broad size distribution is greatly affected by the amount of –44 μm (325 mesh) particles.

Unit operation is determined by particle size range (in micrometers) versus unit operation:

- Screening, 40 μm to large sizes
- Classification, 10–1,000+ μm
- Hydrocyclones, 5–1,000+ μm
- Thickening, 1–1,000 μm
- Clarification, 1–1,000 μm
- Centrifugation, 0.5–600 μm
- Filtration, 0.1–600 μm

To reduce filter cake moisture, consider using surfactants, steam hooding, belt presses, centrifuges, hyperbaric filters, pressure filters, and ceramic filters. And to select an S–L separator, use the following criteria:

- To choose the process, determine what results are expected of the separator.
- Examine the feed:
  - Quantity of solids
  - Temperature
  - pH
  - Viscosity
  - Specific gravity (sg) of solution and solids
  - Percent solids
  - Size distribution of solids
- Calculate the separation rates via Büchner funnel (in liters per minute per square meter) or leaf filter testing. Measure the vacuum (i.e., in mm Hg) and the time required to form a cake of *x* thickness.
  - Consider the rate the solids settle by gravity and determine the percent of total feed volume the settled solids occupy after settling *x* hours.
- Determine if flocculants, filter aids, or anti-foaming agents are needed and what their impact will be.
- Decide if washing is needed and choose temperature and the quantity of wash allowable.
- Consider the clarity of the separated liquid and what percent solids is needed in the underflow or cake.

Thickening

Feed is usually 5% solids or more; however, thickeners perform best at lower feed percent solids. Table 20.11 shows theoretical concentration in solution from final countercurrent decantation (CCD) thickener.

Sizing a Thickener

Sizing a thickener depends on application, percent solids of the feed, and other variables as shown in Table 20.12. The following equation can be used to help size a thickener.

$$W\left(\frac{1}{F}-1\right)=AV\text{sg}_L+W\left(\frac{1}{u}-1\right) \quad \textbf{(EQ 20.32)}$$

where

W = weight of solid, t/d
F = feed slurry pulp density, % solids
A = area of thickener, m^2
V = velocity, m/d
sg_L = specific gravity of liquid
U = underflow slurry pulp density, % solid

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TABLE 20.11 Concentration in solution from final countercurrent decantation thickener*

Number of Stages†	Wash Ratio‡		
	1	2	3
2	0.333	0.143	0.077
3	0.250	0.067	0.025
4	0.200	0.032	0.008
5	0.167	0.016	0.003

* Concentration in feed solution = 1.000. Feed solution flow = solution in each underflow. Leaching is assumed complete before countercurrent decantation (CCD).

† Complete mixing of slurry and wash water in each stage. No evaporation; no other solutions added.

‡ Wash ratio (WR) = wash flow/feed solution flow in dimensionless units. Wash water flow = each thickener overflow. Wash water concentration = 0.000. If wash water has a concentration, that concentration is added to the concentration of every solution in the circuit.

When thickening,

- Determine the thickener area in square meters per metric ton of solids per day versus the feed rate or percent solids in feed.
- Determine the underflow percent solids versus the solids residence time.
- Be sure flocculants are dilute enough and well mixed into the thickener feed.
- Conduct a flocculant comparison study.
- Repair center wells and overflow weirs to stop short circuits.
- Consider an additional CCD stage for reduced soluble loss.
- Increasing the water temperature from 7.2°C (45°F) to 23.9°C (75°F) changes the viscosity of water so much that settling rates increase by about 50%. This implies that flocculant usage can be decreased during warm weather months.
- Conventional thickeners are easy to operate with high underflow densities and high throughput. Feed enters the feed well above the settled solids but has a large footprint.
- High-density thickeners offer extended solids retention time, but high torque requirements limit diameter to 60 m. They may increase underflow density by 10–12 percentage points. Feed enters in the settled solids zone. Flocculants must always be used.
- High-rate thickeners give two to eight times the capacity compared to conventional thickeners. Efficient flocculation is required. Generally, they offer better overflow quality and lower underflow densities than conventional thickeners. Tight control is required, and size is usually limited to 35–40 m in diameter.
- Ultra-high-rate thickeners have deep settling and compaction zones. They are rakeless with internal recycling solution for feed dilution and flocculation. Generally, they give low overflow turbidity and high underflow density with diameters approximately 33%–50% of high-rate thickener diameters for equal capacity.

Clarification

- Clarification is generally used for <5% solids slurries (solids are in free settling mode).
- Sizing is controlled by overflow rate, $m^3/(m^2 \times h)$.
- Determine bulk settling rate, overflow, and underflow solids concentration versus residence time and rheology of underflow.
- Conventional clarifiers are used for very slow settling particles without feed enhancement. Their sizes range from 5 to 25 $m^3/(m^2 \times day)$, and they are used with or without flocculant. Conventional clarifiers are subject to upset with feed or temperature change.

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TABLE 20.12 Typical settling data

Application	Feed, % solids	Underflow	Solids, t/m^{2}d
High-Rate Thickeners and Clarifiers			
Alum sludge	0.5	2–4	0.1–0.25
Alumina, red mud washers	10–15	30–40	0.25–1.0
Coal, clean fines	1–7	25–40	3.5–16.0
Copper concentrate	15–30	60–75	25–50
Copper leach residue	5–15	45–50	6–33
Copper tailings	10–35	50–65	16–50
Gold ore, NaCN leached	10–33	40–60	12–50
Kaolin slurry	1–6	20–30	–
Lime-neutralized pond water			
Partially neutralized	2.7	30–40	4
Second-stage neutralized	2.5	13–20	2.5
$Mg(OH)_2$ from brine	9	25–50	20
$Mg(OH)_2$ from seawater	2–6	10–25	1–10
Magnetite (heavy media)	12–18	60–70	30–50
Metal hydroxides (effluent treatment)	0.1–1.0	2–10	–
Molybdenum tailings	20–30	50–70	30–50
Phosphate slimes	2–8	12–18	6–17
Sand tailings	2–10	30–40	–
Soda ash, primary feed	1–3	8 –20	30–50
Taconite tailings	10–15	55–65	–
Uranium acid-leached ore	10–30	45–65	16–50
Uranium, yellowcake	1–10	15 –40	1–10
Zinc concentrate	15–40	60–75	40–50
Zinc tailings	20–35	50–70	16–50
Conventional Thickeners			
Activated sludge (sewage treatment)	0.2	2 –3	0.025
Alumina, red mud primary	3–4	10–25	0.2–0.5
Coal, refuse	0.5–6.0	20–40	1–2
Copper flotation concentrate	15–40	60–75	2–8
Lead flotation concentrate	15–40	60–85	3–10
Nickel flotation concentrate	15–40	60–85	2–8
TiO_2 pigment clarification	0.1–1.0	20–25	0.06
TiO_2 effluent clarification	0.1–1.0	45–50	0.01
Zinc flotation concentrate	15–40	55–70	2.6

Source: Enviro-Clear 1997

- Reactor clarifiers use an internal solids recycle with draft tube arrangement, which promotes improved settling properties of solids. Flocculation is required. Reactor clarifiers have good overflow quality.
- Inclined plate and tube clarifiers are compact and used for smaller flows and fast settling, nonsticking solids. These clarifiers limit underflow retention time and are usually rakeless.
- Hopper clarifiers are rougher clarifiers used ahead of polishing filters, Flocculation is required. They are rakeless and operate with fluidized solids beds, but control is difficult.

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TABLE 20.13 Hydraulic classifiers and sizers

Type of Classifier*	Particle Size Range, μm	Feed Slurry Density	Typical Feed	Typical Applications
Nonmechanical				
Fluidized bed classifier	10–1,000	20%–60% solids	20–120 t/h/m^2	Fine coal recovery Upgrade specular hematite Heavy mineral sands
Cone	45–600	Not critical	2–100 t/h	Desliming
Mechanical				
Spiral	75–300	Above 50%, efficiency lowers	5–800+ t/h	Desliming concrete aggregate, sand, iron ore Closed-circuit grinding, washing and dewatering, desliming, process feed control
Rake	20–300	Above 50%, efficiency lowers	5–800+ t/h	Same as spiral
Bowl	100–325 mesh	65% solids	5–250 t/h	Closed-circuit grinding

* Other types of classifiers: hydroseparator, jet sizer, super sorter, siphon sizer.

Hydraulic Classifiers and Sizers

Hydraulic classifiers and sizers provide gravitational classification of slurry solids by particle size and density (Table 20.13). An upward-moving column of liquid separates solids into fractions that can be separately collected.

Hydrocyclones

The achievable separation is dictated by diameter of the unit, inlet and outlet dimensions, cone angle, feed pressure, slurry solids concentration, slurry viscosity, liquid density, solids density, particle shape factor, and particle size distribution. The *separation* is the particle size of which 1%–3% reports to overflow; larger sizes report to underflow. Newer hydrocyclone geometries require correction factors that are proprietary to original equipment manufacturers. The following procedure is applicable to older hydrocyclone geometries. Determine separation of a cyclone as follows:

1. Obtain base D_{50} from Figure 20.5.
2. Correct base D_{50} with factors from Figure 20.6 (feed concentration), Figure 20.7 (pressure drop between feed and overflow), and Figure 20.8 (sg of solids).
3. Multiply base D_{50} by each of the three correction factors to get an actual D_{50}, and multiply the actual D_{50} by 2.2 to get the separation size. This is approximate.
4. The flow rate through the cyclone versus the pressure drop is shown in Figure 20.9. Multiple cyclones may be needed to handle the flow.

Centrifugation

- Centrifugation makes finer separations than a hydrocyclone.
- Screening centrifuges are used for solids dewatering and washing.
- Disk stack centrifuges are used for separation of lighter liquid from heavier liquid or very fine solids from liquids (i.e., 1-μm range).
- Solid bowl scroll conveyor centrifuges are the most common type. They can process up to 265 L/min of slurry, make separations in the 2- to 5-μm range, and produce up to 2,500 g (grams).
- Flocculants are used to attain better clarity on centrate.

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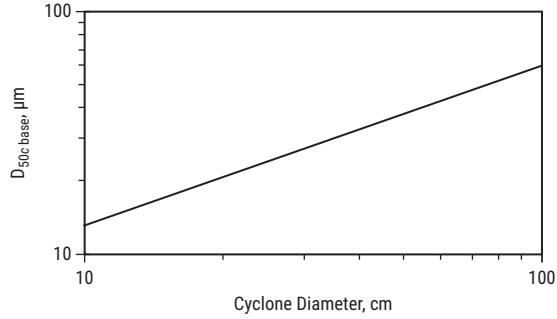
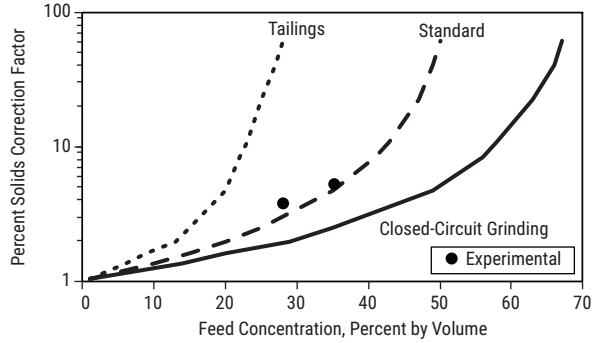


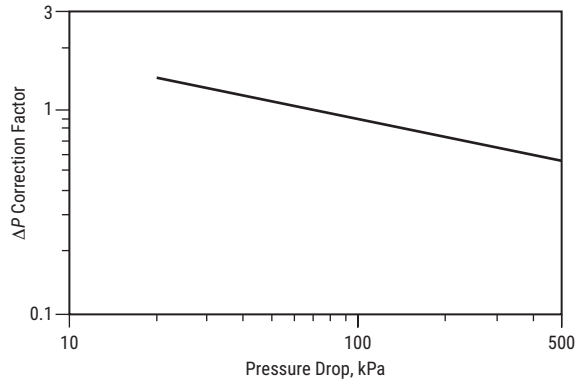
Source: Aterburn 1982

FIGURE 20.5 Cyclone diameter versus D_{50}



Source: Olson and Turner 2002

FIGURE 20.6 Correction for feed concentration



Source: Arterburn 1982

FIGURE 20.7 Correction for pressure drop

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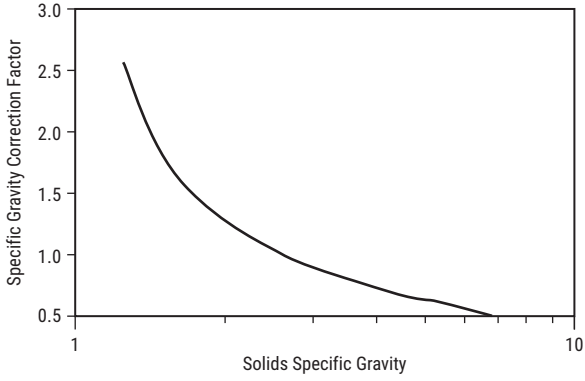
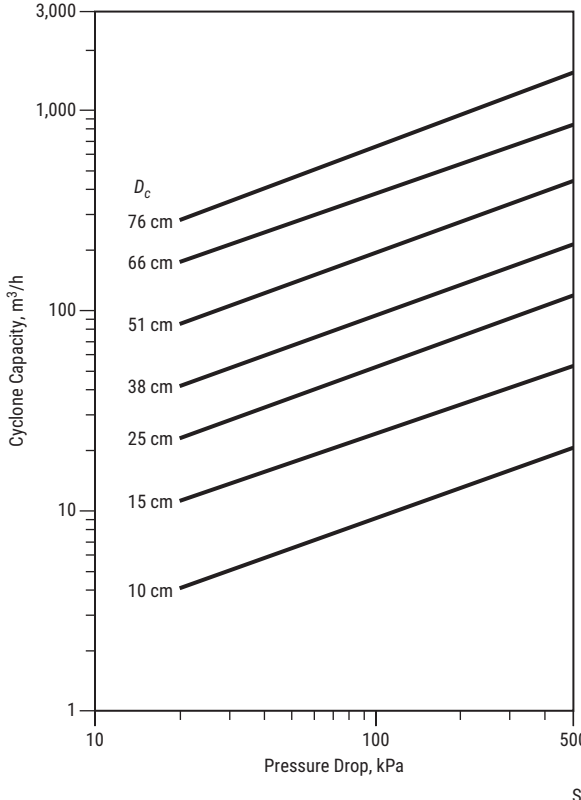


Source: Arterburn 1982

FIGURE 20.8 Correction for solids specific gravity



Source: Arterburn 1982

FIGURE 20.9 Volumetric flow rate versus pressure drop

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Vacuum Filtration

Tables 20.14 and 20.15, respectively, show the characteristics of vacuum filters and typical vacuum filtration results.

- The factors that affect filter selection are particle size distribution in feed, percent solids in feed, temperature, corrosiveness of the slurry, cake moisture, cake washing requirements, wash availability, wash composition, and altitude.
- To obtain reliable results, the following need to be obtained: detailed process knowledge of the separation to be made, separation objectives, bench-scale and/or pilot testing, and representative samples.
- Cake buildup on a laboratory vacuum leaf filter ranges from rapid to slow and filtration is difficult if 0.3-cm cake thickness cannot be formed in less than 5 minutes:
  - Rapid = 0.1–10 cm/s
  - Medium = 0.1–10 cm/min
  - Slow = 0.1–10 cm/h
- Gravity filters are used for free-draining solids. In the case of sand or mixed media filters, they are used to remove trace particulates.
- Other types of filters are precoat drum, drum, disk, horizontal belt, and pan.

TABLE 20.14 Vacuum filter characteristics

Filter Type	Solids Characteristics	Cake Thickness, mm	Wash Application Possible
Precoat drum	Very fine, used for liquor clarification	<1	Yes, but limited by wash arc available
Rotary drum	Must suspend feed solids with mild agitation; can be fine	>5	Yes, but limited by wash arc available
Rotary disk	Must suspend feed solids with moderate agitation; good filter rate	>10	No
Ceramic disk	Must suspend feed solids with moderate agitation; must be discrete, nonbinding solids as cake heel remains below knife	>3	No
Horizontal belt	Broad range; feed flocculation applies; versatile unit	>10	Yes, excellent; can be applied in countercurrent fashion
Pan type	Fast filtering solids; feed can be flocculated	>20	Yes, excellent; can be applied in countercurrent fashion

TABLE 20.15 Typical vacuum filtration results

Feed Solids	Feed, wt % solids	Particle Size	Filter Type	Filter Cake Moisture, wt %	Filtration Rate, kg dry solids/h/m^2
Alumina red mud	25–35	95+% −100 μm	Roller discharge drum	30–40	75–175
Alumina trihydrate seed	30–40	−40 μm	Disk	15–19	800–1,400
Alumina trihydrate product	40–50	30–60 μm	Horizontal belt scroll	12–14 13–16	2,000–5,000 750–1,500
Flotation coal	20–28	30%–35% −200 mesh	Disk	18–23 (surface)	350–450
Nonflotation coal	35–40	−28 mesh	Disk	16–26	250–500
Cement slurry	45–55	80%–90% −200 mesh	Drum disk	18–27	80–400
Copper concentrates	55–65	80%–95% −200 mesh	Disk drum	11–16	300–500

(continues)

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TABLE 20.15 Typical vacuum filtration results (continued)

Feed Solids	Feed, wt % solids	Particle Size	Filter Type	Filter Cake Moisture, wt %	Filtration Rate, kg dry solids/h/m^2
Cyanide slurry	40–50	70%–90% –200 mesh	Drum	24–30	80–400
Fluorspar	55–65	75%–90% –200 mesh	Continuous belt drum	8–13	250–1,000
Iron ore concentrate					
Magnetite	50–60	1,600–2,000 Blaine	Disk	8–10	625–1,500
Hematite	50–65	1,800–2,220 Blaine	Disk	9–11	300–1,000
Kaolin clay	30–35	70%–100% –2 μm	Roller discharge drum	35–40	75–250
Lead concentrates	60–70	80%–95% –325 mesh	Disk continuous belt drum	11–14	350–600
Magnesia precipitate	8–12	—	Drum	—	30–65 (MgO basis)
Digested phosphate rock	30–35	—	Pan	16–22	100–1,000
Sand	55–65	85%–95% +200 mesh	Horizontal belt, scroll	8–11	2,000–4,000
Tailings					
Coal	25–35	80% –200 mesh	Roller discharge drum	30–40	75–125
Iron ore	55–60	10%–15% –200 mesh	Horizontal belt	15–20	1,000–1,500
Uranium ore					
Acid leach	50–55	50%–60% –200 mesh	Continuous belt drum string horizontal belt	20–24	200–500
Carbonate leach	45–50	65%–80% –200 mesh	Horizontal belt	24–30	200–350
Uranium yellow cake	12–20	—	Drum	60–70	25–100
Zinc concentrates	50–65	80%–95% –325 mesh	Disk continuous belt drum	14–17	300–600

Source: Puhr et al. 1985

Pressure Filtration

Pressure filtration is used for fine solids separations that are not economic with vacuum units, to obtain lower cake moisture, and to provide more efficient solute removal from the cake (Table 20.16).

Flocculation

- Flocculation is probably the most used and most abused enhancement tool.
- Common problems are plant feed changes, feed variability, overflocculation, insufficient dilution of flocculant, poor flocculant/feed contact, poor water quality for flocculant dilution, and high shear after flocculation.
- Flocculation benefits are higher settling rates and smaller equipment, but there are trade-offs (i.e., liquor is bound within the floc structure and washing is made more difficult and less predictable). Overflocculation or improper selection of flocculant with respect to charge and molecular weight can dramatically increase slurry viscosity and negate any chance of reducing equipment size or increasing throughput.
- Effective flocculation can usually be maintained through a five-stage CCD thickener circuit but not six or more stages.

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TABLE 20.16 Pressure filtration equipment

Type	Attributes
Membrane (ultra-filtration)	Limited application; sensitive to both liquor and solids characteristics; produces good quality liquor and semi-concentrated pulp; membrane fouling control requires evaluation.
Leaf	Usually for clarification with use of precoat where liquor contains product for recovery; cleaning presents unique problems; batch or semicontinuous.
Horizontal plate	Inexpensive; single-platen types for low tonnages; labor intensive.
Tower press	Small footprint; multiple horizontal platens stacked vertically. Specific applications require sealing, feeding, cloth tracking, and cake discharge impact determination; can be maintenance intensive; relatively complex operation.
Tube	Capable of high pressure; small footprint; cake discharge and resealing make operation labor intensive.
Hyperbaric (fabric or ceramic media)	An adaptation of a disk filter in a pressure vessel; efficacy of mechanical seals on drive and cake discharge are application specific. In situ acid cleaning of media may be required; maintenance intensive; expensive spares.
Plate and frame	Good for small tonnage applications; labor intensive.
Recessed plate	Good for large tonnages if automated, otherwise labor intensive.
Moving belt	Light duty with low driving-force capability; used primarily for clarification; limited application. Needs flocculants.
Cross-flow	Used to concentrate solids; requires pumped or centrifugally induced velocity to preclude cake deposition on media; produced liquor rate and quality good; solids concentration to be discharged limited to pumpable slurry.
Filter/thickener	Used to concentrate difficult-to-dewater solids; limited application and use.

Process and Engineering Considerations

- S–L separation enhancements can be achieved by flocculation, improved upstream particle size control, solids densification, feed concentration alteration, use of admix for viscosity alteration, pH adjustment, and temperature adjustment.
- The use of high-rate and ultra-high-rate thickeners that require flocculation make it imperative to have controls and the ability to rapidly adjust operating conditions.
- Design and control philosophy must be compatible with operating philosophy.
- Samples need to be carefully prepared and evaluated. Sample aging effects, such as oxidation or other chemical or physical changes, can strongly affect results.
- A small change in particle size can dramatically affect results.
- Combining two S–L separation techniques may provide optimal performance (e.g., filtration after thickening). Flocculation is particularly affected by multistage operations, and the effects must be taken into account.
- Slurry rheology must be considered because of its impact on efficient S–L separations, rake mechanisms, pumpability, pipeline design, and mixing. When pumps start to cavitate or slurries do not easily flow, rheology modifiers may be helpful. Rheology of the thickened solids and the underflow can be measured with portable testing units.
- Viscosity cannot be estimated in the field from pressure drop over a known pipeline length. Design parameters need to include particle characteristics, pipe friction, and internal friction of the slurry.

SOLVENT EXTRACTION by John E. Litz

Two types of solvent extraction exist:

1. Liquid–liquid solvent extraction (SX) with organic extractant in an organic diluent
2. Ion-pair SX in which recoverable value is soluble in both the aqueous and organic solution

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Following are some general guidelines for SX:

- Applications include Cu, Zn, Ni, Co, Mo, V, W, Re, Zr, Hf, Be, B, Li, rare earths, Ga, Ge, Nb, Ta, Pt, Pa, and U.
- Extraction provides the most purification because of solvent selectivity. Mass transfer from aqueous solution to organic solution is governed by the laws of chemistry and by the practical aspects of mixing two immiscible liquids.
- Stripping provides the most concentration and recovers solvent in a form ready for recycle.
- Washing may be needed; for example, wash the loaded organic to remove Cl^- if saline water is used in the leach because electrowinning (EW) cannot tolerate much Cl^-.
- Regeneration may be needed to put the organic into chemical form needed for extraction; for example, if an amine is ammonia (NH_3) stripped, it is in free-base form and must be washed with weak acid to put it into hydrogen-bisulfate form.
- Favorable extractant properties are high selectivity for the element to be recovered, high stability, low toxicity, high flash point, high capacity, low solubility in aqueous solutions, low sg, high interfacial tension, noncorrosivity, low viscosity, and low cost.
- Extractant selection is based on literature reviews, batch tests, and continuous laboratory testing. General examples are amines for uranium, oximes for copper, methyl isobutyl ketone for hydrofluoric acid (HF) solutions of Ta and Nb, and alcohols for boric acid.
- Most diluents are aliphatic rather than aromatic. The solvent power of diluent can be exceeded by having too concentrated an extractant; molybdenum-amine complexes are notorious for this.
- Viscosity has significant effect on phase separation time (Table 20.17).

TABLE 20.17 Effect of viscosity on separation time

Aliphatic Diluents*	Kinematic Viscosity, cSt at 25°C	Settling Time, s
Aromatic 150	1.3	93
Escaid 100	1.8	94
Mining kerosene	2.1	114
Escaid 110	2.5	132
Isopar M	3.1	148

* Aliphatic diluents by Exxon.

Mixers

- Mixers have the most impact on efficiency and operating cost of the overall SX circuit.
- Configuration is cylindrical with baffles (to eliminate vortexing) or rectangular without baffles. Discharge to the settler should approach the full width of the mixer.
- Radial impellers give more shear than axial at the same revolutions per minute and tip speed.
- Pump mixers are preferred as they reduce capital and operating costs by eliminating the need for pumping between mixing stages. Entrainment can increase because of high-speed impeller shear. The impeller should be placed at the mixer bottom or above a draft tube. If many shutdowns are expected, placement above a draft tube allows the mixer to start in the dominant phase (the phase that most of the liquid surrounding the impeller is), making it the continuous phase rather than waiting for the mixed phase to invert to the desired continuous phase. The mixer head is controlled by the impeller speed and recirculation rate; thus, the impeller should be overdesigned so that recirculation can be used to control the head. If a draft tube is used and the aqueous phase has high sg, the tube should have holes that allow recirculation from the bottom of the mixer.
- Pump efficiency is approximately proportional to the impeller tip speed, for example, in cylindrical tanks with a height measuring the same as the diameter (Table 20.18).
- Entrainment is minimized when the impeller diameter is one-third of the tank diameter; it also is related to tip speed. Tip speed should be 150–300 m/min.
- The residence time for transfer equals the time in the mixer plus the time in the settler dispersion band.

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TABLE 20.18 Pump efficiency versus impeller tip speed

Speed, m/min	1.0/0.8 Aqueous-to-Organic Specific Gravity, %	1.2/0.8 Aqueous-to-Organic Specific Gravity, %
240	50	40
210	40	—
180	30	—

- Installing a top baffle at approximately 80% of the distance from the bottom to the overflow improves time in the mixer. The baffle should have a centered circular hole measuring about 50% of the impeller diameter.
- Phase continuity
  - If the impeller starts in aqueous (dominant) phase, aqueous is continuous. If it starts in organic phase, organic is continuous. Inverting continuity is difficult; a draft tube makes it easier because, at rest, the impeller can be in the dominant phase.
  - Continuity is hard to maintain as the mixer volume increases; for example, for organic continuous, organic flow as a percentage of total flow needs to increase.
  - Aqueous continuous (1) entrains less aqueous in the organic phase, (2) can minimize transfer of strip solution to the extraction circuit, and (3) can take more horsepower for the same transfer rate.
  - Organic continuous (1) entrains less organic in aqueous phase and (2) is good for raffinate and strip product stages.

Settlers

- The objective with settlers is to produce two phases with minimum entrainment.
- Settler sizing is done as part of a laboratory test program to get approximate sizes; approximately 81 L/min/m^2 is the general rule for oxime systems, but this can widely vary. Also, settlers must be designed larger to handle two problems that occur with cooler solutions:

  1. Flooding the settler with mixed phases that are slow to disengage
  2. Stabilizing the interface with solids or fungus, which slows phase disengagement

- The concentration of extractant in diluent can reduce the rate of phase separation if it is too high. Diluents with a higher solvent power may help. Alcohol or phosphate modifiers can help.
- The entry of the dispersion into the settler should be full width at the level of the dispersion band. A horizontal baffle directs the dispersion velocity horizontally.
- Minimize linear velocity of solutions through the settler. The maximum length-to-width (L/W) ratio is 4. L/W should decrease as throughput increases.
- Baffles are needed to reduce solution velocities. Picket fences work best. The first fence is close to the inlet to hold back the dispersion band. The picket is usually 50-mm wide. The fence is composed of two rows of pickets, one picket-width apart, with picket spacing in each row about one-third to one-quarter of a picket-width apart. If considerable dispersion still occurs downstream, a second picket fence at the settler midpoint helps.
- The flow rate of aqueous and organic phases should be <9 cm/s. If flow is higher, increase the thickness of that phase in the settler.

Controls and Testing

- Improve clarification of the pregnant leach solution.
- Check mixing efficiency at each stage. Take two samples from the mixer. Allow one to separate immediately and mix the other for 10 minutes more. Assume the extra mixing achieves 100% mixing efficiency. If the transfer in sample 1 is <80% of the transfer in sample 2, the mass transfer in that stage of mixing can be improved. Recirculation can improve efficiency. If adequate total mass transfer is

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being achieved, improved stage efficiency may allow elimination of one stage of mixing, thus reducing the power cost.
- Check dispersion band thickness throughout the settler's length to determine if the settler is nearing maximum capacity.
- Check for organic entrainment. Use a centrifuge with Babcock bottles. For oxime systems, >0.1 L/m^3 of aqueous is probably a problem; for other systems, the acceptable loss is proportional to the cost of extractant.
- Maintain constant solution flow to extraction. Use interface level controls to check on flow rates.
- Use a conductivity meter to check for phase continuity in mixers on a periodic basis.
- Monitor the mixer phase ratios on a periodic basis.
- Aqueous-to-organic (A/O) ratios are very system dependent and need to be optimized for each system in a continuous pilot plant.
- The settler area is very system dependent and needs to be determined by laboratory or pilot plant testing, especially if the system is operating near mechanical capacity.
- For oxime systems, treat crud with activated clay. For amine systems, adjust pH to acidic and use filtration.
- Install flotation columns on pregnant electrolyte and raffinate.
- Consider frequency-controlled adjustable speed drives for raffinate return pumps.

PRECIOUS AND BASE METAL SPENT-CATALYST RECYCLING

by Rahul S. Bhaduri, P.E.

Chemical and petrochemical processes use precious and base metal–bearing catalysts for facilitating and/or speeding chemical reactions without themselves undergoing any permanent chemical change.

Crude Oil and Chemicals Refining Catalysts

Refining catalysts are used to upgrade crude oil and play a major role in aiding refiners to (1) meet fuel standards; (2) better handle operational efficiency, including enhancing conversion and selectivity; and (3) keep pace with green trends. Precious metal–bearing catalysts are also used for pollution abatement applications, including eliminating atmospheric emissions of volatile organic compounds (VOCs) and other harmful pollutants.

Petroleum refineries use hydroprocessing units to prepare feedstocks for cracking and to polish final products. Hydroprocessing reduces the boiling range of petroleum feedstock and removes substantial amounts of impurities from the feed (Gary and Handwerk 1994). *Hydroprocessing* is a broad term encompassing the more specific processes of hydrotreating, hydrorefining, and hydrocracking. During hydroprocessing, molecules are split or saturated in the presence of hydrogen. Hydroprocessing reactors (Gerber et al. 1999) that hydrotreat petroleum feedstock stabilize the feed, remove impurities catalytically, and react the feed with hydrogen.

Hydrotreating includes the removal of sulfur, nitrogen, oxygen, metals, and other impurities from petroleum feedstocks. Hydrotreating processes are also specified as hydrodesulfurization (HDS), hydrodenitrogenation (HDN), hydrogenolysis (HDL), and hydrodemetallization (HDM) depending on the characteristics of the feedstock, although all subprocesses may take place to varying degrees on different hydrotreating catalysts. Spent catalysts removed from hydrotreating reactors are listed as Resource Conservation and Recovery Act (RCRA) hazardous K171 wastes (EPA Office of Solid Waste 2002b; EPA 2003).

Hydrorefining also removes impurities, but uses more severe operating conditions than hydrotreating, and treats heavier molecular weight petroleum fractions (e.g., residual fuel oil and heavy gas oil). Spent catalysts removed from hydrorefining reactors also are listed as RCRA hazardous K172 wastes (EPA Office of Solid Waste 2002a; EPA 2003; EPA Office of Solid Waste 2002b).

Hydrocracking is a process in which the primary purpose is to reduce the boiling range of petroleum feedstocks containing high levels of impurities and improve the yield of high-quality light oil fractions. Hydrocracking processes can be tailored to maximize different products such as low-sulfur fuel oil, lubricating oils, jet fuel, and diesel fuel. The U.S. Environmental Protection Agency (EPA) has not made a

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hazardous waste listing determination regarding the status of spent catalysts removed from petroleum hydrocracking reactors.

Various types of catalysts are used in refineries to augment the efficiency of refining operations including the following:

- **Hydrotreating** or removal of about 90% of contaminants such as nitrogen, sulfur, oxygen, and metals from liquid petroleum distillate fractions such as naphtha, gasoline, kerosene, jet fuel, and diesel oil includes saturation of carbon–carbon, olefinic, and aromatic bonds. The primary catalysts used are Mo, Ni, and Co based on an alumina substrate.
- **Hydrorefining** is similar to hydrotreating in that sulfur-containing hydrocarbons are converted into low-sulfur liquids and hydrogen sulfide. Nitrogen and oxygen compounds are also dissociated under a process operated at high temperatures and pressures. Co-Mo catalysts are selective for sulfur removal and Ni-Mo catalysts are selective for nitrogen removal.
- **Hydrocracking** is the conversion of high-boiling constituent hydrocarbons in petroleum crude oils to more valuable lower-boiling products such as gasoline, kerosene, jet fuel, and diesel oil. Hydrocracking includes splitting of carbon–carbon bonds and significant reduction of molecular weight. Catalysts used are Mo, W, Ni, and Co based on zeolite substrates.
- **Fluid catalytic cracking** (FCC) converts high-boiling, high-molecular-weight hydrocarbon fractions of petroleum crude oils to more valuable higher octane rated gasoline, olefinic gases, and other products. FCC catalysts do not operate in a hydrogen environment and work by promoting cracking to produce branched hydrocarbons that are advantageous in gasoline blends. Catalysts include powdered zeolites and rare-earth–exchanged Y-zeolites in an amorphous silica-alumina support.
- **Alkylation** converts low-molecular-weight alkenes (a mixture of propene and butene) into alkylates or high-octane gasoline components. The process occurs in the presence of a strong acid catalyst such as HF or sulfuric acid (H_2SO_4).
- **Reforming** converts naphtha distilled from crude oil into high-octane liquid products termed *reformates*. This route of dehydrogenation also generates significant amounts of by-product hydrogen gas, which is fed into other refinery processes such as hydrocracking. Catalysts used are platinum and/or rhenium on zeolite or amorphous silica-alumina support base.
- **Catalytic** dewaxing improves cold flow properties of diesel fuels and lube oils. Catalysts include Pt and Pd based on zeolite substrates.
- **RDS/VRDS** (i.e., atmospheric residual desulfurization and vacuum residual desulfurization) is used for atmospheric and vacuum residuals desulfurization and generation of transportation fuels and fuel oils; catalysts include Ni-Mo on gamma alumina support.
- **LC-Fining** (which is used for desulfurization, demetallization, Conradson carbon reduction, and hydroprocessing of atmospheric and vacuum residuals) is an ebullated or expanded bed process (as opposed to fixed) for atmospheric and vacuum residuals desulfurization, demetallization, and reduction in coke forming propensity; catalysts include Ni-Mo on a γ-alumina substrate.

Figure 20.10 represents a simplified block flow schematic of a generic oil refinery depicting the use of various catalysts during refining; the diagram omits several unit operations too complex for this review.

Catalyst Deactivation

The size and shape of the catalyst particles are usually well defined to minimize pore diffusion effects in the catalyst configurations and pressure drop across the reactor. During hydrotreating of heavy feeds, the catalyst exhibits a certain degree of deactivation that is dependent on the nature of the feed, the type of reactor, and reaction conditions.

The three basic deactivation mechanisms for refining catalyst are (1) coke deposition, (2) metals buildup and agglomeration, and (3) poisoning via feed contaminants (Gerber et al. 1999; Marafi and Stanislaus 2003; Furimsky and Massoth 1999). At deactivation, spent catalysts may contain nickel, vanadium, sulfur, and minor impurities such as phosphorus and arsenic (Table 20.19). To compensate for catalyst deactivation, the reactor temperature needs to be continuously increased to keep the product quality at the level desired. However, exposing the catalytic bed to high temperatures may cause catalyst support sintering, which is another reason for regeneration or recycling of the spent catalyst.

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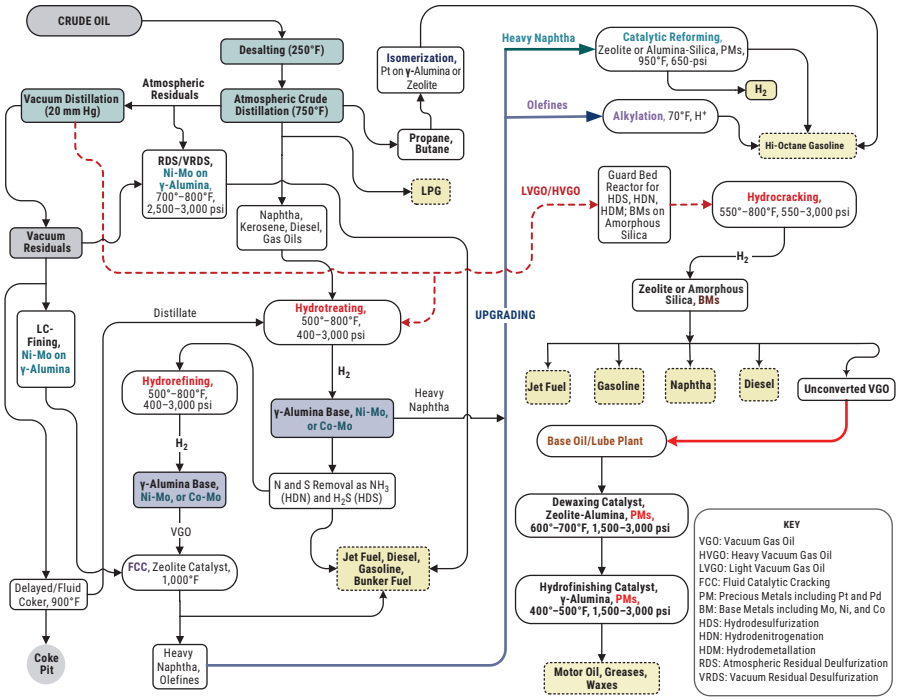

Source: Bhaduri 2016

FIGURE 20.10 Generic oil refinery

Irrespective of how catalysts are applied, or whether they take the form of monolithic structures, pellets, spheres, cylinders, or lobular conformations, most companies in the chemical and refining process industries rely on precious and base metal refiners to recover the valuable metals from their spent catalysts.

Waste Movement Control and Logistics

The Organization for Economic Cooperation and Development (OECD) forum includes a consortium of countries that adopted the Basel Convention in 1992 (OECD 2009). Briefly, it is an international treaty on the control of transboundary movements of hazardous wastes and their disposal. The Convention's main goal is to protect human health and the environment from hazards posed by transboundary movements of hazardous waste.

OECD categorizes wastes to be transported across borders into the following:

- Green covers wastes subject only to controls normally applied in commercial transactions.
- Amber covers wastes that contain hazardous components or exhibit hazardous characteristics. Advance notification of movement and a tracking document are required. Additionally, transport can take place only after a 30-day objection period has passed.
- Red covers wastes that require the same notification as for the Amber list, but for which written consent must be obtained from all concerned authorities before transport can take place.

Although the United States has signed the Basel Convention, it has not ratified the Convention; therefore, the United States cannot participate in waste transfers with Basel parties without a separate and equivalent bilateral or multilateral agreement.

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TABLE 20.19 Hydrotreating spent catalyst

Characteristics	Weight Percent
Moisture	8.0
Oil	12.0
C, fixed	12.0
S	6.0
Mo	4.0
V	5.0
P	0.3
Fe	0.4
Ni	2.5
Co	0.6
SiO_2	1.0
Al_2O_3	34.0
Other	14.2

Spent catalysts arriving by train, truck, rail, barge, and ship are contained in assorted vessels including bulk railcars and trucks, roll-off boxes, flow-bins, supersacks, steel drums, and a variety of totes. The choice of containers depends on the characteristics of the spent catalyst and the method of loading at the customer's location.

Spent catalysts are classified as K171 and K172 listed wastes by the EPA, which may be considered hazardous or nonhazardous depending on the source of oil and type of refining process. Spent catalysts are hazardous primarily because of their ignitable or pyrophoric character and/or the presence of arsenic or benzene (EPA 2003).

Spent-Catalyst Sampling

The information on spent-catalyst sampling in this section is taken from Jacobsen (2005a, 2005b, 2004), Hagelüken and Verhelst (2004), Hagelüken (2006), and Beirne (2018). The entire process begins with the spent catalyst being classified and stored according to the characteristics of the material. Table 20.19 shows a typical composition for hydrotreating spent catalyst. Some of the characteristics may be inherent to the original catalyst itself, including molybdenum, nickel, cobalt, and alumina. Other residuals are imparted to the spent material because of additives and materials that are naturally occurring in the crude oil, including VOCs, P, As, V, Ni, Fe, Na, and Si.

The fundamental principle of sampling involves reducing large quantities of precious or base metal–bearing material (up to several tons) into small representative samples. Sampling begins by converting precious or base metal–bearing scrap into as homogeneous a state as possible so that the concentration of precious metals and other constituents is evenly distributed. Results of sampling the homogeneous mass thus represent an accurate ratio of the precious or base metal content in the overall matrix.

The owner of the material and the refiner usually assay the quality samples (on a loss on ignition [LOI] basis) independently for the metals of concern. If these assays agree within predetermined limits, they are simply averaged to arrive at the payable settlement. If they do not agree, then a sealed sample (called the *umpire sample*) is sent to an independent laboratory (the umpire). The three resulting assays are used by an agreed-on procedure to determine the settlement. This might involve averaging the two closest assays or using the middle assay to determine the final settlement. The reserve samples (usually sealed by both the catalyst owner and the refiner) are held as backup to cover any possible irregularities during the sampling procedures.

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Pretreatment of Spent Catalyst

The information on spent-catalyst pretreatment in this section is taken from Jacobsen (2005a, 2005b, 2004), Hagelüken and Verhelst (2004), Hagelüken (2006), Beirne (2018), and J.K. Litz (personal communication). Any sampling system, especially one designed to handle large lots, requires materials that are free-flowing. This generally is not the state of the spent catalyst when it is removed from the process unit at the refinery, thus it requires pretreatment.

The spent catalyst may require deoiling to remove hydrocarbon feed and product oils. One means of deoiling is solvent deoiling, which removes the petroleum hydrocarbons without removing the coke. Deep deoiling of greater than 98% is accomplished by deoiling the spent oiled catalyst in the presence of an organic solvent such as toluene, xylene, or kerosene. During deoiling, the spent catalyst may become air reactive so the operation could be performed under an inert atmosphere such as nitrogen. Deoiling may be expedited by increasing temperature, but this increases the cost as use of a pressure vessel becomes necessary. Following deoiling, the solvent is stripped and separated from the hydrocarbon oil and recycled. The recovered oil is recycled to the upstream hydroprocessing unit and may account for up to 30 wt % of the original spent catalyst.

Alternatively, the spent catalyst may frequently be moist or sticky and cannot flow through an automatic sampler. Heavy carbon deposits, termed *coke balls*, are often present and can skew sample homogeneity. Catalyst contaminated with wet hydrocarbon displays serious safety and handling issues. In addition, contaminated catalyst cannot be sampled and tested properly and therefore requires pretreatment to oxidize organic matter. The spent-catalyst load may also contain ceramic carrier media, which requires screening and segregation prior to pretreatment.

Pre-burn temperatures of up to 500°C in indirectly fired rotary kilns may remove these physical impediments to achieving a representative sample. The typical rotary kiln can remove carbon and sulfur contaminants at catalyst throughput rates of 300–1,000 lb/h. The contaminants may also be removed in multiple-hearth or fluidized-bed furnaces. An indirectly fired rotary kiln (calciner) is more versatile and controllable than other methods, in that it is easier to control the residence time, the temperature, and the oxygen supply to the reaction. The heat conducts through the shell of the rotary kiln and heat transfer occurs from the feed solids being in contact with the rotary kiln shell (Le Capitaine and Carlson 2018).

Roasting Plant

The roasting plant is used to produce calcine that provides feedstock for the remaining downstream metal separation and recovery operations. Spent catalysts may be mixed with chemicals, such as soda ash, and thermally treated using controlled amounts of heat and oxygen to burn residual oil present in the catalysts and to oxidize molybdenum and vanadium to their highest valence states. These reactions produce soluble and insoluble metal compounds depending on the types of catalyst and their associated contaminants.

Direct fired rotary kilns (Figure 20.11) introduce heat directly into the internals of the kiln via a stream of hot gas. Although greater heat transfer occurs as compared to indirect fired kilns, there is more gas exiting the rotary kiln that requires processing and treatment. Operating temperatures of up to 900°C are generally sufficient for the oxidation roast of most spent catalysts.

Multiple-hearth furnaces and fluid bed roasters provide excellent thermal properties while remaining efficient for the high volumes of material that are processed. Temperature in the reaction hearths or fluid bed may be in the range of 600–900°C; generally, retention time of up to 8 hours is required for multiple hearths and up to 2 hours for fluid bed roasters.

Multiple-hearth furnaces (Figure 20.12) consist of a series of circular hearths, stacked one above the other and enclosed in a refractory-lined steel shell (IFC, n.d.). A vertical variable-speed drive rotating shaft through the center of the furnace carries arms with rabble blades that stir the charge and move it in a spiral path across each hearth. Material is fed to the top hearth and rabbled across it to pass through drop holes to the hearth below. It passes in this way over and across each hearth to the bottom where the product is discharged through one or more ports. Heated gases flow counter- or cocurrently to heat the charge to reaction temperature and to continue the desired reaction.

Fluidized-bed reactors (Figure 20.13) are used not only for roasting, but also for drying and calcination. Gas at a certain critical velocity (Gupta and Sathiyamoorthy 1999) is bubbled up through a bed of particles,

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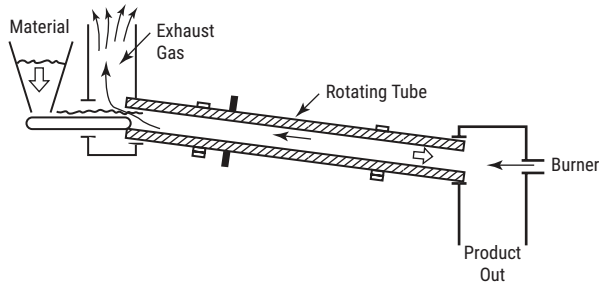
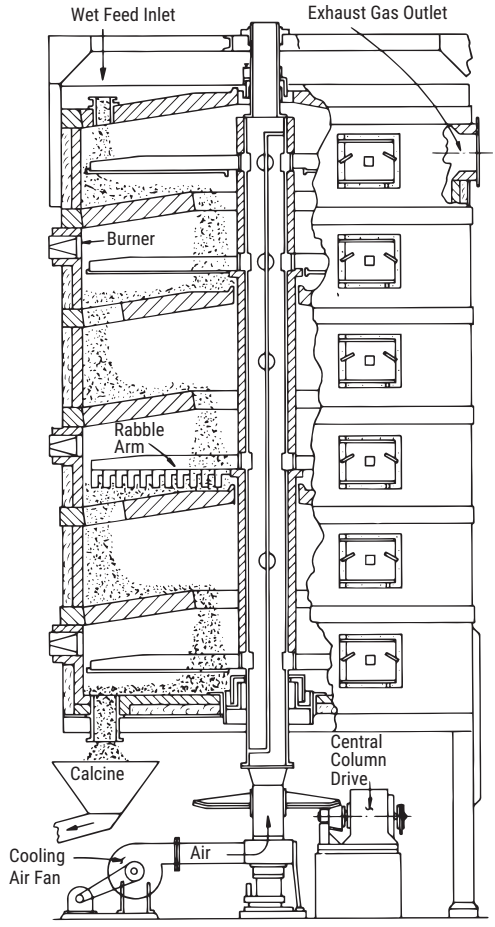


Courtesy of 911 Metallurgist

FIGURE 20.11 Direct fired rotary kiln



Courtesy of M.S.I. Industries, Inc.

FIGURE 20.12 Multiple-hearth roaster

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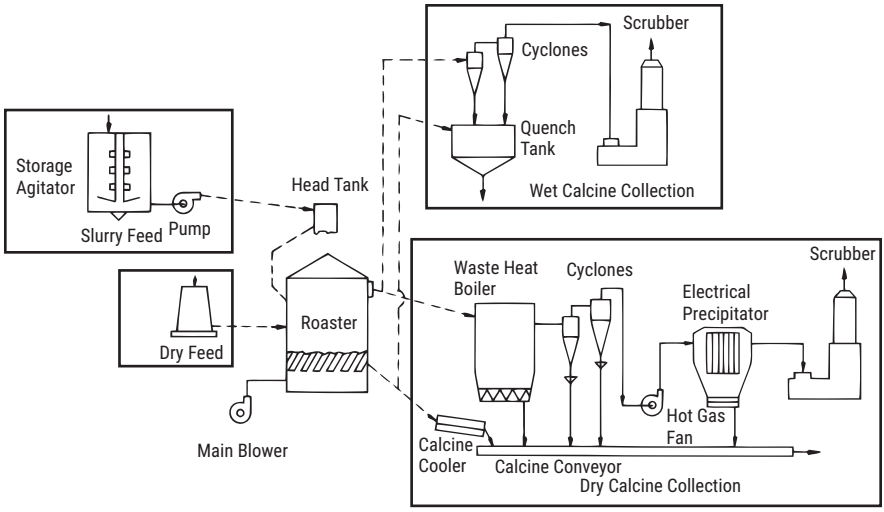

Courtesy of Dorr-Oliver, Inc.

FIGURE 20.13 Fluidized-bed roaster

with the particles large enough that they are not swept from the furnace, but small enough that the gas can expand the bed so that it behaves as a fluid. These reactors provide excellent gas/solid contact without requiring the presence of extremely fine particles.

Some general sulfide oxidations are depicted in Table 20.20. Table 20.21 displays water-soluble base metal reactions formed with an alkali metal compound such as soda ash.

Sulfur Dioxide, Carbon Dioxide, and Particulate Capture

The roasting process converts metal sulfides into their oxide forms in addition to liberating particulate matter, hydrocarbons, and oxides of sulfur from the spent catalyst (Table 20.20; Reactions 1 through 5). The roasters may operate in conjunction with a post-combustion chamber (PCC), gas cooling, cyclones, and electrostatic precipitators (ESPs).

TABLE 20.20 Roasting reactions

1. $4FeS_2 + 11O_2 \rightarrow 2Fe_2O_3 + 8SO_2\uparrow$
2. $As_2S_3 + 9/2O_2 \rightarrow As_2O_3 + 3SO_2\uparrow$
3. $MoS_2 + 7/2O_2 \rightarrow MoO_3 + 2SO_2\uparrow$
4. $NiS + 3/2O_2 \rightarrow NiO + SO_2\uparrow$
5. $V_2S_3 + 11/2O_2 \rightarrow V_2O_5 + 3SO_2\uparrow$
6. $C + O_2 \rightarrow CO_2\uparrow$

TABLE 20.21 Soda ash roasting reactions

1. $Na_2CO_3 + MoO_3 \rightarrow Na_2MoO_4 + CO_2\uparrow$
2. $Na_2CO_3 + V_2O_5 \rightarrow 2NaVO_3 + CO_2\uparrow$
3. $3Na_2CO_3 + As_2O_3 \rightarrow 2Na_3AsO_3 + 3CO_2\uparrow$
4. $Na_2CO_3 + SO_2 + 1/2O_2 \rightarrow Na_2SO_4 + CO_2\uparrow$

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During roasting, hydrocarbons are combusted (Table 20.20; Reaction 6) and sulfides are oxidized to sulfur dioxide and sulfur trioxide (SO_3), which require scrubbing or conversion into sodium sulfate (Table 20.21; Reaction 4) in the presence of soda ash or caustic. Metal oxides react with soda ash to form the corresponding water-soluble sodium salt (Table 20.21; Reactions 1–3). Because SO_3 forms sulfuric acid in the presence of water vapor at cooling, temperatures in the gas handling system must be maintained well above the dew point of sulfuric acid to mitigate corrosion.

The PCC incinerates residual hydrocarbons at 900°C (Llanos and Deering 1995). Cyclones and/or ESPs may be utilized in conjunction with high-temperature baghouses and circulating dry scrubbers to capture particulates and sulfur. The off-gases are humidified with water sprays and contacted with hydrated lime or magnesium hydroxide, $Mg(OH)_2$, in a countercurrent entrained bed reactor where the reaction consumes the water and generates a dry by-product such as in Table 20.22.

TABLE 20.22 Hydrated lime dry by-products

1. $Ca(OH)_2 + SO_2 + 1/2O_2 \rightarrow CaSO_4 + H_2O$
2. $Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O$

Hydrometallurgical Plant

Figure 20.14 depicts a generic block flow of unit operations for reclaiming Mo-Ni-V from spent hydroprocessing alumina catalyst under a combination of both hydro- and pyrometallurgical routes. The roasting scenarios include the presence and the absence of soda ash, with the potential to directly generate ferroalloy product under the latter setting.

Base Metals

The information on base metals in this section is taken from Llanos et al. (1986). The calcine generated during the roasting process (see the preceding "Roasting Plant" subsection) may be ground and leached in 70°C hot water (calcine roasted with soda ash) or caustic (calcine roasted without soda ash) to produce a slurry containing all of the remaining metals and contaminants. Reactions 1 and 2 in Table 20.23 depict solubilization of the oxides of Mo and V in a 50 wt % caustic leach.

Depending on roasting conditions, the γ-aluminum oxide (Al_2O_3) support may react with caustic during the leach to form partially water-soluble aluminate species. Aluminum dissolution is minimal with a soda ash roast at 700°C, increasing significantly with caustic roasting at 500°C (Marafi et al. 2014).

Grinding may be necessary to ensure that the solids have a uniform particle size distribution to increase the dissolution rate of soluble metals and to improve the efficiencies of mechanical decantation and/or filtration. Using countercurrent thickening and filtration techniques, a solid concentrate containing alumina

TABLE 20.23 Base metals reactions

1. $V_2O_5 + 2NaOH \rightarrow 2NaVO_3 + H_2O$
2. $MoO_3 + 2NaOH \rightarrow Na_2MoO_4 + H_2O$
3. $Al_2O_3 + 2NaOH \rightarrow 2NaAlO_2 + H_2O$
4. $MgSO_4{\cdot}7H_2O + PO_4^{3-} + NH_4^+ \rightarrow MgNH_4PO_4{\cdot}6H_2O\downarrow + SO_4^{2-} + H_2O$
5. $MgSO_4{\cdot}7H_2O + AsO_4^{3-} + NH_4^+ \rightarrow MgNH_4AsO_4{\cdot}6H_2O\downarrow + SO_4^{2-} + H_2O$
6. $NaAlO_2 + 2H_2O \rightarrow Al(OH)3\downarrow + NaOH$
7. $2NaVO_3 + (NH_4)_2SO_4 \rightarrow 2NH_4VO_3 + 2Na_2SO_4$
8. $NH_4VO_3 + 2NH_4OH \rightarrow (NH_4)_3VO_4 + H_2O$
9. $2NH_4VO_3 \rightarrow V_2O_5 + 2NH_3 + H_2O$
10. $Na_2MoO_4 + (NH_4)_2SO_4 \rightarrow (NH_4)_2MoO_4 + Na_2SO_4$
11. $(NH_4)_2MoO_4 + H_2SO_4 \rightarrow MoO_3{\cdot}H_2O + (NH_4)_2SO_4$

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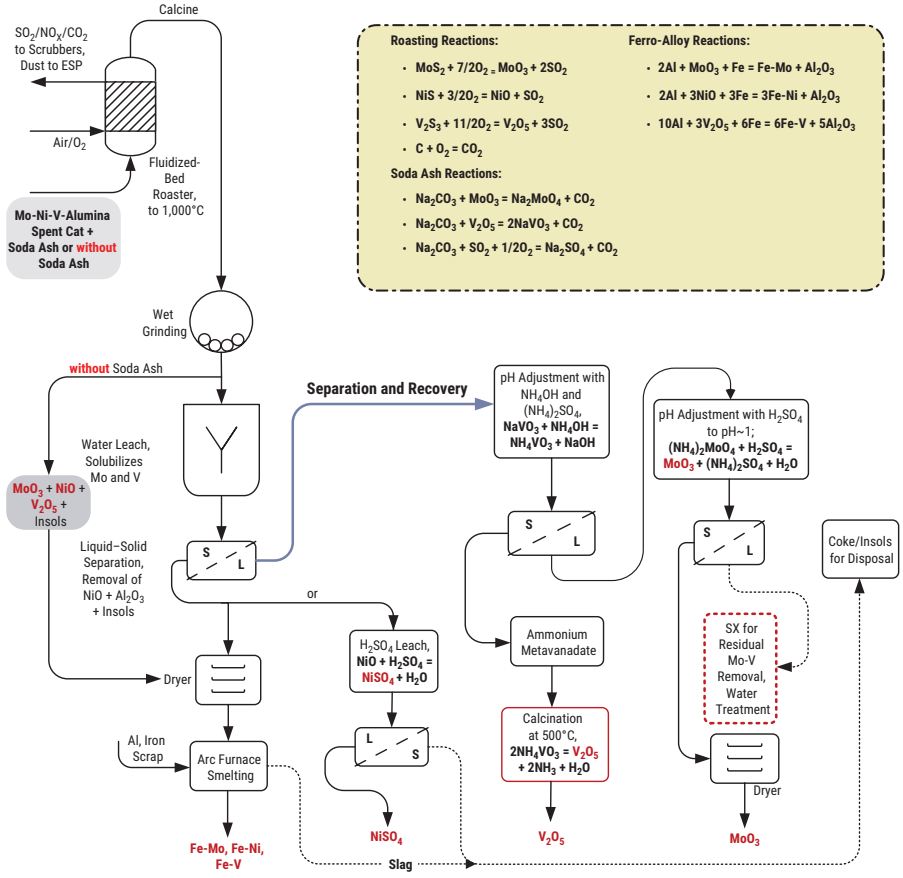
Calcine
$SO_2/NO_X/CO_2$ to Scrubbers, Dust to ESP
Air/O_2
Fluidized-Bed Roaster, to 1,000°C
Mo-Ni-V-Alumina Spent Cat + Soda Ash or without Soda Ash
Roasting Reactions:
$MoS_2 + 7/2O_2 = MoO_3 + 2SO_2$
$NiS + 3/2O_2 = NiO + SO_2$
$V_2S_3 + 11/2O_2 = V_2O_5 + 3SO_2$
$C + O_2 = CO_2$
Soda Ash Reactions:
$Na_2CO_3 + MoO_3 = Na_2MoO_4 + CO_2$
$Na_2CO_3 + V_2O_5 = 2NaVO_3 + CO_2$
$Na_2CO_3 + SO_2 + 1/2O_2 = Na_2SO_4 + CO_2$
Ferro-Alloy Reactions:
$2Al + MoO_3 + Fe = Fe\text{-}Mo + Al_2O_3$
$2Al + 3NiO + 3Fe = 3Fe\text{-}Ni + Al_2O_3$
$10Al + 3V_2O_5 + 6Fe = 6Fe\text{-}V + 5Al_2O_3$
Wet Grinding
without Soda Ash
Separation and Recovery
pH Adjustment with NH_4OH and $(NH_4)_2SO_4$, $NaVO_3 + NH_4OH = NH_4VO_3 + NaOH$
pH Adjustment with H_2SO_4 to pH~1; $(NH_4)_2MoO_4 + H_2SO_4 = MoO_3 + (NH_4)_2SO_4 + H_2O$
Water Leach, Solubilizes Mo and V
$MoO_3 + NiO + V_2O_5$ + Insols
Liquid–Solid Separation, Removal of NiO + Al_2O_3 + Insols
S
L
or
Coke/Insols for Disposal
H_2SO_4 Leach, $NiO + H_2SO_4 = NiSO_4 + H_2O$
Ammonium Metavanadate
SX for Residual Mo-V Removal, Water Treatment
Dryer
Al, Iron Scrap
Arc Furnace Smelting
Calcination at 500°C, $2NH_4VO_3 = V_2O_5 + 2NH_3 + H_2O$
$NiSO_4$
V_2O_5
MoO_3
Fe-Mo, Fe-Ni, Fe-V
Slag

Courtesy of R.S. Bhaduri

FIGURE 20.14 Base metals (Mo, Ni, V) spent-catalyst reclaim

along with oxides of nickel and/or cobalt are produced. This material provides the base feedstock for the pyrometallurgical plant (see the "Pyrometallurgical Plant" section).

The clarified filtrate may be treated with magnesium sulfate heptahydrate ($MgSO_4 \cdot 7H_2O$) at slightly ammoniacal pH to remove soluble phosphorus, arsenic, and any solubilized aluminum (Reactions 4 and 5 in Table 20.23). The purity of vanadium pentoxide (V_2O_5) and molybdenum trioxide (MoO_3) is reduced when these contaminants are present, ultimately reducing yield.

The filtered leach solution containing high concentrations of Mo and V is mixed with ammonium sulfate and ammonia solution at ~50°C to exceed the solubility product of ammonium metavanadate (AMV) and precipitate at pH between 8 and 9 (Reaction 7 in Table 20.23). The pH is controlled by acidification to below 9.0 to obviate V conversion into the soluble orthovanadate (VO_4^{3-}) species per Reaction 8 in Table 20.23 (J.K. Litz, personal communication). Chilling the solution to ~10°C increases V yield and lowers precipitation residence time (Grimley 2013).

The filtered and cold water–washed AMV may be calcined at 500°C to decompose it into ammonia and V_2O_5 (Reaction 9 in Table 20.23).

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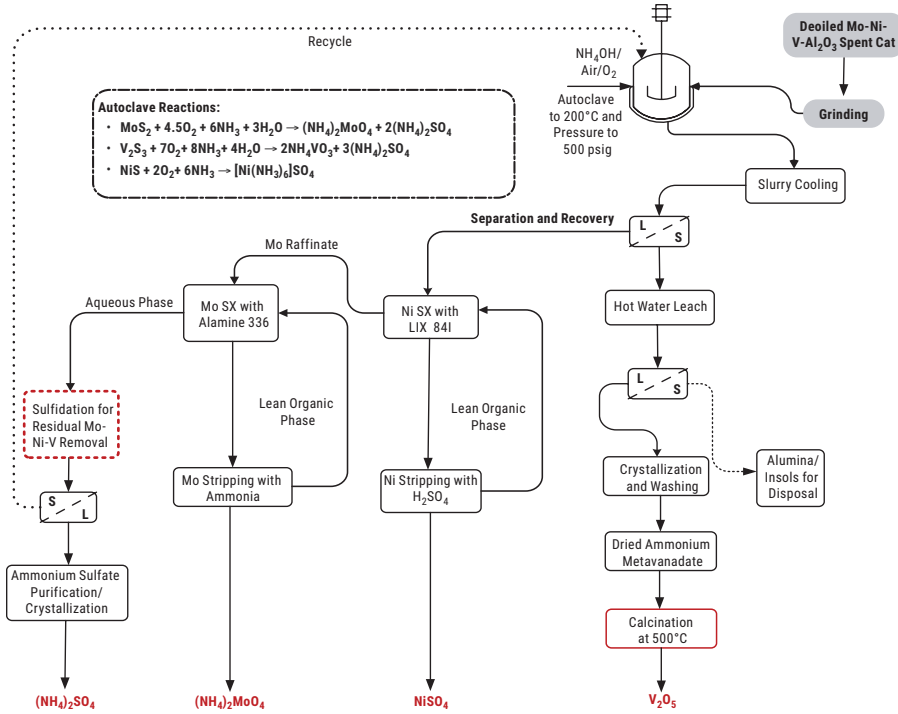

Adapted from Marcantonio 2009; Hubred and Van Leirsburg 1985

FIGURE 20.15 Conceptual base metals (Mo, Ni, V) spent-catalyst reclaim

The granular V_2O_5 is fused and quenched on a rotating wheel to produce flakes typically containing more than 99% V_2O_5. Particulate matter and ammonia are further recovered in a series of baghouses and scrubbers.

Molybdenum is the primary component of the filtrate from the AMV precipitation sequence and is present as ammonium molybdate (Reaction 10 in Table 20.23). This filtrate is heated to between 80° and 85°C and acidified with sulfuric or nitric acid to precipitate molybdic acid (Reaction 11 in Table 20.23) at approximately pH 1. After filtration and washing, the molybdic acid is calcined to produce MoO_3 at >98% purity.

Soluble molybdenum and vanadium residuals in the filtrate may be recovered through SX with Aliquat 336, a quaternary ammonium salt; the Mo may be recovered as sodium molybdate with caustic stripping. The strip liquor is recycled and the raffinate is sent to the ammonia recovery circuit. The ammonia gas is recovered by scrubbing with dilute sulfuric acid to regenerate ammonium sulfate.

Alternate Conceptual Base Metal Process

This section is taken from Marcantonio (2009) and Hubred and Van Leirsburg (1985). Roasting is the primary commercial means of sulfur and carbon removal, followed by hydrometallurgical techniques to recover metals of relevance; however, interest on a bench and pilot scale have been engaged toward direct pressure leaching of deoiled and ground as-is feed material with ammonia at temperatures of up to 200°C and oxygen overpressures up to 500 psig (Figure 20.15).

Reactions 1 through 3 in Table 20.24 depict metal sulfide oxidation in ammoniacal solutions. Vanadium as AMV may be suppressed into the unreactive alumina-coke phase with excess ammonium sulfate during pressure oxidation; this is established by the common ion effect inducing lowering of the solubility product of AMV.

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TABLE 20.24 Metal sulfide oxidation in ammoniacal solutions

1. $MoS_2 + 4.5O_2 + 6NH_3 + 3H_2O \rightarrow (NH_4)_2MoO_4 + 2(NH_4)_2SO_4$
2. $V_2S_3 + 7O_2 + 8NH_3 + 4H_2O \rightarrow 2NH_4VO_3 + 3(NH_4)_2SO_4$
3. $NiS + 2O_2 + 6NH_3 \rightarrow [Ni(NH_3)_6]SO_4$

TABLE 20.25 Precious metals reactions

1. $Al_2O_3 + 2NaOH \rightarrow 2NaAlO_2 + H_2O$
2. $Pt + 3H_2O_2 + 6HCl \rightarrow H_2PtCl_6 + 1/2O_2 + 5H_2O$
3. $Pd + 2H_2O_2 + 4HCl \rightarrow H_2PdCl_4 + 1/2O_2 + 3H_2O$
4. $Pt + 4HNO_3 + 6HCl \rightarrow H_2PtCl_6 + 4NO_2^{\uparrow} + 4H_2O$
5. $Pd + 2HNO_3 + 4HCl \rightarrow H_2PdCl_4 + 2NO_2^{\uparrow} + 2H_2O$

The filtered pressure leach solution at pH ~9.5, containing soluble Mo, Ni, and residual V, can be routed to an SX mixer-settler circuit where Ni can be extracted into a ketoxime such as LIX 84-I. Ni can be stripped from the organic phase with 10% H_2SO_4 acid, as nickel sulfate ($NiSO_4$). The Mo raffinate can be routed for Mo (and V) extraction by a straight chain amine solvent such as Alamine 336. Mo can be stripped from the organic phase with ammonia solution as ammonium orthomolybdate, $(NH_4)2MoO_4$. Raffinate from the Mo-SX circuit containing traces of soluble Mo, Ni, and V may undergo sulfidation and filtration to cleanse the ammonium sulfate stream to marketable fertilizer standards.

Precious Metals

For Pt-Pd spent catalysts on a γ-alumina support, caustic treatment of the deoiled, pretreated, and ground material via hydrometallurgical routes solubilizes aluminum (Reaction 1 in Table 20.25) and releases platinum group metals (PGMs) for recovery. Dissolution of the Pt-Pd residue can be achieved with a combination of hydrochloric-peroxide (HCl-H_2O_2; Reactions 2 and 3 in Table 20.25) or aqua regia (HNO_3-HCl; Reactions 4 and 5 in Table 20.25) acid, or the PGM residue can be purified by smelting.

The solubilized PGMs can be separated from each other (Lee et al. 2010) via SX using tri-n-butyl phosphate (TBP)-thiourea strip for Pd and Aliquat 336-thiourea strip for Pt or ion-exchange (IX) resins (Gaita and Al-Bazi 1995) such as Amberlite IRA-93.

Pyrometallurgical Plant

As discussed in the preceding "Base Metals" subsection, a feedstock for the pyrometallurgical plant is a concentrate containing alumina along with oxides of nickel and cobalt produced in the hydrometallurgical plant.

Base Metals in the Pyrometallurgical Plant

The concentrate is dried and reduced in the presence of carbon deploying an electric arc furnace (Figure 20.16) to produce a high-grade alloy of nickel, cobalt, and fused alumina. Nickel and cobalt are sold as an alloy directly to major commodity consumers, and alumina is recycled and used for applications in the abrasive and steel industries.

The production of ferrovanadium (FeV) is normally based on vanadium trioxide (V_2O_3) or V_2O_5 using Al or ferrosilicon (FeSi) as a reductant (Figure 20.15). When Al is used, the exotherms generated during the reduction are sufficient to eliminate the need for external heating as shown in the following reaction. When FeSi is used as a reductant, an electric furnace is normally applied for the melting (Figure 20.16). For modern FeV plants, Al is normally used as a reductant and an electric furnace is harnessed to ensure even better control of the process.

$$10Al + 3V_2O_5 + 6Fe \longrightarrow 6Fe\text{-}V + 5Al_2O_3 \qquad \textbf{(EQ 20.33)}$$

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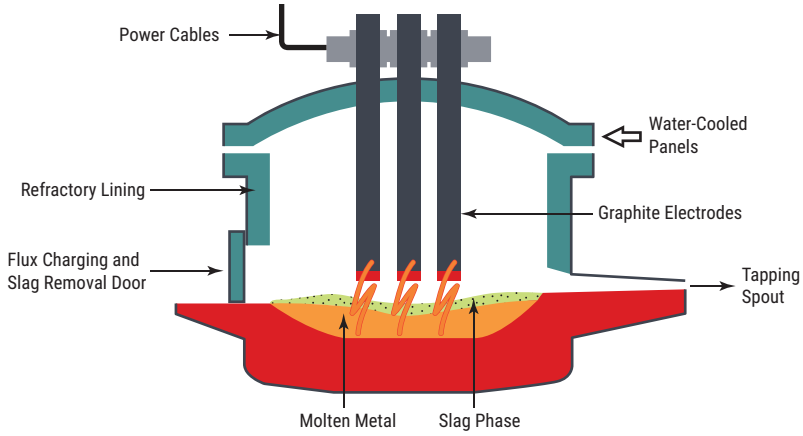

Adapted from CIEC 2016

FIGURE 20.16 Electric arc furnace

Precious Metals in the Pyrometallurgical Plant

When sampling and pretreatment are complete, the spent-catalyst lot is blended with a mix of flux and a carrier metal such as copper or iron (see the subsections "Spent-Catalyst Sampling," "Pretreatment of Spent Catalyst," and "Roasting Plant"). The proportions in the mix are determined by the calculated concentration of recoverable precious metals in the lot and the desired slag chemistry; the latter includes accounting for the slag electrical conductivity, corrosivity, morphology, and melting temperature.

The charge mix is then smelted, usually in an electric arc furnace (Figure 20.16). The material is directly exposed to an electric arc. With the current in the furnace terminals passing through the charged material at arc temperatures reaching in excess of 2,000°C (Rumpold and Antrekowitsch 2012), the furnace produces a two-layer molten pool. One layer is slag, which is the reaction product of the catalyst support and the added fluxes. This layer floats on a pool of collector metal in which the precious metals have dissolved. After the slag is poured off, the remaining layer of metal is either tapped into molds of a suitable size or the liquid metal alloy is fragmented in air or water into smaller pellets to augment surface area for rapid dissolution of the collector metal in acid.

The resulting ingots or pellets are then sent to a conventional hydrometallurgical or electrorefining (ER) facility for separation and production of market-grade precious metals. The slag, containing trace amounts of precious metals, may be processed for further recovery and refining. The off-gases are treated in baghouses and electrostatic precipitators to remove solids, and then scrubbed to remove sulfur and nitrogen oxides (SO_x and NO_x).

HEAP LEACHING by Thom Seal, P.E.

Heap leaching is common for oxide ores of Ag, Au, and Cu with growing applications for Co, U, V, and Zn and bio-oxidation of sulfides (Figure 20.17). Following are common terms and their definitions:

- **Run of mine (ROM).** The size distribution of rock as mined; the size varies geologically in the deposit, by the mining method, and if blasted.
- **Agglomeration.** The binding of fine particles and clay with a binder(s) to larger particles to reduce migration of fines and improve permeability. Dosage is measured in kilograms per metric ton of ore or in pounds per short ton.

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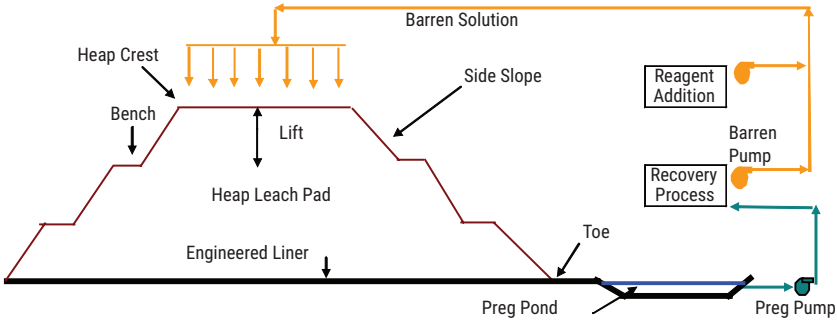

FIGURE 20.17 SX-EW process

- **Solution**
  - Solution with leaching reagents pumped to the top of the heap for irrigation is called *barren solution* for precious metal leaching and *raffinate* for copper leaching.
  - Solution with dissolved metals flowing from the heap leach pad is called *pregnant leach solution* (PLS).
- **Cell.** Ore is staked in lifts of 3–20 m depending on permeability, solution balance, and stacking method. A *cell* is a surface area of a lift of ore under irrigation fed from a header pipe.
- **Microporosity (ε).** Included in the voidage volume, microporosity represents the small cracks in the ore where leaching solution may diffuse.
- **Liner.** The containment layer at the bottom of the heap to contain and collect the PLS in a French drain pipe system and transfer it to a pond for recovery. Although usually made of high-density polyethylene (HDPE) with clay, other geotextile materials are also used.
- **Carbon in columns (CIC).** Fluidized activated carbon is configured to capture the dissolved metals from the PLS to make barren solution. Carbon is advanced countercurrent to the leach solution from new or regenerated carbon in the bottom of the CIC tank to loaded carbon in the top of the CIC tank train.
- **Merrill–Crowe process.** This process removes precious metal from the deaerated PLS by cementation with powdered elemental Zn. The process is used when the concentration of Ag is greater than 10 times the concentration of Ag in the pregnant solution, as shown by the following reaction:

$$2Au(CN)^{2-} + Zn^{o} = 2Au^{o} + Zn(CN)_4^{2-} \quad \textbf{(EQ 20.34)}$$

- **Solvent extraction–electrowinning (SX-EW).** In this process, base metals are dissolved in the PLS and are subjected to mixing with selective miscible organic reagents to collect the dissolved metals. The mixture is allowed to settle, then separated, and the metal-containing organic phase (SX) is then mixed with stronger aqueous reagents to strip the dissolved metals into an aqueous phase for direct EW. See Figure 20.17 for the process description.

Material Properties

The following are used to determine material properties (Black 1965):

- percent moisture (%M) = (wet weight − dry weight)/dry weight (× 100)
- density (γ) = mass/volume
- bulk density = mass/(volume of material + volume of voids)
- specific gravity (sg) = mass at a specific volume/mass of an equal volume of pure water

The following rules of material properties are from Poirier and Geiger (1973):

- To determine voidage or void fraction (V), $V = 1$ − bulk density/true density = volume voids/(volume voids + volume solids).

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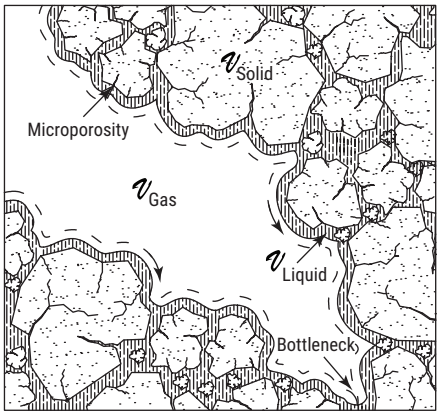

Source: Schlitt 1984

FIGURE 20.18 Relationship of volumes

- Eighty percent pass (P_{80}) is the size of a theoretical screen opening in which 80% of a sample mass is smaller or passes that size.
- Average particle diameter (D_{50}) is determined when half of the particles are smaller and the other half are larger.

Figure 20.18 shows the various types of volumes; solid, liquid, and gas.

Heap Leach Chemical Equations

The following heap leach chemical equations are divided between those for precious metals and those for copper minerals.

Precious Metals

$$4Au + 8CN^- + O_2 + 2H_2O \rightarrow 4Au(CN)^{2-} + 4OH^- \quad \textbf{(EQ 20.35)}$$

$$4Ag + 8CN^- + O_2 + 2H_2O \rightarrow 4Ag(CN)^{2-} + 4OH^- \quad \textbf{(EQ 20.36)}$$

Copper Minerals

Malachite: $Cu_2CO_3(OH)2 + 4H^+ \rightarrow 2Cu^{2+} + CO_2(g) + 3H_2O$ **(EQ 20.37)**

Azurite: $Cu_3(CO_3)2(OH)2 + 6H^+ \rightarrow 3Cu^{2+} + 2CO_2(g) + 4H_2O$ **(EQ 20.38)**

Chrysocolla: $Cu_2H_2Si_2O_5(OH)4 + 4H^+ \rightarrow 2Cu^{2+} + 2SiO_2 + 5H_2O$ **(EQ 20.39)**

Chalcocite: $Cu_2S + 2Fe^{3+} \rightarrow Cu^{2+} + CuS + 2Fe^{2+}$ **(EQ 20.40)**

Covellite: $CuS + 2\ Fe^{3+} \rightarrow Cu^{2+} + 2Fe^{2+} + So$ **(EQ 20.41)**

Chalcopyrite: $CuFeS_2 + 4Fe^{3+} \rightarrow Cu^{2+} + 5Fe^{2+} + 2So$ **(EQ 20.42)**

Pyrite oxidation: $FeS_2 + 14Fe^{3+} + 8H_2O \rightarrow 15\ Fe^{2+} + 16H^+ + 2SO_4^{2-}$ **(EQ 20.43)**

Ferric oxidation: $Fe^{2+} + ¼O_2 + H^+ \rightarrow Fe^{3+} + ½H_2O$ **(EQ 20.44)**

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Typical Heap Operating Conditions

Table 20.26 shows typical heap operating conditions.

TABLE 20.26 Typical heap operation conditions

	Gold	Copper
Application rate, L/(h·m^2)	2.5–12.0	12.0–35
Crush size, mm	1.7 to run of mine	6.0 to run of mine
Agglomeration	From H_2O only to as much as 20 kg/t cement; from conveyors only to pug mill or agglomeration drum agglomerator	H_2SO_4 cure; gypsum from H_2SO_4 + $Ca(OH)_2$
Heap heights, m	6–200	—
Lift heights, m	6–12	Shorter, 3–8
Preg	0.5–2.0, g/t	<1 to 6 g/L
Barren	<0.03 g/t	0.14–1.8 g/L
Evaporation, % of applied	2–10	2–10
Overall heap slope	2:1	2:1
Agglomerates, dry t/m^3	1.28–1.92	—

Rules for Column Testing of Gold Ores

- Acquire a representative core without drilling muds. If muds are needed, test them for preg robbing. Keep core moist in plastic sleeves; if sulfides are present, then freeze core to prevent oxidation. Get rock quality designation (RQD) and photograph of core. Determine size distribution for column test ore.
- Test the head assay as well as the cyanide soluble assay via a shake test or bottle roll to compare column test data and generate recovery models. Refractory ores have a low percentage of precious metal extraction when compared to the head assay.
- Primary leach time in commercial heap is approximately one-and-a-half times the leach time in a 6-m-high column test and approximately two to four times the leach time in 2-m-high column test.
- About the same gold extraction is obtained if the same number of tons of leaching solution is applied to a ton of ore in either a column test or a commercial heap.
- The approximate U.S. economic limit for 0.2 g/t of ore is 0.5% gold extraction per month.
- Cyanide (CN) consumption in a 96-h, 1.7-mm crush-size bottle leach test is an approximation of commercial heap consumption.
- Gold extraction from a 96-h bottle leach test of –1.7-mm ore is typically 10% less than gold extraction after 60 days from column leached ore that is –12.7 mm; this may be a quick way to forecast commercial heap leach results.
- Typical heap detoxification time to reach 0.2 ppm CN_w is 50% of leach time.
- Dissolution is kinetically controlled by diffusion with adequate reagents available.
- Measurements of permeability and reagent consumption are important.
- Use 6-m-high column tests for ROM testing; 2-m-high column tests for scoping tests.
- To obtain “ultimate gold extraction,” perform weekly regression analysis of column test data and extrapolate to 180–360 days. Continue tests until extraction remains constant.

Rules for Column Testing Copper Ores

- Test the head assay as well as the acid soluble assay via a shake test or bottle roll to compare column test data.
- Pretreat with H_2SO_4 (acid cure, 5–12 kg/t).

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- Do not allow solution pH to rise above 3–4.
- Be aware of assaying problems for determination of free acid concentration.
- Crush ores to permit greater control of pH within the particles; generally <2.5 cm.
- Solution effectively fills all of the voids for rock sizes <0.3 mm and air is excluded; if rock is 1–2 mm, drainage is almost complete and voids are filled with air.
- Thin layer leaching avoids acid depletion.
- Column diameter should be four to six times the largest ore particle dimension.
- Solution irrigation rates are 12–35 L/(h·m^2).
- Examine mineral decrepitation or reactions with leaching reagents for changes in permeability and migration of fines.
- Dissolution is kinetically controlled by the chemical reaction and or diffusion.
- Measurements of permeability and reagent consumption are important.

Heap Leach Mathematical Equations

Column leach tests can be modeled with the fraction extracted F in time t with k a fitting parameter constant.

$$F(t) = e^{-kt} \quad \textbf{(EQ 20.45)}$$

The superficial velocity of solution percolating through a heap or specific discharge is measured as flow Q per unit area A or velocity v = distance/time = L/t.

$$v = \frac{Q}{A} = \left(\frac{L^3}{t}\right) \div L^2 = \frac{L}{t} \quad \textbf{(EQ 20.46)}$$

Hydraulic conductivity K also has units of L/t and is proportional to the hydraulic gradient $h/\Delta x$, where h is the hydrostatic head and Δx is the distance over which the hydraulic is expressed.

$$v = \frac{Q}{A} = K\left(\frac{h}{\Delta x}\right) \quad \textbf{(EQ 20.47)}$$

Darcy's law for flow in one direction or L/t is also a function of intrinsic permeability ki, solution fluid viscosity u, the change in pressure p, and the distance x (Bartlett 1998), so:

$$v = \frac{Q}{A} = K\left(\frac{h}{\Delta x}\right) = ki \div u\left(\frac{\Delta p}{\Delta x}\right) \quad \textbf{(EQ 20.48)}$$

Application rate is equal to the flow, Q per area A, or half the distance between emitters on both sides of the solution line and half the distance on both sides of the line spacing.

French drains spacing (L) above the liner to collect the pregnant solution is designed to reduce the saturated zone height h and is a function of the infiltration rate W and the hydraulic conductivity of the ore K (McWhorter and Sunada 2010).

$$h = \frac{1}{2} L \sqrt{\frac{W}{K}} \quad \textbf{(EQ 20.49)}$$

The volume of a heap can be estimated by breaking up the formation into smaller geometric shapes such as rectangular boxes, pyramids, and truncated pyramids (Figure 20.19). The volume of a regular pointed pyramid is equal to the base width w, the base length l, and the height.

$$h \times V = \frac{1}{3} w \times l \times h \quad \textbf{(EQ 20.50)}$$

The volume of a truncated pyramid V is the base width a, top length b, and height h:

$$V = \frac{1}{3}(a^2 + ab + b^2)h \quad \textbf{(EQ 20.51)}$$

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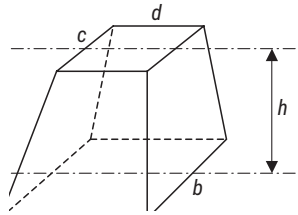

Source: Zipf 2001

FIGURE 20.19 Frustum of a pyramid that emulates a heap leach volume

Design Features

Solution flow through a heap should be downward parallel and just coat all the ore particles with a thin film of barren solution containing all the required reagents that are renewed over time with new solution, thereby sweeping away the PLS. Too much fluid fills the voids above bottlenecks, reducing the dissolving of air from the voids, and promotes horizontal flow.

- Use balls or covers on ponds to save sodium cyanide (NaCN), calcium oxide (CaO), water, and bird kills.
- Use drip emitter irrigation to minimize destruction of agglomerates by droplets, or use fine sprays, or cover the heap with porous plastic cloth to absorb the shock of droplets. Take into account evaporation and reagent destruction in droplets.
- Use preg ponds with dividers or storm ponds, so pond repairs can be made without shutting down.
- Arrange the crush circuit to allow bypass of secondary and tertiary crushing if both refractory and very leachable ores are processed.
- Scalp –25 mm fines from ore ahead of the primary crusher; agglomerate only fines.
- Spray or add CaO into trucks hauling ROM ore to the heap or onto the heap slope. Try to mix with the ore.
- Use less than 5% slope on the heap's surface to minimize runoff and make reclamation easier.
- Consider whether a containment berm is needed for heap stability. Remember friction angles can be very low for plastic on plastic or some ores on plastic, especially when saturated. Benches contain slumps and raveling of side slopes. Use telescoping well casing with breakable welds when using submersible pumps inside the heap.
- Use perforated and corrugated HDPE for drainage piping under the heap: usually 100 mm ø on 6-m to 10-m centers and at a 45-degree angle to the fall line. The drainage piping is connected to a header that discharges to a sump, pond, or ditch.
- Surfactants have had variable success in the past.
- Scale inhibitors are used to protect pumps and the recovery systems. Solution quality is determined using the Langelier saturation index.
- Use gravity separation to recover gold from the fines fraction if the ore requires fine crushing; then recombine wet fines with dry coarser fraction ahead of agglomeration.

Operation of Pads

The permeability of ore in a composite pad should have a hydraulic conductivity K of 10^{-6} cm/s and normally be in the 10^{-2} to 10^{-3} cm/s range; geomembranes are theoretically 10^{-14} cm/s or lower but have a much higher permeability because of punctures and imperfect installation. See Figure 20.20 for a generalized cross section of a heap leach with flow characteristics.

- Stackers can operate on up to 8% slopes if done carefully, which means the first lift only.
- The minimum slope of a pad should be >2% to account for settlement of material under it.

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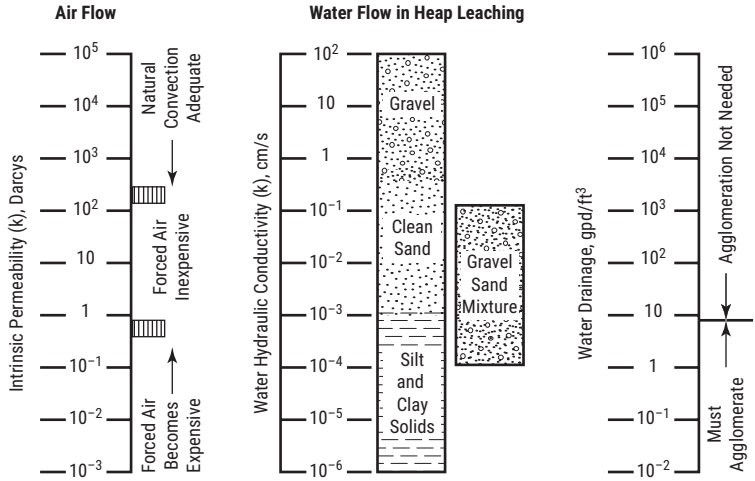
- A drainage or cushion layer of crushed gravel on top of the pad permits good drainage from under the heap (prevents buildup of hydraulic head and high seepage rates) and prevents puncturing the liner when placing the first lift of ore. A 300-mm-thick layer must be placed with small equipment; a 1-m-thick layer can be placed with small mine equipment. If a 100-ton haul truck, such as the Caterpillar 777, is used, then the thickness should be at least 1.5 m.
- HDPE is a crystalline material and sharp rocks may puncture it over time. Generally, HDPE is the material of choice for liners under heaps and ponds. Cushion layers over the liner are used to prevent rock punctures.
- Polyvinyl chloride (PVC) has high plasticity and will stretch over sharp rocks.
- Be aware of friction angles, especially between synthetics and between clays and smooth geomembranes.
- Adequately rip the top of a placed lift at least two different directions to a depth of 2.4 m (8 ft) before irrigation and before stacking new ore on top of a lift. Reduced compaction yields better permeability and higher extraction.
- Determine the effect of additional lifts on the compaction of material below. Compaction reduces voidage and permeability and promotes horizontal channeling of solution leaving under leached zones. Voidage in the ore on top of a heap may be 30% and at the bottom of the heap 3%–5%.
- Increasing solution application beyond adequate wetting does not increase metal dissolution and production, but promotes solution channeling, migration of fines, reduced pregnant grade by dilution, and reduced metal recovery efficiency from solutions.

Symptoms of Percolation Problems

- Seepage from the heap slope more than 1 m above the pad liner
- Ponding on the top of the heap
- Washouts, slumping, cracks, or irregular slope of the heap's side slopes
- Ore that has slid off the pad
- A perched water table within the heap
- Very low application flow rates



Source: Bartlett 1998

FIGURE 20.20 Generalized cross section of a heap leach

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TABLE 20.27 Dissolved oxygen per liter of distilled water in milligrams

Pressure, mm Hg	Temperature, °C			
	5	20	35	50
760	12.7	9	6.9	5.6
700	11.8	8.4	6.4	5
650	11	7.7	6	4.8
600	10.1	7.2	5.5	4.3
550	9.3	6.7	5	4
500	8.5	6	4.6	3.7
450	7.8	5.5	4.2	3.3

Source: Liddel 1922

To Prevent Problems

- Examine all the leach equations for reagent requirements; dissolved oxygen is often required (Table 20.27).
- High cyanide concentration in CIC PLS makes $Cu(CN)^{3-}$; it does not load on carbon very well.
- High Cu or Zn concentrations may give free-cyanide assays that are too high when using silver chloride (AgCl) titration.
- Because of dissolved salts (total dissolved solids [TDS]), the density of the leach solutions are often not 1.000 g/mL.
- Daily monitoring of solution grades and flows for PLS and barren solution is essential. Periodic monitoring of solution chemistry (pH, TDS, temperature, hardness, unwanted metals, etc.) for trends can prevent problems.

Water Balance Techniques

- Reduce evaporation by (1) covering ponds, (2) using drip irrigation rather than sprays, and (3) using pipe to conduct the preg solution from heap to pond rather than open ditches.
- Enhance evaporation by (1) using sprays on top of the heap rather than emitters, (2) spraying barren solution on black plastic (up to 50% evaporation and heat solution to 35˚C), (3) installing snow-making fogging machines on heaps (up to 30% evaporation), and (4) installing sprays in ponds. Sprays may affect reagent concentration in solution.
- Air naturally diffuses about 3 m in 3 years into a heap with a 30% voidage, and only 1.5 m in the same time for material with 10% voidage.
- Low recovery percentages during the first year of operation may be caused by the following:
  - Not irrigating nearly all of heap's surface or too high an irrigation rate, promoting channeling and uneven wetting
  - Ore permeability or preg-rob issues
  - Minerals in ore that consume reagents or chemically break down ore, thereby changing permeability and solution flow through the heap
  - An overly optimistic forecast of the rate at which values will be extracted
  - Different ore than forecast
  - Poor blasthole sampling that gives the wrong ore grade and not a representative sample

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CARBON PLANTS by Vernon F. (Fred) Swanson, P.E.

Carbon expansion characteristics are shown in Table 20.28.

Carbon in Leach and Carbon in Pulp

- High slurry density creates carbon concentrate at the top of the tank.
- High viscosity traps carbon in pockets of pulp, which has poor contact with new solution.
- Only sampling throughout the tanks, and testing, show if carbon is uniformly suspended. To sample the slurry/carbon pulp:
  1. Use a rod to reach almost to the agitator with a 1-L small-neck bottle attached to the rod with a cork attached to the cord.
  2. Submerge the bottle to the desired depth and pull the cork for the sample.
  3. Pour pulp through a 500- to 600-μm sieve and wash out the bottle to get all the solids.
  4. Wash the slurry from the carbon, dry the carbon, and weigh it to determine grams of carbon per liter of pulp.
  5. Assay the carbon for metal loading.
  6. Assay the solution and solids for metallurgical balance.
- Carbon concentration should range from 10 to 25 g/L.
- Use high concentration for Ag or preg-robbing ores.
- For high loading, use low concentration in the first tank and high concentration in the middle tanks.
- Check residence time.
- Institute step loading efficiencies.
- Check the carbon inventory.
- Carbon contamination can occur with oil spills and flotation reagents. Cure contamination with regeneration.

TABLE 20.28 Carbon expansion characteristics

	Temperature = 35°C		Temperature = 25°C		Temperature = 15°C		Temperature = 5°C	
	Flow		Flow		Flow		Flow	
Bed expansion, %	L/m^2	gpm/ft^2	L/m^2	gpm/ft^2	L/m^2	gpm/ft^2	L/m^2	gpm/ft^2
Carbon, −2.8 × 3.0 mm								
25	922	22.62	835	20.50	296	7.26	668	16.40
50	1,360	33.38	1,223	30.03	247	6.05	1,068	26.22
75	1,648	40.46	1,583	38.86	211	5.19	1,350	33.14
100	1,950	47.86	1,833	45.00	185	4.54	1,612	39.56
Carbon, −1.0 × 0.72 mm								
25	360	8.84	328	8.06	296	7.26	207	5.07
50	585	14.36	525	12.89	247	6.05	362	8.88
75	783	19.23	738	18.12	211	5.19	517	12.68
100	937	22.99	888	21.80	185	4.54	647	15.86
125					164	4.04	762	18.69
Carbon, −0.5 × 0.3 mm								
25	130	3.20	109	2.68	296	7.26	71	1.74
50	218	5.36	183	4.5	247	6.05	132	3.23
75	282	6.91	242	5.93	211	5.62	167	4.09
100	343	8.43	287	7.04	185	5.19	197	4.83
125	390	9.57	337	8.26	164	4.54	257	6.30

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Carbon in Columns

- The amount of carbon in each tank should be equal (if not, check transfer facilities).
- Dead bed depth (no flow) should be approximately 1.2 m.
- Typically, the number of stages is four to six adsorption tanks.
- Uniform distribution of solution throughout the tank requires >6,895 N/m^2 (1 psi) across the bed plate.
- Lower carbon attrition with a recessed impeller, Hidrostal, or screw pumps, and lower it more with eductors.
- Carbon kinetics and loading tests are needed for proper design.
- If the Ag/Au ratio has increased over design, a higher stripping frequency or higher tonnage of carbon per strip may be needed. High barrens (>0.001 oz/st or 0.028 g/ t) may mean a high Ag/Au ratio.
- Carbon loads Au, Ag, Cu, Hg, and Ni.
- Reduce Cu in dore with a cold cyanide strip.
- Remove Ni from carbon with hot acid wash (90°C), which may require tank redesign.
- Analyze carbon for metallics (Ni, Fe, Co, etc.) if regeneration is not working well.

Acid Washing

- Use 1% HCl for 30 minutes, 1 BV (bed volumes) at 2 BV/h.
- Acid washing removes calcium carbonate ($CaCO_3$).
- Perform acid wash before stripping to enhance strip efficiency.
- Reduce wash frequency to minimum for good carbon activity, which also saves on costs.

Stripping (Elution)

- Many methods are used for stripping activated carbon from gold and silver ores:
  - Zadra, atmospheric with or without alcohol
  - Pressure stripping
  - Integrated pressure stripping (IPS)
  - Stripping that has pressurized EW and no heat exchangers (tough to remove cathodes from EW)
  - Anglo American Research Laboratory (AARL) method with either a single pass of approximately 10 BV or 10 BV with the last 5 BV used in the next strip and all flows at approximately 2 BV/h with the usual sequence
    - 1 BV of hot 3%–5% NaCN
    - 4–5 BV of strip solution from the previous strip combined with the first BV to make pregnant solution
    - 4–5 BV of fresh water (<200 ppm TDS) saved for the next strip
    - 1 BV of cold fresh water to cool the carbon and transfer it
    - Preg batched through EW
- Usually, a solution of 1% NaCN and 3% sodium hydroxide (NaOH) is used.
- If strip times are too slow, check the solution temperature and composition. Higher temperature speeds up stripping, but check that the strip vessel can handle the pressure and the heat exchangers handle the extra heat.
- The strip column is usually a height-to-diameter ratio of 6:1. Hotter solution temperature can allow a lower ratio.

Regeneration

- Regeneration is the most problem-prone process in carbon plants. Check mechanicals and fabrication.
- Heating is indirect, endothermic, with an O_2-free atmosphere with steam: $C + H_2O = CO + H_2$.
- The operating temperature should be more than 600°C and less than 750°C. If there are many organic contaminants on the carbon, run at 800°C.

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- Feed wet carbon (water makes the steam) and add water spray at the kiln discharge if needed.
- Check the temperature profile along the kiln's length.
- Two fans are required: (1) for combustion gases, which need to be reducing if they are allowed to enter the kiln for heat recuperation, and (2) for regeneration products.
- Regenerate only as needed, not necessarily every time.

BACTERIAL OXIDATION by Andrew P. Briggs, P.E.

Table 20.29 gives a list of bacteria used for oxidation and includes approximate operating temperatures for the specific bacteria. The typical cell formula (Mintek) is $CH_{1.67}N_{0.2}P_{0.014}O_{0.27}$. The size of the mesophiles is 0.5 × 1.0 μm (rod or comma shaped). The concentration is 106/mL in heap leach solutions and 108–109/mL in stirred tank solutions.

TABLE 20.29 Bacteria

Bacteria	Oxidation of	Doubling Time, h	Approximate Operating Temperature, °C
Mesophiles			
Thiobacillus ferrooxidans	Fe^{2+} and sulfide minerals	3.6–12.0	25–45
Thiobacillus thiooxidans	S^0 and soluble sulfide compounds	10–24	25–45
Leptospirillum ferrooxidans	Fe^{2+}	9–36	25–45
Moderate thermophiles	(diverse group) Fe^{2+}, S^0, sulfide minerals	1	45–55
Sulfobacillus thermosulfidooxidans	—	—	—
Extreme thermophiles	Fe^{2+}, S^0, sulfide minerals	≈42	>50
Sulfolobus species	—	—	—
Acidianus species	—	—	—

Toxicity

Toxicity is variable and not known with any accuracy. Following are the approximate minimum inhibitory concentrations:

- SX reagents, 16 mg/L
- Other organics, <24 mg/L
- Cl^-, 1,500–7,000 inhibited and 19,000 toxic
- Nitrate (NO_3), 200 mg/L
- Thiocyanate (SCN^-), 2 mg/L
- Cyanate (CNO^-), 10 mg/L
- Metal CN^- complexes, 10 mg/L
- As^{3+}, 6,000 mg/L
- Hg, ~4 mg/L

Minerals Oxidized

The sequence of mineral oxidation is thus: FeS > Cu_2S > FeAsS > ZnS > CuS > FeS_2 > $CuFeS_2$ (passivates). The reactions are as follows:

Pyrite: $4FeS_2 + 15O_2 + 2H_2O \rightarrow 2Fe_2(SO_4)_3 + 2H_2SO_4$ **(EQ 20.52)**

Arsenopyrite: $2FeAsS + 7O_2 + 2H_2O + H_2SO_4 \rightarrow 2H_3AsO_4 + Fe_2(SO_4)_3$ **(EQ 20.53)**

Pyrrhotite: $2FeS + 4.5O_2 + H_2SO_4 \rightarrow 2Fe^{3+} + 3SO_4^{2-} + H_2O$ **(EQ 20.54)**

Chalcopyrite: $4CuFeS_2 + 2H_2SO_4 + 17O_2 \rightarrow 4Cu^{2+} + 4Fe^{3+} + 10SO_4^{2-} + 2H_2O$ **(EQ 20.55)**

Chalcocite: $Cu_2S + H_2SO_4 + 2.5O_2 \rightarrow 2Cu^{2+} + 2SO_4^{2-} + H_2O$ **(EQ 20.56)**

Covellite: $CuS + H_2SO_4 + 2O_2 \rightarrow Cu^{2+} + 2SO_4^{2-} + 2H^+$ **(EQ 20.57)**

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Bornite: $Cu_5FeS_4 + 4H_2SO_4 + 6O_2 \rightarrow FeSO_4 + 5CuSO_4 + 2S^o + 4H_2O$ **(EQ 20.58)**

Sphalerite: $ZnS + 2.5O_2 + 2H^+ \rightarrow Zn^{2+} + SO_4^{2-} + H_2O$ **(EQ 20.59)**

Galena: $PbS + 2O_2 = Pb^{2+} + SO_4^{2-} \rightarrow PbSO_4$ **(EQ 20.60)**

Siegenite: $(Co,Ni)_3S_4 + 7.5O_2 + H_2O \rightarrow xCo^{2+} + (3\text{-}x)Ni^{2+} + 4SO_4^{2-} + 2H^+$ **(EQ 20.61)**

Calculations

- O_2 utilization efficiency is 30%–40% for stirred tanks and 20%–25% for heaps.
- In stirred tanks, the hydrostatic head pressure should be more than 15 kPa plus line loss. Heaps need approximately 8 kPa plus line losses (the heap is geometry dependent).
- Gas held up in reactors is 7%–15% by volume.
- Agitator power (stirred tank reactors) should be approximately 27 W/Nm^{3}h of air added (agitator dependent).
- Blower efficiency should be 74%–78%.
- For blower power, use the following equation:

$$\text{power, kW} = 1.0195\ WT\left(\left(\frac{P_2}{P_1}\right)^{0.283} - 1\right) \quad \textbf{(EQ 20.62)}$$

where

W = air flow, kg/s
T = inlet air temperature, K
P_1 = inlet pressure
P_2 = delivery pressure, P_1 and P_2 absolute (not gauge) pressure

Table 20.30 gives reaction heat, oxygen requirements, and sulfuric acid demand for various minerals.

TABLE 20.30 Reactions and requirements

		Heat of Reaction		Oxygen Requirement		H_2SO_4 Demand, kg/kg Mineral
		Mineral, kJ/kg	Sulfide, kJ/kg S^{2-}	mol O_2/ mol Mineral	kg O_2/ kg S^{2-}	
Pyrrhotite	FeS	−11,373	−31,245	2.25	2.25	0.557
Arsenopyrite	FeAsS	−9,415	−48,036	3.5	3.5	0.301
Pyrite	FeS_2	−12,884	−24,173	3.75	1.88	−0.408
Chalcopyrite	$CuFeS_2$	−9,593	−27,505	4.25	2.13	—
Chalcocite	Cu_2S	−6,201	−30,811	2.5	2.5	—
Covellite	CuS	−792	−24,756	2	2	—
Pentlandite	$(Ni,Fe)_9S_8$	−10,174	−30,644	17.63	2.2	—
Ankerite	$Ca(Fe,Mg)(CO_3)_2$	−219.2	—	—	—	0.979
Siderite	$FeCO_3$	−326.7	—	0.069*	—	1.267

* kg/kg mineral for $Fe^{2+} \rightarrow Fe^{3+}$.

Heat Balances

- Stirred tank reactors
  - Gains can be achieved through reactions, agitator power, and air addition.
  - Losses can happen through evaporation (air leaves reactors at operating temperature, saturated in moisture) and tank walls. Increased heat in the incoming slurry and the Joule–Thomson effect of air expansion can also cause loss.

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  - Warm cooling water approximately 4°C less than reactor temperature can affect heat balance:

$$\text{flow rate} = \frac{\text{heat load (kW)} \times 3.6}{\Delta T \times 4.186} \ (\text{m}^3/\text{h}) \quad \text{(EQ 20.63)}$$

- Heaps
  - Gains are achieved through reactions and solar radiation (day).
  - Losses occur through evaporation (air addition leaves heap saturated in moisture) and evaporation of irrigation solutions. Convection from heap surfaces, radiation (night), and increased heat in irrigation solutions also cause loss.
  - At 1% S^{2-} oxidized in the ore, the challenge is to maintain heap temperature; at 3% S^{2-} oxidized, the challenge is to remove the heat.
  - Temperature control in heaps can be managed by rest periods, irrigation rates, air addition rates, and the use of covers.

Typical Circuit Conditions

Table 20.31 is a comparison of plant operations for gold recovery in stirred tank reactors. The following is a list of typical circuit conditions:

- Slurry density, 15%–20% solids
- Residence time, 4–6 days
- Temperature, 40°C
- pH range, approximately 1.2–1.6
- Dissolved O_2, 2 ppm

TABLE 20.31 Gold recovery in stirred tank reactors—plant comparisons

Item	Unit	Fairview	São Bento*	Harbour Lights	Wiluna	Ashanti (Sansu)	Youanmi†	Tamburaque
Arsenopyrite	%	13	18	17	22	17	9.4	56
Pyrite	%	33	16	28	33	6	49	35
Pyrrhotite	%	Negligible	19	NIL	NIL	13	NIL	NIL
Gold	g/t	109	30	86.6	92.9	76.4	64	23
S^{2-}	%	20	18.7	18.6	22	11.4	28	30
S^{2-} oxidized	%	89	—	87	95	94	32	86
Concentrate feed	t/d	35	—	40	158	960	120	60
Total reactor volume (aerated)	m^3	913	1,374	880	3,391	18,278	3,000	1,336
Heat of reaction	MW	2	—	2.2	11.1	41.1	4	6.1
Oxygen demand	kg/h	540	—	612	3,024	10,872	987	1,656
Oxygen demand	$kg/h/m^3$	0.59	—	0.69	0.89	0.59	0.33	1.24
S^{2-} oxidation rate	$kg/m^3/d$	6.8	—	7.4	9.7	5.6	3.3	11.6
Specific power consumption	$kW{\cdot}h/kgS^{2-}$	1.9	1.8	1.9	1.5	1.9	2.3	—
Gold leach	h	—	—	—	—	24	32	—
CN consumption	kg/t cons	—	—	—	30	10	7.5	—
Lime addition	kg/t cons	—	—	—	50	30	25	—
Gold recovery	%	—	—	—	96	94	90–95	—

* At São Bento, pretreatment is done by bacterial oxidation followed by pressure oxidation; the throughput is very elastic.

† Youanmi is Bactech technology.

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Nutrient Additions

Table 20.32 shows bacterial oxidation control and troubleshooting. Nutrient additions are measured in kilograms per metric ton of concentrates and are as follows:

- Standard biological oxidation addition:
  - N, 1.7 kg/t
  - P, 0.3 kg/t
  - K, 0.9 kg/t
- Calculated from cell requirements:
  - N, 0.11 kg/t
  - P, 0.016 kg/t
  - K, zero
- Additional parameters to check
  - Fe^{2+}/Fe^{3+}ratio (separate from redox potential)
  - Oxygen uptake rate
  - Presence of poisons in solution
  - Reagents (shale flask tests)
- In most cases, when the circuit is operating poorly, the concentrate feed to the circuit should be reduced or stopped until the parameters have stabilized.

TABLE 20.32 Bacterial oxidation control and troubleshooting

Control Parameter	Response Time	Comments
Dissolved oxygen (DO) level	Within minutes	If DO is high and same air addition, then oxidation rates have dropped. If DO is low, not enough air is being added.
Redox potential	Within a day	If redox potential is low, oxidation rate is low.
Cooling water addition	Within hours	If cooling water addition is low, oxidation rate drops. If temperature is high, not enough water is being added.
pH	Within a day	If pH is low, oxidation of pyrite has increased and more limestone is required. If pH is high, oxidation rate is decreasing.
Bacterial counts	A few days	Lower counts indicate reproduction rate lower than washout rate—slow feed down.
Froth	Within minutes	Excessive froth indicates bacteria under stress—watch other parameters. Froth should be made up of fine bubbles, not coarse.
Smell	Ongoing	A strong acid smell should be evolved from the top of the slurry.
Color	Per process	The color should change from dark gray (feed) to yellowish green (discharge). Final leach solution should be reddish (Fe^{3+}).

Solid–Liquid Separation

S–L separation is usually performed in three CCD high-rate thickeners:

- The typical required thickener area is 4.2– 9.0 $m^2/t{\cdot}h$ (depends on particle size and level of oxidation).
- The solids content in the underflow is approximately 33%.
- Usually, the wash ratio is 6 t solution per metric ton of bio-oxidation residue solution.
- Flocculant is added at approximately 80 g/t for the first stage and 25 g/t for the second and third stages.

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Neutralization

For a two-stage pH-controlled circuit for arsenic precipitate stability,

- Stage 1 neutralizes to pH 4–5 in the following reactions:

$$H_2SO_4 + Ca(OH)_2 \rightarrow CaSO_4 \cdot 2H_2O \quad \textbf{(EQ 20.64)}$$

$$2Fe_2(SO_4)_3 + 2H_3AsO_4 + H_2SO_4 + 7CaCO_3 + 13H_2O \rightarrow 2FeAsO_4 + 2Fe(OH)_3 + 7CaSO_4 \cdot 2H_2O + 7CO_2 \quad \textbf{(EQ 20.65)}$$

- Stage 2 neutralizes to pH 6–8 in the following reaction:

$$MSO_4 + Ca(OH)_2 + 2H_2O \rightarrow M(OH)_2 + CaSO_4 \cdot 2H_2O \quad \textbf{(EQ 20.66)}$$

- The number of stages can be four to six, and the residence time at each stage is 1.5–1.0 hours.
- Arsenic stability factors are as follows:
  - The optimum Fe/As ratio is ≥3:1.
  - The optimum pH is 4.5–5.0.
  - Higher temperature increases stability.
  - Co-precipitation of other compounds (gypsum, Zn, Cu, Cd salts) increases stability.
- Products, depending on pH and Fe/As molar ratio, may be scorodite ($FeAsO_4 \cdot 2H_2O$), basic ferric arsenate ($FeAsO_4 \cdot xFe(OH)_3 \cdot CaHAsO_4$), amorphous calcium arsenate ($Ca_3(AsO_4)_2$), or hydrated calcium arsenate ($Ca_2AsO_4 \cdot OH$).

Stirred Tank Biological Reactors for Base Metals

- No commercial operations exist whereby the primary goal is recovery of base metals.
- Agitated biological oxidation systems for gold and silver recovery exist in many countries for gold and silver ores (Marsden 2019).

Bacteria Heap Leach Operations

- All commercial operations primarily recover Cu from Cu_2S and CuS, but other metals have opportunities for recovery.
- Gold operations are at the development phase.

COAL CLEANING by Gary F. Meenan

Cleaning is defined as separation of low specific gravity, low ash coal (product) from high specific gravity, high ash coal (refuse) to give the yield and quality required. Raw coal characteristics are determined by a specific gravity analysis on a dry weight basis at specific gravities of 1.3, 1.35, 1.4, 1.45, 1.5, 1.55, 1.6, 1.65, and 1.7. Each fraction, including float and sink, is weighed and assayed for percentage of ash and sulfur. Higher specific gravity fractions have higher ash contents. Table 20.33 gives the relative difficulty of separation for different values of the specific gravity distribution.

Coal cleaning performance criteria are yield, coal recovery, and distribution curves (probable error, E_p, and error area).

- Yield depends on ROM coal's specific gravity distribution. Use ash balances on feed, product, and refuse streams.

$$Y_p \times 100\% = \frac{(A_r - A_f)}{(A_r - A_p)} \quad \textbf{(EQ 20.67)}$$

where

Y_p = yield of product
A_p = ash in product, %
A_f = ash in feed, %
A_r = ash in refuse, %

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TABLE 20.33 Separation difficulty and specific gravity distribution

Weight Percent Feed in the ±0.10 Specific Gravity Units	Degree of Difficulty
0–7	Simple
7–10	Moderately difficult
10–15	Difficult
15–20	Very difficult
20–25	Exceedingly difficult
>25	Formidable

Source: Bird 1928

- Recovery is the amount of ash-free coal in the product divided by the amount of ash-free coal in the feed and can be expressed as

$$CR_p = (Y_p \times 100\%)\frac{(100 - A_p)}{(100 - A_f)} \quad \textbf{(EQ 20.68)}$$

where

CR_p = ash-free coal recovery of product
CR_r = ash-free coal recovery of refuse

- The distribution curve is the method for assessing the performance of coal cleaning equipment.
  - The distribution curve equals the percent recovery to product versus the mean density of each specific gravity fraction.
  - The distribution curve is independent of ROM coal distribution by specific gravity; it depends on the cleaning equipment.
  - Construct the distribution curve as follows:
    1. Perform specific gravity analysis of the product and refuse and get an ash assay of the feed as shown in Table 20.34.
    2. Determine product yield: $Y_p = 100(77.65 - 36.61) / (77.65 - 5.88) = 57.19\%$
    3. Determine refuse yield: $Y_r = 100\% - 57.19\% = 42.81\%$
    4. Multiply the weight percent of each specific gravity fraction of product by $Y_p/100$ for a table of adjusted weight percents.
    5. Multiply the weight percent of each specific gravity fraction of refuse by $Y_p/100$ for a table of adjusted weight percent.
    6. Add the adjusted weight percent of the product and refuse in each specific gravity fraction (Tables 20.35 and 20.36).
- For each sg fraction of product, divide the weight percent of the product by the weight percent of the feed for that fraction.
- For each sg fraction of refuse, divide the weight percent of the refuse by the weight percent of the feed for that fraction.
- Plot the weight fraction to the product coal versus the mean specific gravity of each specific gravity fraction (Figure 20.21). This is the distribution curve. The mean specific gravity of 1.3 float and 1.7 sink gravity fractions are approximately 1.25 and 1.90, respectively.
- Performance criteria derived from the distribution curve are probable error, E_p, and the error area. Sharp separations have small error and small area. E_p gives no information about the recovery of low ash clean coal or the rejection of high ash impurities. Typical E_p for heavy media = 0.020; for hydrocyclones = 0.225.

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TABLE 20.34 Example of coal feed data*

Specific Gravity Fraction	Wt, %	Ash, %	Sulfur, %	Cumulative Weight, %	Cumulative Ash, %	Cumulative Sulfur, %
Product						
Float 1.30	74.26	4.02	1.17	74.26	4.02	1.17
1.30 × 1.35	15.70	8.41	2.19	89.96	4.79	1.35
1.35 × 1.40	6.59	13.40	2.97	96.55	5.37	1.46
1.40 × 1.45	2.58	18.44	3.31	99.13	5.71	1.51
1.45 × 1.50	0.62	22.52	3.78	99.75	5.82	1.52
1.50 × 1.55	0.15	27.04	3.98	99.90	5.85	1.53
1.55 × 1.60	0.05	32.99	4.62	99.95	5.86	1.53
1.60 × 1.70	0.03	37.19	4.24	99.98	5.87	1.53
1.70 sink	0.02	53.92	5.07	100.0	5.88	1.53
Refuse						
Float 1.30	0.50	4.04	1.11	0.50	4.04	1.11
1.30 × 1.35	0.23	8.33	2.32	0.73	5.39	1.49
1.35 × 1.40	0.24	14.20	2.58	0.97	7.57	1.76
1.40 × 1.45	0.53	19.68	2.89	1.50	11.85	2.16
1.45 × 1.50	1.61	23.85	3.27	3.11	18.06	2.73
1.50 × 1.55	1.58	28.42	3.49	4.69	21.55	2.99
1.55 × 1.60	1.15	33.55	3.19	5.84	23.91	3.03
1.60 × 1.70	2.02	39.21	2.98	7.86	27.85	3.02
1.70 sink	92.14	81.90	1.80	100.0	77.65	1.90

* The ash content is 36.61%.

TABLE 20.35 Weight percent of feed

Specific Gravity Fraction	Weight Percent of Product	+	Weight Percent of Refuse	=	Weight Percent of Feed
Float 1.30	42.47		0.21		42.68
1.30 × 1.35	8.98		0.10		9.08
1.35 × 1.40	3.77		0.10		3.87
1.40 × 1.45	1.48		0.23		1.70
1.45 × 1.50	0.35		0.69		1.04
1.50 × 1.55	0.09		0.68		0.76
1.55 × 1.60	0.03		0.49		0.52
1.60 × 1.70	0.02		0.86		0.88
1.70 Sink	0.02		39.44		39.46

$$E_p = \frac{(D_{25} - D_{75})}{2} = \frac{(1.50 - 1.44)}{2} = 0.030 \quad \text{(EQ 20.69)}$$

where

E_p = probable error
D_{25} = specific gravity at 25% recovery to product
D_{75} = specific gravity at 75% recovery to product

- The error area determined by the planimeter is approximately 20 for the curve shown in Figure 20.21. Typical error areas for heavy media are 10, and for hydrocyclones, they are as high as 125. Jigs and tables are between.

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TABLE 20.36 Weight fraction to product

Specific Gravity Fraction	Weight Percent Product/Weight Percent Feed	=	Weight Fraction to Product
Float 1.30	42.47/42.68		0.995
1.30 × 1.35	8.98/9.08		0.989
1.35 × 1.40	3.77/3.87		0.974
1.40 × 1.45	1.48/1.70		0.871
1.45 × 1.50	0.35/1.04		0.337
1.50 × 1.55	0.09/0.76		0.118
1.55 × 1.60	0.03/0.52		0.058
1.60 × 1.70	0.02/0.88		0.023
1.70 Sink	0.02/39.46		0.001

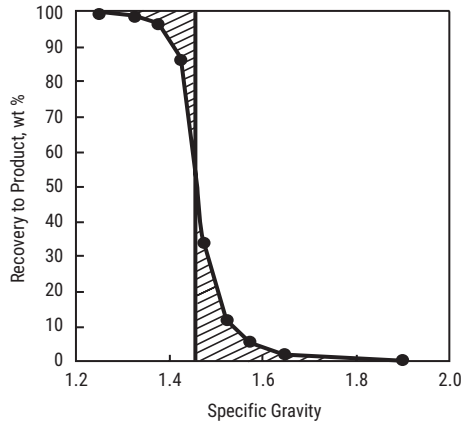

FIGURE 20.21 Density distribution curve showing error area of the example

Operations and Troubleshooting

- Typical equipment for cleaning coarse coal (+9.5 mm) are heavy-media vessels and Baum jigs; for intermediate coal (9.5 mm × 0.6 mm), heavy-media cyclones, Batac jigs, and Deister tables; for fine coal (−0.6 mm), mechanical flotation cells; for ultra-fine coal (−50 μm) that is high in clay, column flotation cells.
- Most new coal plants have a spiral circuit in them. The feed stream must be properly sized and constant. The optimal size range is 1.18 mm × 0.25 mm (16 × 60 mesh). Re-treating the primary middlings in secondary spirals is more common, and it minimizes coal losses. Consider re-treating the secondary spiral middlings in a heavy-media cyclone circuit.
- Coarse and intermediate-size bituminous coals are dewatered with screens and basket centrifuges to a final moisture content of approximately 5%. The fines are dewatered with vacuum disk filters (≈25% moisture) and screen-bowl centrifuges (≈20%–21% moisture). Feed size of fines is typically 0.6 mm × 0 with 55% greater than 0.15 mm. Filtration rate ($kg/h{\cdot}m^2$) increases with (1) the square root of pressure, (2) increasing feed solids content, and (3) quicker cake formation or filter cycle.
- Vacuum filter maintenance procedure:
  1. Change sectors on a routine basis.
  2. Use high-pressure sprays to minimize the internal buildup of solids and keep screen cloth clean; provide a minimum of 5 cfm/ft^2 of air suction to maximize throughput and moisture removal.

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3. Maintain the slurry agitators to minimize the "donut" effect and avoid sanding out in the tub (air agitation with simple mushroom valves effectively keep the solids in suspension).
4. Use a snap blow cake discharge mechanism to avoid sector buildup.
5. Control tub level to avoid conditions where the minimum submergence level to maintain vacuum is exceeded.
6. Maintain a minimum vacuum of 10–12 in. of Hg.

- Limits on impurities are dictated by the end user or contract specifications. Product sulfur content can range from 0.2% to 5.0%; the heating value can range from 8,000 Btu/lb for lignite to more than 13,000 Btu/lb for bituminous coals. The product ash content range can be from 5% to 13%. The moisture content also varies with the rank of coal. The total moisture of bituminous coal (plant product) ranges from 6% to 12%. Thermally dried bituminous coal is usually less than 6%. Western or lignite coal can have +40% moisture. The metallurgical coal must have a sulfur content of 1% or less, high heating value (about 13,000 Btu/lb), an ash content of around 5%, and a moisture content of about 5%.
- Anthracite and bituminous coals are treated similarly. Lignite is not cleaned.
- The grindability of a coal is a measure of its resistance to crushing. Grindability is a composite property embracing hardness, strength, fracture, mineral ash content of the coal, inherent moisture, and the type of coal. Coal types range from soft to hard, easy grind to tough grind, and low volatile content to high volatile content. Generally, coals are easier to grind that fall in the lower ranges of the coal types, which are peat, lignite, sub-bituminous, bituminous, and anthracite. A method of measuring coal grindability is to run a sample through a Hardgrove grindability machine in which a definite amount of grinding energy is applied. The percent by weight of the coal that passes through a 200-mesh sieve (a screen with openings of 74 μm, or 0.003 in.) is used to calculate the Hardgrove grindability index (HGI). For example, a coal having a Hardgrove index of 40 can be pulverized at 80% of the mill capacity of a coal having an index of 55 when both are being pulverized to 70% through a 200-mesh sieve. The index is used as a guideline for sizing the grinding equipment in a coal-preparation plant.

DUST COLLECTION by Paul D. Chamberlin, P.E.

Figure 20.22 shows suggested dust collection equipment versus particle size. Figure 20.23 shows power requirements versus particle size for various collectors. Generally, dust becomes a problem when moisture content is <4%. Chemical control of dust is by (1) wet suppression with treated or untreated water, (2) foam suppression, and (3) crusting of storage piles.

- Wet suppression generally requires 4–8 L of treated water (0.05%–2.0% wetting agent) per metric ton when wetting ore on a conveyor; without a wetting agent, water use is three to four times more.
- Foam suppression usually requires 0.8–1.6 L of treated water per metric ton of ore; it does a better job of reducing respirable dust than wet suppression.
- Crusting agents are applied by spraying or with a hydroseeder.

Mechanical control of dust is by (1) dust collectors, (2) hoods and ducts, and (3) tight control of conveyor systems. Conveyor systems should run at <150 m/min with transfer chutes that drop ore onto receiving belts with the same velocity and direction. Install skirtboard seals, two belt cleaners in series, and slider beds or cradles at loading points to avoid belt sag.

- Appropriate venturi scrubber pressure drops remove 100% of particles: approximately 25.4 cm water gauge for 2-μm particles; 50 cm for 0.8 μm; 90 cm for 0.6 μm; 140 cm for 0.4 μm; and 216 cm for 0.2 μm.
- Cooling hot gases from 650° to 290°C using injected water sprays increases needed baghouse capacity by 25% (it is a straight-line relationship). Water injection must be far enough upstream to be sure it evaporates before reaching the baghouse. Also beware of fan erosion, scale buildup, metal distortion, refractory failures, and nozzle failures.
- Gas velocity to avoid material buildup in the bottom of ducts should range from 1,066 to 1,827 m/min.

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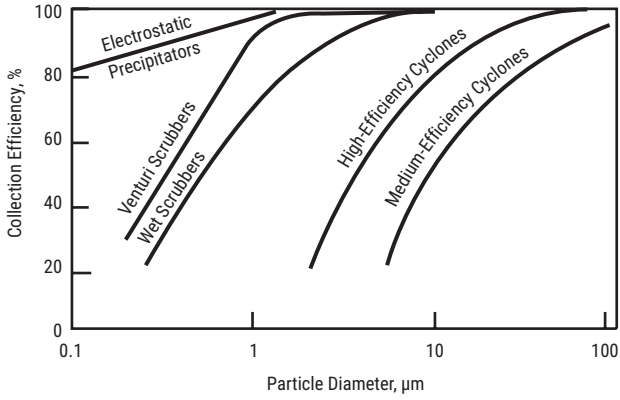
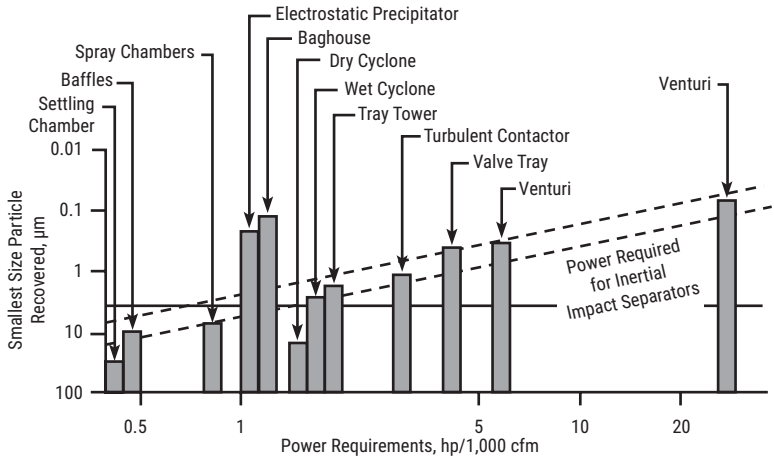


Source: Rossano 1969

FIGURE 20.22 Collection efficiency versus particle size



Source: Agarwal et al. 1974

FIGURE 20.23 Power requirements versus particle size

Fabric Dust Collectors

The suggested cubic feet of air volume in the bag to square foot area of the bag cloth surface are as follows:

- Cement = 1.5 to 2
- Coal = 2 to 2.5
- Limestone = 2 to 2.5
- Sand = 3 to 3.5

Floating dust can usually be made to flow at 61 m/min. Total air flow is 61 m/min multiplied by the total square meters of all openings in the housing on the dust-creating machine. Velocity in ducts should be approximately 1,066 m/min for light dusts to 1,827 m/min for heavy dusts. Fabric bag materials are shown in Table 20.37.

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TABLE 20.37 Fabric bag materials

		Resistance			
	Maximum Long-Term Operating Temperature, °C	Tensile Strength	Abrasion	Acid	Alkali
Cotton	82	Good	Fair	Poor	Excellent
Polypropylene	93	–	–	Excellent	Excellent
Nylon	93	Excellent	Excellent	Poor	Excellent
Acrylic	121	–	Good	Good	Fair
Polyester	135	Excellent	Good	Good	Fair
Nomex felt	218	Excellent	Good	Fair	Excellent
Teflon, Gore-Tex	218	–	–	Excellent	Excellent
Fiberglass	260	Excellent	Poor	Fair	Poor
P84, polyimide	260	–	–	Excellent	–
Nextel 312, ceramic	500	–	–	–	–

Source: Bergmann 1981

Troubleshooting

- For electrostatic precipitators (ESPs), keep flush water below 1,000 mg/L calcium sulfate ($CaSO_4$) and 15 mg/L calcium fluoride (CaF_2) to prevent scaling. Adjust pH to control $CaCO_3$. Oils in gas or flush water lower ESP efficiency. Adequate grounding is needed; check for galvanic corrosion between dissimilar metals. Keep flush water below its adiabatic saturation temperature.
- Needlepoint felts are better than woven fabrics for pulse jet filters.
- Pleated paper cartridge filters are best for control rooms; a dirty air inlet typically is <23 g/m^3 and pH must be 6–8.

ELECTROWINNING AND ELECTROREFINING by Douglas J. Robinson

In EW, the primary cathode reaction is $M^{+z} + ze^- = M°$, and the primary anode reaction is $H_2O = 2H^+ + 0.5\ O_2(gas) + 2e^-$, or in the case of an EW chloride electrolyte, $2Cl^- = Cl_2 + 2e^-$.

In ER, the primary anode reaction is $M° = M^{z+} + ze^-$. In electromotive series (typical voltages), most active metals dissolve first and most noble metals deposit first as shown in Table 20.38.

The following equation shows Faraday's law:

$$W = \frac{I \times t \times M \times CE}{z \times F} \quad \text{(coulombs = amps} \times \text{seconds)} \qquad \textbf{(EQ 20.70)}$$

where

W = metal dissolved or deposited, g
I = current, A
t = time, s
M = molecular weight of the metal, g
CE = current efficiency as a decimal fraction
z = change of valence in the deposition or dissolution reaction, that is, moles electrons per mole metal
F = faraday = 96,500 coulombs per mole electrons coulombs = A × s

TABLE 20.38 Deposition of metals

Noble							Active
Au^{+3}/Au	Ag^+/Ag	Fe^{+3}/Fe^{+2}	Cu^{+2}/Cu	H^+/H_2	Pb^{+2}/Pb	Ni^{+2}/Ni	Zn^{+2}/Zn
1.35V	0.79	0.55	0.335	0.00	−1.26	−0.25	−0.76

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TABLE 20.39 Electrolytic plant operating data

Item	Cu	Zn	Ni	Pb	Ag
Range of plant production, stpd	9–450	270–820	36–180	91–540	0.1–1.6
Cathode current density, A/m^2	215–377	377–807	215–269	161–215	54–215
Cell voltage, EW	1.96	3.2	3.8	–	–
Cell voltage, ER	0.28–0.48	–	–	0.09–0.15	0.13
kW·h/kg produced, EW	2.1	3.5	3.8	–	–
kW·h/kg produced, ER	0.3–0.5	–	–	0.33–0.52	0.5
Cathode material	SS, Cu	Al	SS, Ni	Pb	SS
Cathode size, m^2	1.86–2.32	1.86–3.25	1.86–2.32	1.49–2.32	0.28–0.56
Anode material	Rolled Ca-Sn-Pb	Cast Ag-Pb	Sb-Pb	Gr	Ag
Metal salt	$CuSO_4$	$ZnSO_4$	$NiSO_4$, $NiCl_2$	$PbSiF_6$	$AgNO_3$
Acid concentration, g/L	180	160	pH 3	100	pH 3
Reagents, 5–15 mg/L	Guar	Glue	–	Electrochemical activation (ECA) glue	Glue

For example, 1,000 A produces about 28.5 kg of Cu° from Cu^{++} per day per cell at 100% CE. Table 20.39 shows typical electrolytic plant operating data.

- No adequate method of making a first principles calculation of current density exists that is general for all metals. Current density is selected from operating experience. The practical current density is <40% of the limiting current density to avoid the onset of powdery deposits. Mass transfer theory says that an approximation of the limiting current density (IL) in amperes per square meter is 18.6 × [M^+] in grams per liter; that is, for a 40 g/L Cu^{++} solution, IL is approximately 744 A/m^2.
- Deposition generally should be under activation or charge transfer control. The cathode overvoltage typically exceeds 30 mV. Temperatures often exceed 40°C.
- Generally, a surface-active agent is required to control crystal growth if the electrolyte temperature exceeds 45°C. Concentrations in the milligram per liter range are suitable.
- The submerged area of the cathodes should be as large as possible to maximize the current carried by each electrode (this minimizes the total number of cathodes and the overall capital cost needed to meet design production).
- The submerged anode dimensions should be 2.5–3.8 cm less than the cathode on each side and at the bottom when the anode–cathode spacing is about 3.8 cm. If the anode is too wide or long, the cathode edges become rough. Anode edges radiate current cylindrically, with good throwing power at angles up to 45 degrees. Therefore, cathode overlap should approximately equal anode–cathode spacing.
- Solution resistance primarily depends on electrode spacing with total plant voltage = (number of cells × cell voltage) + busbar losses of 1–2 V maximum:

$$R = r\left(\frac{L}{A}\right) \qquad \textbf{(EQ 20.71)}$$

where

R = resistance, ohms
r = resistivity of the electrolyte, ohm-cm
A = submerged cathode area, cm^2
L = electrode spacing, cm

- The number of electrodes per cell is a compromise. Typical values for any metal are 20–30 cathodes/cell at a production rate of 20 stpd, 50–60 at 100–300 stpd, and 70–90 for larger plants. An increase in the number of cathodes/cell decreases operating labor costs, whereas a decrease in the number of cathodes/cell decreases capital cost by reducing both the rectifier current and the cross-sectional area of busbars.

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- Cathode width is determined by the width of cathode material that can be purchased and by the door opening size on the melting furnace. The total number of cathodes = total current/(current density × submerged cathode area). The number of cells = total cathodes/(cathodes/cell). Cells should be adjacent with their tops all at the same elevation and the ends of all cells in perfect alignment. Each row of cells should have many cells in it and the number of rows should be an even number.
- Backup rectifiers and transformers should be installed.
- Busbars should not be operated >40°C above ambient air. Short circuits lead to overheating of the intercell busbar, which can lead to two warped electrodes in the adjacent cells. Their current-carrying capacity, *ampacity*, depends on the material; that is, copper ampacity is three times that of aluminum. The ampacity also depends on busbar shape; a perfect shape is a hollow bar with coolant running through the bar. Typical shapes are rectangular, and the ampacity varies with orientation, which affects cooling, for example:
  - A bar 6-mm wide × 100-mm tall (vertical orientation) can carry 155 A/cm^2.
  - A bar 100-mm wide × 6-mm tall (horizontal orientation) can carry 77 A/cm^2.
  - Four bars 6-mm wide × 100-mm tall at 6-mm spacing can carry 109 A/cm^2.
- Pull cathodes in gangs of every second or third cathode so that at least one surface of each anode is always active. Pull as large a gang as possible to minimize crane trips. Wash electrolyte from cathodes before it dries and crystallizes; once crystallized, it can never be totally washed off. Remove deposits from blanks by flexing and high-frequency low-amplitude hammering. High-impact hammering can deform and damage the blanks.
- The reactive film on anodes—lead dioxide (PbO_2) on Pb alloy anodes and ruthenium(IV) (RuO_2) or other Pt group oxide on titanium anodes—must be maintained while repairing or bypassing cells to prevent chemical contamination of the plated cathode metal. This is done with properly designed shorting frames or by draining the electrolyte. Ramping the current down (15 minutes) and then back up (45 minutes) is another way, but it loses production and can be uneconomical. Power failures can ruin the reactive film and contaminate cathodes; such problems can be avoided with backup generators.
- Electrode envelopes (diaphragms)
  - Up to 15 g/L metal concentration change can be obtained if diaphragms are put around cathodes and a solution that has been purified by hydrometallurgical methods is fed into the cathode bag and allowed to permeate through the diaphragm into the common anode chamber, for example, nickel refining.
  - Up to 90 g/L acid change can be obtained across diaphragms that are put around anodes when high pH electrolyte is fed into the cathode chamber and some of it flows through the diaphragms into the individual anode chambers, for example, Co EW.
  - Chlorine gas from chloride electrolysis can be totally captured by enclosing the anodes in diaphragm bags.
- Control acid mist generated at the anodes in sulfate electrolysis by enclosing anodes in a bag; fitting anodes with hoods; covering each cell with hydrophobic beads, balls, blankets; providing cross-flow ventilation to carry the mist outside the building or to a scrubber; and by using alternative anode reactions, such as the ferrous-ferric couple.
- Uniform electrolyte flow is essential for (1) each cell, (2) the flow of current between each pair of electrodes, and (3) the electrolyte concentration throughout the interelectrode gaps. Items 2 and 3 are achieved by creating a uniform reservoir of electrolyte beneath the electrodes and then allowing the natural convective flow generated by the flow current to the electrodes to pump the electrolyte into the gap. Some plants incorporate a flow distributor in each cell; others use a 0.3-m-deep space below the electrodes to naturally create the reservoir. A sufficient flow is generally created with a change in metal ion concentration of 3–5 g/L between the feed and the spent electrolytes. The flow rate of spent electrolyte overflowing each cell can be determined by use of a rectangular weir plate in the overflow box.
- Uniform electrode spacing is essential and is established by insulating-spacing capblocks on the intercell walls and by insulators on the anode body.
- Electrode bars and hanger bars
  - The connection between the electrode blade and bar must be metallurgically sound to minimize voltage drop and ensure uniform current distribution. In Zn EW, the cathode blade is Al alloy and the bar is Al° fitted with a copper contact tip and Al° lifting hooks. In copper, the blade is stainless steel (SS) and bars are either a slotted solid Cu° or an SS tube electroplated with a 3-mm-thick coating of Cu°. Some metal processes use Ti cathode blades welded to a copper-cored Ti bar.

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- Materials of construction
  - Generally, new cells are of cast polymer concrete, a mixture of 12%–15% vinyl ester resin with at least two sizes of silica sand grains, 0.3 mm and 6 mm.
  - Buildings are reinforced concrete.
  - Floors are concrete-coated with a 12-mm-thick layer of polymer concrete.
  - Steel columns are coated with epoxy paint. Sometimes an SS or fiber-reinforced plastic (FRP) sheathing is attached to the inside of the columns for increased corrosion protection.

Checklist

- ☐ Tidy plant appearance
- ☐ Cells in alignment—tops, spacing, front and back walls
- ☐ Electrodes in perfect straight-line rows and with uniform spacing
- ☐ Electrodes flat and straight
- ☐ Electrode contacts cleaned regularly
- ☐ Busbars cool to the touch
- ☐ Salt spills promptly cleaned up
- ☐ Working platforms unobstructed
- ☐ Flow rates to all cells uniform
- ☐ Spent solution concentrations on target
- ☐ Additive flow rates and concentrations properly set
- ☐ Every pair of anode–cathode surfaces identical with every other pair
- ☐ Handrails maintained in perfect condition (ensures that everything else is under control)

PYROMETALLURGY by Terry P. McNulty, P.E.

This section covers chemistry, offers troubleshooting advice, and outlines operations.

Physical Chemistry

Figure 20.24 shows heat capacities of gases and liquids. Tables 20.40 through 20.42, respectively, show heat capacity ratios, heat capacities of solids, and saturated steam tables. Enthalpy (heat content), heat of formation, and free energy are all measured in kilojoules per mole. Following are other important measurements to remember:

- 1 calorie (small) = 4.184 J (joules); mol = gram molecular weight
- Heat capacity = heat energy needed to raise temperature of *x* weight by 1.0°C
- Specific heat of water = 1 cal/g°C = 4.184 J/g°C = 1 Btu/lb°F
- Specific heat of many minerals and rocks is approximately 0.25 cal/g°C

Free-energy diagrams (Ellingham or Richardson diagrams) consolidate a wealth of thermodynamic information on a single page. They are useful for rapidly estimating thermodynamic data. Figures 20.25 and 20.26, respectively, are for oxides and sulfides. Some useful features of the diagrams are as follows:

- Changes of state for elements and their oxides are shown on the diagram (see legend). Temperatures (°C) at which changes occur are read from the x-axis.
- The standard Gibbs free energy of formation ($\Delta G°_f$ per mole of gas) for the oxidation reactions is read directly from the y-axis at the temperature of interest.
- Standard entropy of reaction is determined by measuring the slope of the line for the reaction of interest; the slope is equal to the negative entropy ($\Delta S°$).
- Knowing the standard free energy and standard entropy of reaction, the standard enthalpy of reaction ($\Delta H°$) can be calculated by the Gibbs–Helmholtz equation:

$$t\Delta G° = \Delta H° - T\Delta S° \quad \textbf{(EQ 20.72)}$$

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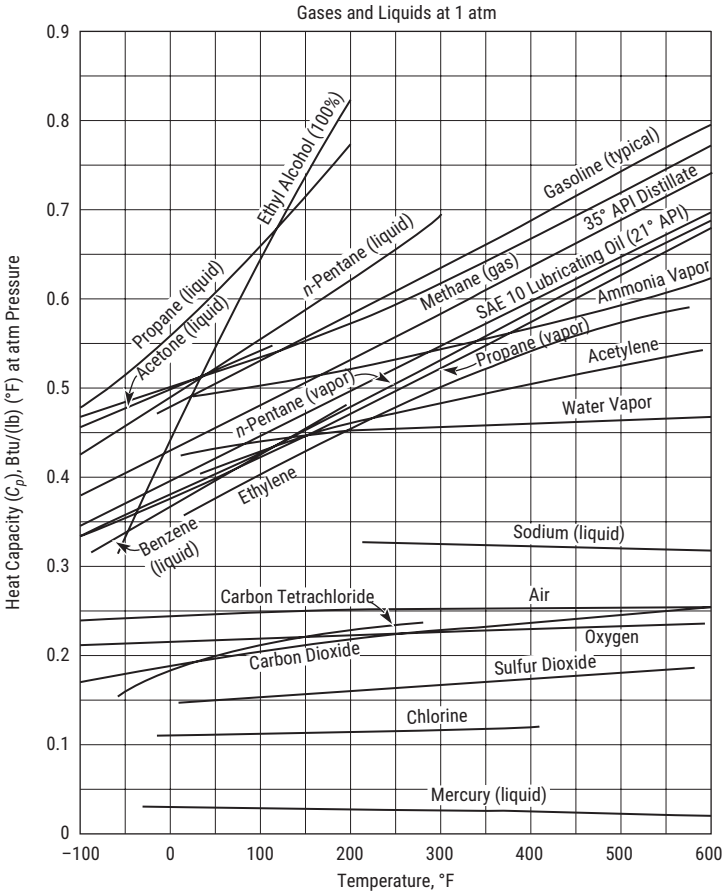

Source: Foust 1990

FIGURE 20.24 Heat capacities of gases and liquids

This approximation is valid up to the first-phase transition.

- Equilibrium oxygen partial pressure (P_{02}) and corresponding carbon monoxide-to-carbon dioxide (CO/CO_2) and hydrogen-to-water (H_2/H_2O) ratios can be determined for specific oxidation reactions and temperatures by first drawing a line from the appropriate symbol (O, C, or H on the vertical line to the left of the diagram) to the point on the desired reaction line that corresponds to the temperature of interest, and then extending the construction line to the appropriate scale to determine the P_{02}, CO/CO_2, and H_2/H_2O ratios. One can determine whether the reaction will spontaneously proceed under a nonequilibrium gas composition by applying the van't Hoff reaction isotherm:

$$\Delta G^\circ = \Delta G^\circ + RT \ln K \qquad \textbf{(EQ 20.73)}$$

where

K = dimensionless equilibrium constant defined as ratio of the molar concentrations of reactants and products
R = ideal gas constant
T = temperature

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TABLE 20.40 Ratio of heat capacities*

Compound	Formula	Temperature, °C	Ratio of Specific Heats, $\gamma = Cp/Cv$
Acetylene	C_2H_2	15	1.26
		–71	1.31
Air	—	925	1.36
		17	1.403
		–78	1.408
		–118	1.415
Ammonia	NH_3	15	1.310
Argon	Ar	15	1.668
		–180	1.76
		0–100	1.67
Benzene	C_6H_6	90	1.10
Carbon dioxide	CO_2	15	1.304
		–75	1.37
		–180	1.41
Chlorine	Cl_2	15	1.355
Dichlorodifluoromethane	CCl_2F_2	25	1.139
Ethane	C_2H_6	100	1.19
		15	1.22
		–82	1.28
Ethyl alcohol	C_2H_6O	90	1.13
Ethyl ether	$C_4H_{10}O$	35	1.08
		80	1.086
Ethylene	C_2H_4	100	1.18
		15	1.255
		–91	1.35
Helium	He	–180	1.660
Hexane (*n*–)	C_6H_{14}	80	1.08
Hydrogen	H_2	15	1.410
		–76	1.453
		–181	1.597
Methane	CH_4	600	1.113
		300	1.16
		15	1.31
		–80	1.34
		–115	1.41
Methyl alcohol	CH_4O	77	1.203
Nitrogen	N_2	15	1.404
		–181	1.47
Oxygen	O_2	15	1.401
		–76	1.415
		–181	1.45
Pentane (*n*–)	C_5H_{12}	86	1.086
Sulfur dioxide	SO_2	15	1.29

* At 1 atm (C_p/C_v).

Source: Foust 1990

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TABLE 20.41 Heat capacities of solids

Metals	Heat Capacity, Btu/lb °F
Aluminum	0.374 (0°C), 0.405 (100°C)
Copper	0.164 (0°C), 0.169 (100°C)
Iron, cast	0.214 (20–100°C)
Lead	0.0535 (0°C), 0.0575 (100°C)
Nickel	0.0186 (0°C), 0.206 (100°C)
Steel	0.12
Tin	0.096 (0°C), 0.104 (100°C)
Zinc	0.164 (0°C), 0.172 (100°C)
Miscellaneous	**Heat Capacity, Btu/lb °F**
Alumina	0.2 (100°C), 0.274 (1,500°C)
Asbestos	0.25
Brickwork	About 0.2
Carbon (mean values)	0.168 (26–76°C) 0.314 (40–892°C); 0.387 (56–1,450°C)
Cellulose	0.32
Cement, portland clinker	0.186
Charcoal (wood)	0.242
Chrome brick	0.17
Clay	0.224
Coal	0.26–0.37
Coke (mean values)	0.265 (21–400°C) 0.359 (21–800°C); 0.403 (21–1,300°C)
Concrete	0.156 (70–312° F), 0.219 (72–1,472° F)
Fireclay brick	0.198 (100°C), 0.298 (1,500°C)
Fluorspar	0.21 (30°C)
Glass (crown)	0.16 to 0.20
Flint	0.117
Pyrex	0.20
Silicate	0.188–0.204 (0–100°C); 0.24 to 0.26 (0–700°C)
Wool	0.157
Graphite	0.165 (26–76°C), 0.390 (56–1,450°C)
Gypsum	0.259 (16–46°C)
Limestone	0.217
Magnesia	0.234 (100°C), 0.188 (1,500°C)
Magnesite brick	0.222 (100°C), 0.195 (1,500°C)
Marble	0.21 (18°C)
Quartz	0.17 (0°C), 0.28 (350°C)
Sand	0.19
Stone	About 0.2
Wood (oak)	0.570
Woods (other)	Most woods vary between 0.45 and 0.65

Source: Foust 1990

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TABLE 20.42 Saturated steam tables

Temperature, °F	Absolute Pressure, lb/in.^{2}	Volume, ft^3/lb		Enthalpy, Btu/lb		Entropy, Btu/(lb)(°R)	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
32	0.08854	0.01602	3,306	0.00	1,075.8	0.0000	2.1877
35	0.09995	0.01602	2,947	3.02	1,077.1	0.0061	2.1770
40	0.12170	0.01602	2,444	8.05	1,079.3	0.0162	2.1597
45	0.14752	0.01602	2,036.4	13.06	1,081.5	0.0262	2.1429
50	0.17811	0.01603	1,703.2	18.07	1,083.7	0.0361	2.1264
60	0.2563	0.01604	1,206.7	28.06	1,088.0	0.0555	2.0948
70	0.3631	0.01606	867.9	38.04	1,092.3	0.0745	2.0647
80	0.5069	0.01608	633.1	48.02	1,096.6	0.0932	2.0360
90	0.6982	0.01610	468.0	57.99	1,100.9	0.1115	2.0087
100	0.9492	0.01613	350.4	67.97	1,105.2	0.1295	1.9826
110	1.2748	0.01617	265.4	77.94	1,109.5	0.1471	1.9577
120	1.6924	0.01620	203.27	87.92	1,113.7	0.1645	1.9339
130	2.2225	0.01625	157.34	97.90	1,117.9	0.1816	1.9112
140	2.8886	0.01629	123.01	107.89	1,122.0	0.1984	1.8894
150	3.718	0.01634	97.07	117.89	1,126.1	0.2149	1.8685
160	4.741	0.01639	77.29	127.89	1,130.2	0.2311	1.8485
170	5.992	0.01645	62.06	137.90	1,134.2	0.2472	1.8293
180	7.510	0.01651	50.23	147.92	1,138.1	0.2630	1.8109
190	9.339	0.01657	40.96	157.95	1,142.0	0.2785	1.7932
200	11.526	0.01663	33.64	167.99	1,145.9	0.2938	1.7762
210	14.123	0.01670	27.82	178.05	1,149.7	0.3090	1.7598
212	14.696	0.01672	26.80	180.07	1,150.4	0.3120	1.7566
220	17.186	0.01677	23.15	188.13	1,153.4	0.3239	1.7440
230	20.780	0.01684	19.382	198.23	1,157.0	0.3387	1.7288
240	24.969	0.01692	16.323	208.34	1,160.5	0.3531	1.7140
250	29.825	0.01700	13.821	216.48	1,164.0	0.3675	1.6998
260	35.429	0.01709	11.763	228.64	1,167.3	0.3817	1.6860
270	41.858	0.01717	10.061	238.84	1,170.6	0.3958	1.6727
280	49.203	0.01726	8.645	249.06	1,173.8	0.4096	1.6597
290	57.556	0.01735	7.461	259.31	1,176.8	0.4234	1.6472
300	67.013	0.01745	6.466	269.59	1,179.7	0.4369	1.6350
310	77.68	0.01755	5.626	279.92	1,182.5	0.4504	1.6231
320	89.66	0.01765	4.914	290.28	1,185.2	0.4637	1.6115
330	103.06	0.01776	4.307	300.68	1,187.7	0.4769	1.6002
340	118.01	0.01787	3.788	311.13	1,190.1	0.4900	1.5891
350	134.63	0.01799	3.342	321.63	1,192.3	0.5029	1.5783
360	153.04	0.01811	2.957	332.18	1,194.4	0.5158	1.5677
370	173.37	0.01823	2.625	342.79	1,196.3	0.5286	1.5573
380	195.77	0.01836	2.335	353.45	1,198.1	0.5413	1.5471
390	220.37	0.01850	2.0836	364.17	1,199.6	0.5539	1.5371
400	247.31	0.01864	1.8633	374.97	1,201.0	0.5664	1.5272

(continues)

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TABLE 20.42 **Saturated steam tables** (continued)

Temperature, °F	Absolute Pressure, lb/in.^{2}	Volume, ft^3/lb		Enthalpy, Btu/lb		Entropy, Btu/(lb)(°R)	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
410	276.75	0.01878	1.6700	385.83	1,202.1	0.5788	1.5174
420	308.83	0.01894	1.5000	396.77	1,203.1	0.5912	1.5078
430	343.72	0.01910	1.3499	407.79	1,203.8	0.6035	1.4982
440	381.59	0.01926	1.2171	418.90	1,204.3	0.6158	1.4887
450	422.6	0.0194	1.0993	430.1	1,204.6	0.6280	1.4793
460	466.9	0.0196	0.9944	441.4	1,204.6	0.6402	1.4700
470	514.7	0.0198	0.9009	452.8	1,204.3	0.6523	1.4606
480	566.1	0.0200	0.8172	464.4	1,203.7	0.6645	1.4513
490	621.4	0.0202	0.7423	476.0	1,202.8	0.6766	1.4419
500	680.8	0.0204	0.6749	487.8	1,201.7	0.6887	1.4325
520	812.4	0.0209	0.5594	511.9	1,198.2	0.7130	1.4136
540	962.5	0.0215	0.4649	536.6	1,193.2	0.7374	1.3942
560	1,133.1	0.0221	0.3868	562.2	1,186.4	0.7621	1.3742
580	1,325.8	0.0228	0.3217	588.9	1,177.3	0.7872	1.3532
600	1,542.9	0.0236	0.2668	617.0	1,165.5	0.8131	1.3307
620	1,786.6	0.0247	0.2201	646.7	1,150.3	0.8398	1.3062
640	2,059.7	0.0260	0.1798	678.6	1,130.5	0.8679	1.2789
660	2,365.4	0.0278	0.1442	714.2	1,104.4	0.8987	1.2472
680	2,708.1	0.0305	0.1115	757.3	1,067.2	0.9351	1.2071
700	3,093.7	0.0369	0.0761	823.3	995.4	0.9905	1.1389
705.4	3,206.2	0.0503	0.0503	902.7	902.7	1.0580	1.0580

Source: Keenan and Keyes 1936

Troubleshooting

- Look for obvious hot spots, such as buildup in the area with refractories. Check on the chemistry of reactions and determine if there has been a change in feed composition.
- For burners, determine if the net heat content of the fuel has changed. Perform exhaust gas analysis to see if the correct amount of air is flowing through the burner. Determine if the flame is being "thrown" too far.

Typical Operations and Emissions

Following are typical pyrometallurgical operations and a chart of emissions is shown in Table 20.43.

Drying

- Reduce filter cake moisture (e.g., filter aids, air blow, hot water/vacuum evaporation in plate and frame filters, pressure filtration, belt press filters, ceramic disk filters).
- Use infrared or induction heat to eliminate natural gas (NOx) and to reduce dusting.
- To evaporate 1 g of water requires approximately 666–1,221 calories, which is equal to 2,787–5,109 J.

Sintering

- Sintering is used to prepare an agglomerated calcine for blast furnaces.
- The feed size is approximately 12.7 mm or less.

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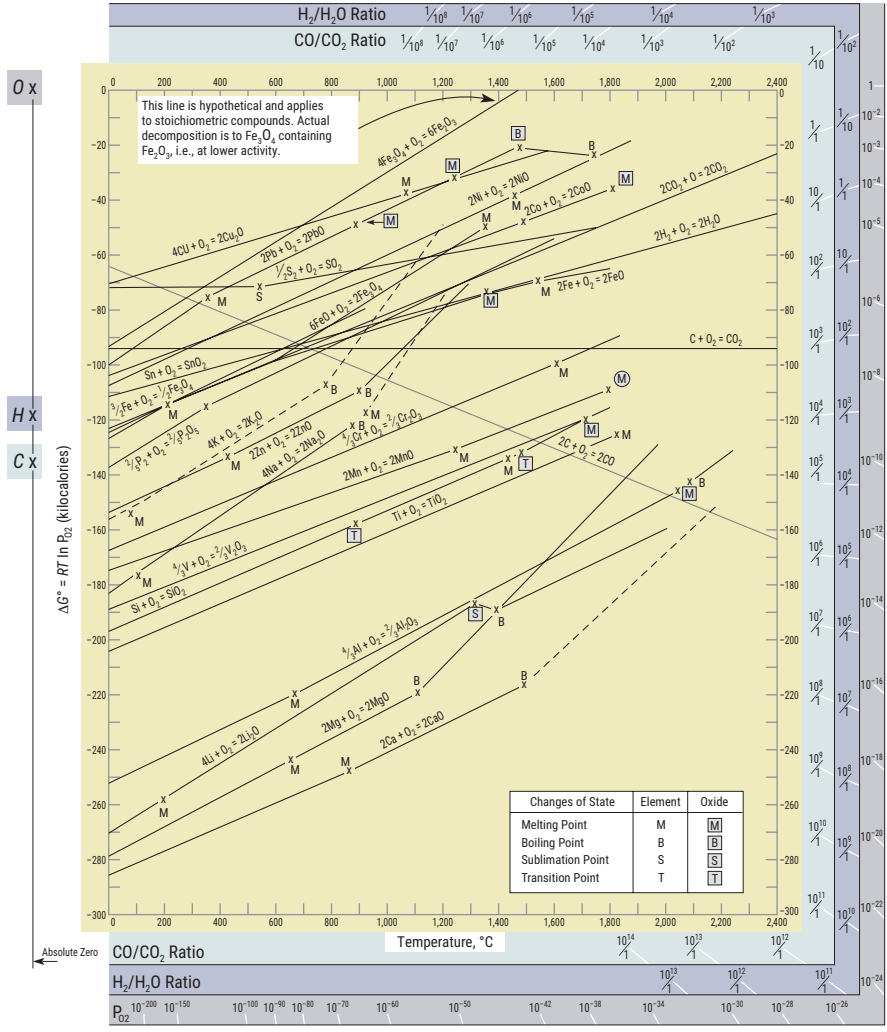
H2/H2O Ratio
CO/CO2 Ratio
This line is hypothetical and applies to stoichiometric compounds. Actual decomposition is to Fe3O4 containing Fe2O3, i.e., at lower activity.
4Fe3O4 + O2 = 6Fe2O3
4Cu + O2 = 2Cu2O
2Pb + O2 = 2PbO
1/2S2 + O2 = SO2
6FeO + O2 = 2Fe3O4
2Ni + O2 = 2NiO
2Co + O2 = 2CoO
2CO2 + O = 2CO2
2H2 + O2 = 2H2O
2Fe + O2 = 2FeO
C + O2 = CO2
Sn + O2 = SnO2
3/4Fe + O2 = 1/2Fe2O3
1/2K2 + O2 = 1/2K2O2
4K + O2 = 2K2O
2Zn + O2 = 2ZnO
4Na + O2 = 2Na2O
2Mn + O2 = 2MnO
2/3Cr + O2 = 1/3Cr2O3
2C + O2 = 2CO
Ti + O2 = TiO2
4/3V + O2 = 2/3V2O3
Si + O2 = SiO2
4/3Al + O2 = 2/3Al2O3
4Li + O2 = 2Li2O
2Mg + O2 = 2MgO
2Ca + O2 = 2CaO
Changes of State | Element | Oxide
Melting Point | M | M
Boiling Point | B | B
Sublimation Point | S | S
Transition Point | T | T
O x
H x
C x
ΔG° = RT ln PO2 (kilocalories)
Absolute Zero
Temperature, °C
CO/CO2 Ratio
H2/H2O Ratio
PO2

Source: Darken and Gurry 1953

FIGURE 20.25 Free-energy diagram for oxides

- For ignition, feed should be approximately 3-cm deep; up to 0.3-m deep on the traveling grate for sintering.
- Grates travel 25–120 cm/min.
- Between 2.9 and 16.6 t/m^2 of grate area per day is the usual capacity.
- Generally, updraft machines are used in the Pb industry, and downdraft machines are used in the Zn industry.
- Monitor the product fines, the sulfur burning rate in metric tons per square meter per day, the sulfur dioxide (SO_2) concentration in the exhaust gas, changes in the chemical analysis of the feed material and the final sinter product, and grate clogging.

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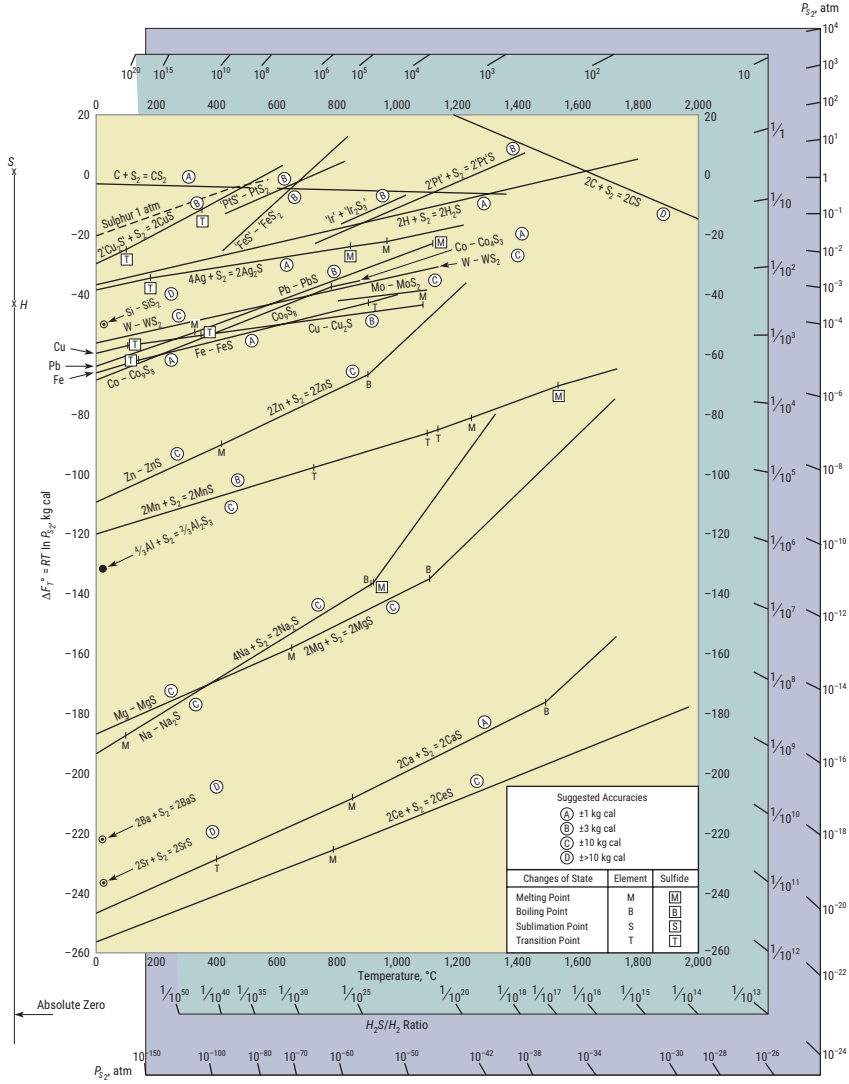

Source: Richardson and Jeffes 1952

FIGURE 20.26 Free energy of formation

Fluid Bed Reactors

- A temperature profile across the bed should be nearly constant; ±3°C in circulating bed fluid reactors (CBFRs). This allows more precise roasting and, possibly, 5% better gold extraction.
- Autogenous roasting requires about 4% sulfide sulfur in the feed.
- To burn off organic carbon, 700°C is needed.
- Dead roasted material should contain <0.1% sulfur.
- Feed should typically be <6.3 mm.

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TABLE 20.43 Emissions from typical pyrometallurgical operations

Metal	Type of Operation	Particulates		Sulfur Oxides		Carbon Monoxide		Gaseous Hydrofluoric Acid	
		kg/t	lb/st	kg/t	lb/st	kg/t	lb/st	kg/t	lb/st
Copper	Roasting	22.5	45	30	60	—	—	—	—
	Reverb, furnace smelting	10	20	160	320	—	—	—	—
	Converting	30	60	435	870	—	—	—	—
Lead	Sintering (updraft)	106.5	213	275	550	—	—	—	—
	Blast furnace	180.5	361	22.5	45	—	—	—	—
	Dross reverb furnace	10	20	Negligible	Negligible	—	—	—	—
Zinc	Roasting (multiple hearth)	60	120	550	1,100	—	—	—	—
	Sintering	45	90	Included in above	—	—	—	—	—
Pig Iron	Blast furnace	82.5	165	—	—	875	1,750	—	—
	Sintering	10	20	—	—	—	—	—	—
Steel	Open hearth furnace (no oxygen lance)	4.15	8.3	—	—	—	—	0.05	0.1
	Open hearth furnace (oxygen lance)	8.7	17.4	—	—	—	—	0.05	0.1
	Basic oxygen furnace	25.5	51	—	—	69.5	139	Negligible	Negligible
	Electric arc furnace (no oxygen lance)	4.6	9.2	—	—	9	18	0.006	0.012
	Electric arc furnace (oxygen lance)	5.5	11	—	—	9	18	0.006	0.012

Source: Sittig 1975

- Organic carbon in calcine should be 40% of the amount in the feed.
- CBFRs are replacing many stationary bed reactors.

Following are design considerations for fluid bed reactors:

- Dry or slurry feed
- Preheating of the roasting air
- Heat recovery circuit from the calcine
- Feed preheating
- Direct injection of additional fuel
- Reactor cooling method: cooling surfaces, water injection, or recycling of calcine
- Calcine cooling by a fluid bed cooler or by direct quenching
- Heat recovery in a waste heat boiler or by direct/indirect gas cooling
- Pollution control by a baghouse or an electrostatic precipitator

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Rotary Kilns

- Sizing and retention times are best calculated by the supplier's computer models. Data needed about the feed material are feed rate, bulk density, and angle of repose. Data needed about the kiln are inside diameter, length, slope, dam geometry, and speed in rotations per minute.
- The U.S. Bureau of Mines formula, simplified by Allis-Chalmers and shown next, varies by as much as 40% from actual operations and is best for lightly loaded kilns; that is, 5% loading rather than a more typical 15%–20% loading (loading = percent of cross-sectional area of kiln's interior occupied by bed of material).

$$\frac{L}{T} = V_{avg} = \frac{SDN}{1.77\beta^{0.5}} \quad \textbf{(EQ 20.74)}$$

where

L = kiln length, m
T = total retention time, minutes
V_{avg} = average axial solids velocity, m/min
S = kiln slope, degrees
D = kiln diameter, m
N = kiln rotation rate, rpm
β = dynamic angle of repose, degrees

Flash Smelter

- Slag contains about 2.5% Cu (about 10 times the Cu in the reverb furnace slag). It is reprocessed by flotation to yield tailings with approximately 0.3%–0.5% Cu (0.1%–0.3% Ni). High slag assays may be from pushing the capacity. Metal assays alone do not allow complete diagnosis of problems; optical microscopy is needed for characterization of losses.
- Matte grade is about 60%–70% Cu.
- Fusion is often incomplete because of (1) lack of understanding of slags and buildup of magnetite on the furnace bottom resulting in less retention time, (2) flux is too coarse, and (3) there is inadequate blending of furnace feed.
- Failure to obtain and analyze frequent, concurrent, and representative samples of feed, slag, metal, and dust results in a poor understanding of impurity distribution.
- Sulfur distribution (typical) is 97.4% to H_2SO_4, 0.5% to atmosphere from the acid plant, 0.2% to atmosphere from the dryer ahead of the smelter, 0.3% to atmosphere from the flash smelter, 0.8% to the atmosphere from converting and anode casting operations, and 0.8% in slag flotation tailings.

INDUSTRIAL MINERALS by Edwin H. Bentzen III

Industrial minerals are sold on the basis of their physical and chemical properties. They can be sold into more than one market or for more than one end-product use, often at two extremely different prices based only on the change in one chemical or physical characteristic. A product may sell in one market but not in another because of the differing needs of the buyer. Many tests can be used to assess the quality of an end product; some are written and some are not. Table 20.44 shows some of the tests used for a few industrial minerals. It is not all-inclusive. These tests often force the use of certain unit operations in an effort to make a salable product.

GOLD LEACHING by Paul D. Chamberlin, P.E.

Following is the reaction for gold leaching, and Table 20.45 shows the solubility of minerals in cyanide:

$$4Au + 8NaCN + O_2 + 2H_2O = 4NaAu(CN)_2 + 4NaOH \quad \textbf{(EQ 20.75)}$$

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TABLE 20.44 Common industrial mineral tests

		Chemical Analysis		Physical Characteristics										
Mineral/ Commodity	Major Markets	Major Value	Major Impurities	Moisture Criteria	Bacteria Count	Particle Size Distribution	Particle Shape	Mineralogical/Chemical Composition	Optical Factors (1)	Oil Absorption	Odor/Gas Absorption	Rheology Properties (2)	Specific Gravity	Pyro Factors; Other
Chemical Industries														
Alumina	C	Al_2O_3	Fe, Si	Y	X	N	N	Y						(3)
Beryllium	C	Be		N		N	N	Y						
Borates	C, F, A	B	As	N		Y	N	Y						
Bromine	C	Br	Cl, I, H_2O, organics			N	N	Y						
Chromium	C	Cr	Fe, Mg, S	N		Y	N	Y						
Fluorine	C	F		N		N	N	Y						
Iodine	C	I	Br, Cl			N	N	Y						
Lime	C, F, A	Ca	Mg, Fe	N		S	N	Y						
Lithium	C	Li		N		N	N	Y						
Phosphate	C, F, A	PO_4	U, V, Ni	N		N	N	Y						
Potash	C, F, A	K_2O	SiO_2, NaCl	Y		Y	N	Y						
Rare earths	C			N		Y	N	Y						
Salt	C	NaCl	As, organics	Y		Y	N	Y						
Soda ash	C, A	Na_2CO_3	Fe, organics, other carbonates, pyrite	Y		Y	N	Y						
Strontium minerals	C	Sr	Ba, Fe, F	N		Y	N	Y						
Sodium sulfate	C, F, A	Na_2SO_4	Fe, organics, other sulfates	Y		Y	N	Y						
Sulfur	C, F, A	S	Organics	N		S	Y	Y						
Zircon	C	Zr	Hf, Th, U, Fe	N		Y	N	Y						
Fillers														
Calcium carbonate	FI, P		Dolomite, iron oxides	Y	N	Y	S	S	Y	N	N	Y		
Common clays	FI, CO		Quartz	Y	N	Y	Y	N	N	N	N	Y		(4)
Diatomite	FI, CO, FA, P		Clays, quartz	Y	N	Y	Y	Y	N	N	N	N		
Iron oxides	P		Quartz	Y	N	Y	Y	N	Y	N	N	Y		
Kaolin	FI, CO, FA, P		Iron oxides, quartz	S	Y	Y	Y	Y	Y	N	N	Y		
Mica	FI, CO		Quartz, feldspar	Y	N	Y	Y	S	Y	S	N	Y		

(continues)

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TABLE 20.44 Common industrial mineral tests (continued)

		Chemical Analysis		Physical Characteristics										
Mineral/ Commodity	**Major Markets**	**Major Value**	**Major Impurities**	**Moisture Criteria**	**Bacteria Count**	**Particle Size Distribution**	**Particle Shape**	**Mineralogical/Chemical Composition**	**Optical Factors (1)**	**Oil Absorption**	**Odor/Gas Absorption**	**Rheology Properties (2)**	**Specific Gravity**	**Pyro Factors; Other**
Fillers (continued)														
Natural zeolites	FI		Quartz, clays	Y	N	Y	N	Y	N	Y	Y	N		
Perlite, expanded	FI		Obsidian	Y	N	Y	Y	N	Y	S	N	N		
Talc	FI, CO, P		Asbestos serpentine	N	Y	Y	Y	Y	Y	S	S	Y		
Titanium minerals	FI, P		Zircon, garnet, quartz, iron oxides, manganese	N	N	S	N	S	N	N	N	N		
Tripoli	FI			Y	N	Y	Y	N	N	N	N	S		
Ceramics														
Asbestos	I			Y		Y	Y	Y	S	S	N	S		
Chromite	R	Cr	Olivine, serpentine	Y		Y	S	Y	N	N	N	N		(5)
Feldspar	CE, G, I, FL	Al_2O_3, alkali	Mica, quartz, iron minerals	Y		Y	N	S	Y	S	N	N		
Graphite	R, I	Carbon, LOI	Quartz	Y		Y	Y	Y	N	S	N	S		
Kyanite	CE, R	Al_2O_3, alkali	Quartz	Y		Y	Y	Y	N	N	N	N		
Magnesite	CE, R	Mg, Ca, LOI	$CaCO_3$, quartz, Fe	Y		Y	N	Y	S	N	N	N		
Nepheline syenite	CE, R, G	Al_2O_3, Fe, alkali	Iron minerals	Y		Y	N	Y	N	N	N	N		
Olivine	R, FL	LOI	Serpentine, talc	Y		Y	S	Y	N	N	N	N		
Perlite, expanded	I	LOI	Obsidian	Y		Y	N	Y	S	S	S	S		
Pyrophyllite	CE, R	Al_2O_3, alkali	Quartz	Y		Y	Y	Y	Y	N	N	N		
Silica sand	CE, R, G, I, FL	SiO_2	Clay, refractory heavy minerals	Y		Y	Y	S	S	N	N	N		
Sillimanite	CE, R	Al_2O_3, alkali	Quartz	Y		Y	Y	Y	Y	N	N	N		
Vermiculite	I	LOI	Quartz, feldspar	Y		Y	Y	S	N	S	S	S		

(continues)

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TABLE 20.44 Common industrial mineral tests (continued)

		Chemical Analysis		Physical Characteristics										
Mineral/ Commodity	Major Markets	Major Value	Major Impurities	Moisture Criteria	Bacteria Count	Particle Size Distribution	Particle Shape	Mineralogical/Chemical Composition	Optical Factors (1)	Oil Absorption	Odor/Gas Absorption	Rheology Properties (2)	Specific Gravity	Pyro Factors; Other
Well Drilling														
Barite	W	BA	Quartz	Y		Y	N	Y	N	N	N	Y	Y	
Bentonite	W, AD			Y		Y	S	Y	N	Y	S	Y	N	
Corundum	AB	Al_2O_3	Quartz	N		Y	S	N	N	N	N	N	S	
Diamonds	AB	C		N		Y	S	N	N	N	N	N	N	
Diatomite	AD, E, AB	Si	Quartz	S		Y	Y	N	S	Y	S	S	N	
Emery	AB		Quartz	N		Y	S	N	N	N	N	N	S	
Fracture sand	W, AB	SiO_2	Clay, feldspar	N		Y	Y	N	N	N	N	N	N	
Garnet	AB		Quartz	N		Y	S	N	N	N	N	N	Y	
Limestone	E	$CaCO_3$	Quartz	S		S	N	N	S	S	S	S	N	
Silica sand	AB	SiO_2	Clay, feldspar	S		S	S	N	N	N	N	N	N	
Zeolite	AD, E, AB			Y		Y	S	Y	N	Y	Y	S	N	
Construction														
Crushed rock			Sand, silt, clay	S		Y	S	S	N	N	N	N		
Dimension stone			Pyrite, iron carbonates	N		Y	S	S	Y	N	N	N		
Gypsum			Anhydrite, quartz	Y		Y	N	Y	Y	N	N	Y		
Pumice, scoria			Quartz	Y		Y	S	S	S	S	N	N		
Sand and gravel			Clay, organics	Y		Y	S	Y	N	N	N	Y		

Legend:

Y = yes; N = no; S = sometimes.

A = agriculture; AB = abrasives; AD = adsorbents; C = chemicals; CE = ceramics; CO = coatings; E = environmental; F = fertilizer; FA = filter aid; FI = fillers; FL = flux; G = glass; I = insulators; P = pigments; R = refractories; W = well drilling.

(1) = optical factors, i.e., opacity, color, brightness, reflectance; (2) = rheology properties, i.e., swelling, loading, adsorbing; (3) = polished stone value, load capacity, thermal expansion; (4) = swelling, adsorption, load capacity; (5) = load capacity, expansion, pyrometallurgical tests.

The reaction requires the following steps:

1. Install dissolved oxygen probes and aerate to >2 ppm dissolved oxygen.
2. Dissolved oxygen in leach solution (up to 6–7 ppm naturally) is enough to dissolve Au, but may not be enough to react with other oxygen consumers.
3. If preg robbers are present, take cyanide out of the grinding circuit.
4. Test carbon-in-leach (CIL) tailings for gravity concentration and regrinding of pyrite.
5. Conduct a recovery versus grind P_{80} series.
6. As little as 100 mg/L of NaCN is enough to dissolve gold (approximately 10 times more for Ag).
7. Copper cyanide, CuCN, and zinc cyanide C_2N_2Zn, can titrate as free cyanide; be careful.

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TABLE 20.45 Solubility of minerals in cyanide

Mineral	Percent Dissolved in 24 hours	Source	Mineral	Percent Dissolved in 24 hours	Source
Calaverite, $AuTe_2$	Readily soluble	Johnstone 1933	Hydrozincite, $3ZnCO_3 \cdot 2H_2O$	35.1	Leaver and Woolf 1931
Argentite, Ag_2S	Readily soluble	Leaver et al. 1931	Franklinite, $(Fe,Mn,Zn)O \cdot (Fe,Mn)_2O_3$	20.2	Leaver and Woolf 1931
Cerargyrite, AgCl	Readily soluble	Leaver et al. 1931	Sphalerite, ZnS	18.4	Leaver and Woolf 1931
Proustite, Ag_3AsS_3	Sparingly soluble	Leaver et al. 1931	Gelamine, $H_2Zn_2SiO_4$	13.4	Leaver and Woolf 1931
Pyrargyrite, Ag_3SbS_3	Sparingly soluble	Leaver et al. 1931	Willemite, Zn_2SiO_4	13.1	Leaver and Woolf 1931
Azurite, $2CuCO_3 \cdot Cu(OH)_2$	94.5	Leaver and Woolf 1931	Pyrrhotite, FeS	Readily soluble	Hedley and Tabachnik 1968
Malachite, $CuCO_3 \cdot Cu(OH)_2$	90.2	Leaver and Woolf 1931	Pyrite, FeS_2	Sparingly soluble	Hedley and Tabachnik 1968
Chalcocite, Cu_2S	90.2	Leaver and Woolf 1931	Hematite, Fe_2O_3	Sparingly soluble	Hedley and Tabachnik 1968
Cuprite, Cu_2O	85.5	Leaver and Woolf 1931	Magnetite, Fe_3O_4	Nearly insoluble	Hedley and Tabachnik 1968
Bornite, $FeS \cdot 2Cu_2S \cdot CuS$	70	Leaver and Woolf 1931	Siderite, $FeCO_3$	Nearly insoluble	Hedley and Tabachnik 1968
Enargite, $3CuS \cdot As_2S_5$	65.8	Leaver and Woolf 1931	Orpiment, As_2S_3	73	Hedley and Tabachnik 1968
Tetrahedrite, $4Cu_2S \cdot Sb_2S_3$	21.9	Leaver and Woolf 1931	Realgar, AsS	9.4	Hedley and Tabachnik 1968
Chrysocolla, $CuSiO_3$	11.8	Leaver and Woolf 1931	Arsenopyrite, FeAsS	0.9	Hedley and Tabachnik 1968
Chalcopyrite, $CuFeS_2$	5.6	Leaver and Woolf 1931	Stibnite, Sb_2S_3	21.1	Hedley and Tabachnik 1968
Smithsonite, $ZnCO_3$	40.2	Leaver and Woolf 1931	Galena, PbS	Soluble at high pH	Lemmon 1940
Zincite, ZnO	35.2	Leaver and Woolf 1931	–	–	–

Merrill–Crowe recovery of gold and silver requires the following:

- Preg solution should be <1 ppm total suspended solids after clarification and <1 ppm dissolved oxygen after deaeration. Eliminate surge capacity between clarification and Zn precipitation.
- Add NaCN into Zn cone to prevent formation of zinc peroxide (ZnO_2).
- Free NaCN concentration should be approximately 0.1–0.5 g/L entering the precipitate filter presses.
- Zinc consumption should be approximately 6 g Zn per gram Ag or 3 g Zn per gram Au.
- Pb acetate (1–10 ppm Pb++) combines with sulfides and reduces NaCN consumption as SCN–.
- An Hg/Au ratio of >5 makes a slimy, hard-to-filter precipitate.
- Keep the filter press feed pump submerged in barren solution.
- Optimal pH is 11–12.
- Excess zinc can collect in the filter presses; at the end of a press run, stop adding zinc through the Zn cone and use up the zinc in the presses until the barrens rise.
- More than 200 ppm Cu in preg retards Au precipitation because it consumes most of the free cyanide; nickel acts similarly.
- Sulfide ions react with zinc and free cyanide, thus inhibiting Au precipitation; arsenic and antimony are also deleterious.
- Good barren assays should be <0.034–0.100 g Au/t solution.
- Use very fine, high-grade zinc dust, not shavings.
- Use antiscalants to minimize lime scaling in the clarifiers and filter presses.

VAT LEACHING by Louis W. Cope, P.E.

Vat leaching is a more controlled, contained version of heap leaching. It is applicable to gold, copper, and other leachable mineral ores. Whereas a heap is contained only on its bottom side, a vat of ore is normally contained from below and on all four sides; that is, a tub (vat). A vat leaching operation can have any number of vats, with piping for flexible movement of solutions. Solution flow through the ore can be upward or downward. Loading and excavating a vat can be with conveyors, bucket-wheel excavators, clamshells, mobile front-end loaders, or by sluicing.

Table 20.46 compares leaching by vat, heap, and agitation. The Ecovat, for example, is a continuous vat in which ore and solution are fed at one end while the spent ore and metal-loaded solution are continuously removed from the vat.

- Advantages are that the ore in a vat can be totally immersed for better contact by the leaching solution. Like heap leaching, it is applicable to ores that do not require fine grinding for metal extraction. Vat leaching operations can be put under a roof or even within a building for use in cold, rainy, high evaporation climates, or where there is a shortage of water.
- Disadvantages are that vat leaching is applicable only to fast-leaching ores. Vat leaching is constrained by time the same as on-off heap leaching. Loading of ore and removal of spent ore must be maintained on a schedule.

PRESSURE OXIDATION OF GOLD ORES by Rahul S. Bhaduri, P.E.

This section covers acidic and alkaline pressure oxidation, as well as chemistry and acid systems.

Refractory Ore

The gradual depletion of free-milling gold deposits has directed the metallurgical industry toward the exploitation of refractory gold ores and the development of suitable processing methods for their treatment. The term *refractory* is used to describe those gold ores that do not yield satisfactory gold extraction by conventional cyanidation methods. Typically, gold recoveries from refractory ores are <80% by gravity/cyanidation (Fraser et al. 1991), therefore, requiring more sophisticated treatment techniques.

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TABLE 20.46 Vat, heap, and agitation leaching

	Vat	Heap	Agitation
Capital cost	1	0.8	2.5
Operating cost	1	0.9	3.0
Treatment time, days	2–3	90–300+	<3
Gold recovery, %	85	70	90+

The main causes of refractoriness are included in Table 20.47 and incorporate

- Locking of gold in sulfide minerals;
- Presence of decomposition products of sulfide minerals in cyanide solutions, which act either as cyanicides or as oxygen consumers;
- Presence of carbonaceous minerals in the ore that tend to adsorb the gold complex from the cyanide solution;
- Formation of protective coatings on the surface of the gold by various chemical compounds (iron oxides, antimony, and lead compounds);
- Existence of gold in the form of insoluble solid–solution alloys ($AuTe_2$, Au_2Te_3, $AuAgTe_2$, $AgAuTe_4$, $AuCuTe_4$, Ag_3AuTe_2) or compounds; and
- Passivation of the anodic dissolution of gold when the metal is in contact with other conducting minerals (Filmer 1982).

Pressure Oxidation

TABLE 20.47 Refractory ore types and treatments

Mineral	Metallurgical Issue	Pretreatment
Pyrrhotite (Fe_7S_8)	Reduced recovery, high reagent consumption	Pre-aeration
Carbonaceous ore	Preg-robbing characteristics (Au associated with or absorbed by organic carbon in ore)	Blanking, chlorination, CIL, roasting, thiosulfate-resin adsorption
Arsenopyrite/Pyrite (FeAsS, FeS_2)	Au encapsulated in sulfides and too fine to liberate by grinding	Autoclaving, roasting, bacterial oxidation
Tellurides ($AuTe_2$, $AgAuTe_4$, Ag_3AuTe_2)	Au present as a component of a telluride compound	Oxidation, roasting
Occluded	Fine Au encapsulated in host silica rock	Fine grinding

Source: Schweigart and Liebenberg 1986; Jha 1987; Kontopoulos and Stefanakis 1990

One of the earliest applications of pressure oxidation (POX) for rendering a wide range of refractory gold ores and flotation concentrates more amenable to cyanidation was demonstrated in an investigation by American Cyanamid Company (Hedley and Tabachnik 1957). The evaluation included materials containing arsenopyrite, pyrite, stibnite, realgar, orpiment, berthierite, and carbonaceous material. Gold extractions from ores and concentrates were improved from the range of 18%–86% to the 92%–99.5% range after pressure oxidation over a temperature range of 225°–245°C and oxygen overpressures between 450 and 600 psig prior to cyanidation.

The mineralogy of the ore and the mode of occurrence and distribution of gold among the various mineralogical phases is of utmost importance for development of an optimal processing method. In pyritic ores, gold is often found in submicrometer-size particles encapsulated in a finely disseminated state within the sulfide matrix. This encapsulation is the main cause of refractoriness, as access of cyanide ions to a gold particle inside a dense sulfide grain is limited. This is the ultimate type of refractoriness, because complete breakdown of the host mineral structure is required to achieve satisfactory gold extraction. A detailed mineralogical examination of the ore to be treated is therefore important, as it can provide the proper guidelines for process selection.

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The presence of arsenic makes processing even more complicated, because of additional imposed environmental constraints. The oxidation of sulfide minerals, such as arsenopyrite, to liberate the gold and render it amenable to cyanidation is accomplished in industrial practice using roasting, aqueous pressure oxidation, or bio-oxidation (Sparrow and Woodcock 1995). Roasting followed by cyanidation was the traditional treatment route for many years, but it is no longer an acceptable method for arsenic-bearing sulfide ores and concentrates because of the high operating costs and environmental regulations regarding sulfur and arsenic emissions (Schraufnagel 1983).

Hydrometallurgical processes, including pressure oxidation and bacterial oxidation followed by cyanidation, have found increasing favor and have become common practice (Thomas 1991; Van Weert et al. 1986; Nagpal et al. 1994; Deng 1995). These oxidative treatments are effective for gold extraction, but usually require a significant consumption of lime to neutralize the acid produced during oxidation of the sulfide minerals and to raise the pH to the levels required for cyanidation. In the case of bio-oxidation, long residence times are required in the oxidation step.

Alkaline oxidation may be the preferred treatment option for those arsenical sulfide ores that contain large amounts of acid-consuming minerals such as carbonates, low-sulfur content, and minimal preg-robbing tendencies. This pretreatment has several technical merits:

- No acid is required to initiate reactions.
- The exclusion of acidulation and neutralization circuits lowers capital costs.
- Expensive exotic materials of construction are absent, including titanium, Ferralium 255, and Alloy 20.
- Silver jarosite formation is absent, and therefore high silver recoveries are possible during subsequent cyanidation.
- Maintenance requirements are reduced.

However, in comparison with many developed acid processes, the alkaline oxidation route has received limited attention for refractory gold ores and only the Barrick Mercur gold mine in Utah used alkaline pressure oxidation (Thomas 1991; St. Louis and Edgecombe 1990) on a commercial scale. This may be because metal recoveries by alkaline autoclaving are typically lower than the acidic route as the reaction products may not be fully dispersed from the oxidized surface resulting in encapsulated gold.

Acidic Pressure Oxidation

Sulfide minerals decompose rapidly in acidic media under conditions of elevated pressure and temperature in the presence of oxygen as the principal oxidizer. This so-called chemical deliverance allows for the release of metals, encapsulated in the sulfide mineral matrix often as a solid–solution, which are not amenable to liberation by physical grinding. The sulfide-bearing ores or concentrates being treated often contain sufficient sulfide sulfur (typically >4 wt %) to allow for an autogenously run reaction; the exothermic sulfur oxidation reaction generates sufficient heat for the oxidation reaction to proceed to completion. Additional heating with steam injection is required with lower sulfur grade materials. With higher S levels, the extra heat of the oxidation reactions cannot be usefully recovered; on the contrary, cooling of the autoclaves is required with direct water addition. The oxygen overpressure required is primarily determined by the mass transfer requirements of the process; higher overpressures tend to shorten residence time in the reactor (Figure 20.27).

The aqueous oxidation of metal sulfides is characterized by complex chemistry at acidic pH <1, with several possible reactions either in series or parallel depending on the physiochemical conditions of the system. The inherent sulfide sulfur content of ores and/or their concentrate derivatives can report in several forms, during acidic POX reactions, such as elemental sulfur, sulfate, basic sulfate, acid, or jarosite.

Formation of elemental sulfur is favored only during the initial stages of pyrite reaction (Equations 20.76 and 20.77), at relatively low temperatures (90°–120°C) and at high acidities. Molten sulfur (temperatures >120°C) is an effective collector of many sulfides and can also collect gold.

$$4FeAsS + 5O_2 + 4H_2SO_4 \rightarrow 4HAsO_2 + 4FeSO_4 + 4S^\circ + 2H_20 \quad \textbf{(EQ 20.76)}$$

$$FeS_2 + 2O_2 \rightarrow FeSO_4 + S^\circ \quad \textbf{(EQ 20.77)}$$

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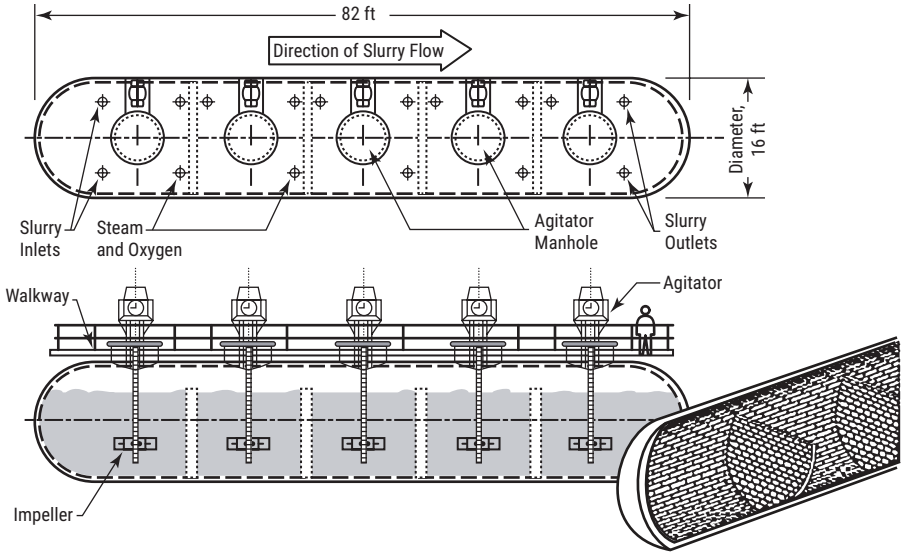

Source: Thomas 2005

FIGURE 20.27 Acidic pressure oxidation autoclave with internal brick lining

Formation of elemental sulfur (Berezowsky and Weir 1984) during pressure oxidation is undesirable for many reasons including the following:

- Occlusion of unreacted sulfides
- Hindering complete oxidation
- Occlusion of gold particles
- Impeding metal extraction during cyanidation
- Increased cyanide consumption during cyanidation
- Increased oxygen consumption during cyanidation

Sulfide oxidation must be conducted at temperatures in excess of 170°C to promote complete oxidation of the sulfides (see the following reaction) to sulfates and/or to oxidize any elemental sulfur formed as an intermediate product.

$$2S^0 + 3O_2 + 2H_2O \rightarrow 2H_2SO_4 \qquad \textbf{(EQ 20.78)}$$

Gold recovery is most sensitive to the type of sulfide mineralization that is oxidized, not the total amount of sulfur oxidation. In some ore types containing gold inclusions in marcasitic sulfur (Simmons 1993), partial oxidation of the sulfide sulfur, in the 50%–75% range, may be successful in providing high gold recovery. It may also result in lowering the amount of sulfuric acid produced, thereby lowering the amount of lime required for neutralization.

The pressure oxidation kinetics of sulfides are strongly dependent on particle size (Berezowsky and Weir 1989). For example, batch oxidation tests conducted on one concentrate showed that a reduction of particle size from P_{80} of 110 to 13 µm decreased the retention time requirement for oxidation of 95% of the sulfur from 92 to 23 minutes. Comparison of the relative capital and operating costs associated with concentrate regrinding versus reduced autoclave size and power requirements indicated an optimum particle size of about a P_{80} of 30 µm for this particular concentrate (Weir et al. 1986). Although a regrind of an ore has a similar effect on the oxidation kinetics, the higher costs associated with regrinding of the ore and the

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possibility of deteriorated solid–liquid separation characteristics may offset some of the benefits derived from pressure oxidation.

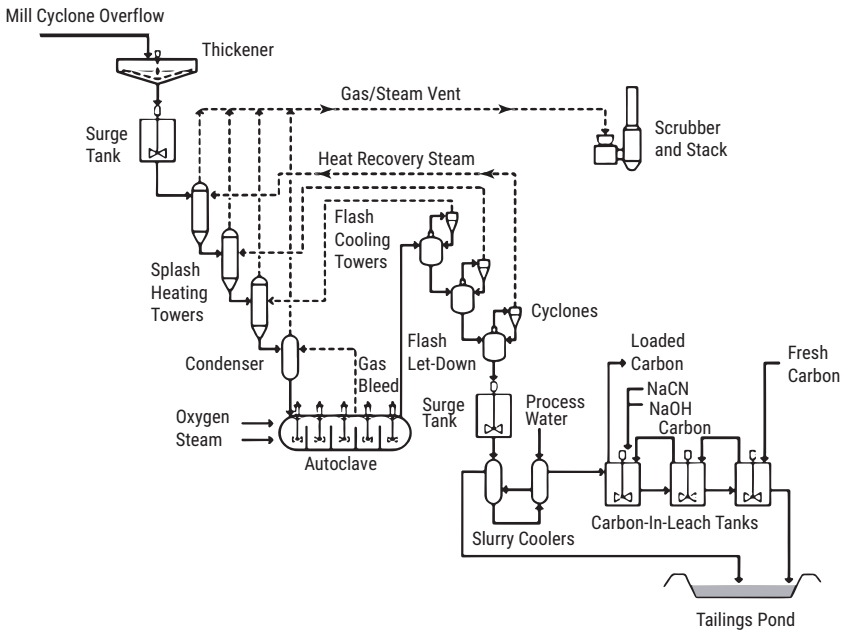
Fine grinding down to P_{80} of 20–22 µm in combination (Simmons 1995) with high-temperature POX at 225°C significantly increased organic carbon oxidation/passivation. Carbon oxidation/passivation correlated well with lower preg-robbing characteristics from feed through each autoclave compartment (four-unit reactor) to the discharge material.

Intensive pre-aeration during neutralization may be instrumental in converting significant amounts of ferrous iron (Fe^{+2}) to ferric iron (Fe^{+3}) in the autoclave discharge slurry (Simmons 1993). Aeration may result in a reduction in cyanide consumption by more than 1,000 g/t (~2.0 lb/st). Two-stage direct neutralization of the acidic autoclave discharge may confirm the ability to produce stable iron and arsenic residues for tailings disposal.

Alkaline Pressure Oxidation

Typically, the as-is pH range of autoclave feed and discharge is between 7.5 and 9.0, a feature unique to high-carbonate content (up to 20 wt %) and low sulfides (<2 wt %) in the ore (St. Louis and Edgecombe 1990). Gold particles entrained in pyrite are released or exposed when the oxidation takes place. The major oxidation reactions are the conversion of pyrite to hydrated iron oxides and hydroxides. Secondary reactions include conversion of arsenic sulfides to hydrated arsenic and iron-arsenic oxides. All sulfuric acid produced in these reactions is immediately neutralized by the limestone in the ore, resulting in the production of calcium sulfate and carbon dioxide (Figure 20.28).

Through bench-scale (Zhang and Nicol 2005; Lulham 1989; Koslides and Ciminelli 1992) experimentation, it has been ventured that gold dissolves during the alkaline oxidation (atmospheric pressure) of both arsenopyrite and pyritic-type refractory concentrates (Equations 20.79 and 20.80), prior to cyanidation, in the presence of copper ions. The dominant products of the oxidation reaction appear to be thiosulfate and monothioarsenate; the thiosulfate is believed to solubilize up to 80% contained gold (Equation 20.81) based on temperature, pulp density, stirring speed, and concentrations of oxygen and caustic.



Source: Thomas 1991. Reprinted with the permission of the Australasian Institute of Mining and Metallurgy

FIGURE 20.28 Simplified acid pressure oxidation circuit

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$$6FeAsS + 13O_2 + 22NaOH \rightarrow 6Fe(OH)_3 + 2Na_3AsO_3S + 4Na_3AsO_4 + 2Na_2S_2O_3 + 2H_2O \quad \textbf{(EQ 20.79)}$$

$$12FeS_2 + 25O_2 + 32NaOH + 2H_2O \rightarrow 12Fe(OH)_3 + 8Na_2S_2O_3 + 8Na_2SO_3 \quad \textbf{(EQ 20.80)}$$

$$4Na_2S_2O_3 + 2Au + H_2O + 1/2O_2 \rightarrow 2Na_3Au(S_2O_3)_2 + 2NaOH \quad \textbf{(EQ 20.81)}$$

The factors affecting oxidation kinetics of alkaline POX are similar to those in acidic POX; higher total pressures may be necessary to achieve desired oxygen partial pressures, because the higher carbonate content may liberate more CO_2 diluting the O_2 content of the gas phase in the autoclave (Mason et al. 1999).

Under alkaline POX conditions, the soluble sodium arsenate in the autoclave discharge (Equation 20.82) may be precipitated with lime as an insoluble calcium arsenate salt with subsequent regeneration of caustic.

$$4CaO + 2Na_3AsO_4 + 4H_2O \rightarrow Ca(OH)_2 + Ca_3(AsO_4)_2 + 6NaOH \quad \textbf{(EQ 20.82)}$$

Pressure Oxidation Chemistry, Acid Systems

Table 20.48 lists heats of reaction for various sulfide minerals under standard temperature and pressure for acidic and alkaline POX leaching as well as roasting. Based on iron and arsenic sulfide mineral oxidation, favorability in terms of free energy correlated from the heat of reaction, apparently alkaline POX > acidic POX > roasting. It is also evident that in all three routes, the lower heats of reaction for arsenopyrite (FeAsS) and ferrous disulfide (FeS_2) render a greater degree of complexity in the oxidation mechanism and in overcoming the activation energy barrier as compared to the other minerals. Arsenic and antimony (metalloids) minerals in the feed (present as realgar, orpiment, arsenopyrite, stibnite, etc.) behave in a similar manner to sulfur, forming arsenate (V) or antimonate (V) species.

Under acidic POX conditions, a significant portion of the solubilized iron and arsenic is precipitated as ferric arsenate and shown in the following reaction:

$$Fe_2(SO_4)_3 + 2H_3AsO_4 \rightarrow FeAsO_4 + 3H_2SO_4 \quad \textbf{(EQ 20.83)}$$

Some of the ferric sulfate may be hydrolyzed and precipitated as hematite (shown in the following reaction), as a basic ferric sulfate, or as hydronium jarosite (Kontopoulos and Stefanakis 1990; Berezowsky and Weir 1984; Long and Dixon 1999).

$$5Fe_2(SO_4)_3 + 19H_2O \rightarrow Fe_2O_3 + 2Fe(OH)SO_4 + 2H_3OFe_3(SO_4)_2(OH)_6 + 9H_2SO_4 \quad \textbf{(EQ 20.84)}$$

In the presence of potassium and sodium, released by dissolution of some of the gangue components, hydrolysis of at least a portion of the ferric sulfate (shown in the following reaction) to the corresponding jarosite may be expected.

$$Fe_2(SO_4)_3 + K_2SO_4 + 12H_2O \rightarrow 2KFe_3(SO_4)_2(OH)_6 + 6H_2SO_4 \quad \textbf{(EQ 20.85)}$$

Many other metals, such as silver, mercury, and lead, are also precipitated as jarosites either through substitution of the potassium, sodium, or hydronium ions or through formation of solid solutions in the precipitated jarosites.

Silver in the argentojarosite phase (as shown in the following reaction) cannot be extracted by subsequent cyanide leaching unless the oxidized material is conditioned with lime at 80°–95°C, to convert the jarosites to iron hydroxides and gypsum prior to cyanidation.

$$2AgFe_3(SO_4)_2(OH)_6 + 3Ca(OH)_2 \rightarrow 6Fe(OH)_3 + 3CaSO_4 + Ag_2SO_4 \quad \textbf{(EQ 20.86)}$$

Gold recovery generally decreases as graphitic or organic carbon content increases in the feed. Fine grinding (<25 µm) has been shown to significantly increase organic carbon oxidation/passivation (Simmons 1995).

Gold remains in its submicrometer elemental state unless chlorides are present; even at low chloride levels of <20 ppm, gold may be oxidized to its soluble chloro-complexes under acidic conditions (Nicol and Liu

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TABLE 20.48 Heats of formation and reaction for selected metal sulfides and products–reactants*

Mineral/Compound	ΔH_f, kJ/g·mol	Reactions	ΔH_r, kJ/g·mol
FeS_2 (pyrite)	−171.5	**Acidic Media Pressure Oxidation, 298 K and 1 atm†**	
Fe_7S_8 (pyrrhotite)	−736.4	$4FeS_2 + 15O_2 + 5H_2O \rightarrow Fe_2(SO_4)_3 + Fe_2O_3 + 5H_2SO_4$	−1,336.4
FeAsS (arsenopyrite)	−41.8	$4Fe_7S_8 + 69O_2 + 10H_2SO_4 \rightarrow 14Fe_2(SO_4)_3 + 10H_2O$	−6,992.6
As_2S_2 (realgar)	−142.5	$2FeAsS + 7O_2 + 6H_2O \rightarrow 2FeAsO_4{\cdot}2H_2O + 2H_2SO_4$	−1,396.4
As_2S_3 (orpiment)	−168.8	$2As_2S_2 + 2Fe_2(SO_4)_3 + 11O_2 + 18H_2O \rightarrow 4FeAsO_4{\cdot}2H_2O + 10H_2SO_4$	−1,724.7
MoS_2 (molybdenite)	−235.1	$2As_2S_3 + 2Fe_2(SO_4)_3 + 14O_2 + 20H_2O \rightarrow 4FeAsO_4{\cdot}2H_2O + 12H_2SO_4$	−2,223.0
NiS (nickel sulfide)	−87.9	**Caustic Media Pressure Oxidation, 298 K and 1 atm†**	
V_2S_3 (vanadium sulfide)	−1,484.8	$2FeS_2 + 15/2O_2 + 8NaOH \rightarrow Fe_2O_3 + 4Na_2SO_4 + 4H_2O$	−2,157.3
$Fe_2(SO_4)_3$ (ferric sulfate)	−2,583.0	$2Fe_7S_8 + 69/2O_2 + 32NaOH \rightarrow 7Fe_2O_3 + 16Na_2SO_4 + 16H_2O$	−8,673.7
Fe_2O_3 (hematite)	−825.5	$2FeAsS + 7O_2 + 10NaOH \rightarrow Fe_2O_3 + 2Na_3AsO_4 + 2Na_2SO_4 + 5H_2O$	−1,876.5
$FeAsO_4.2H_2O$ (scorodite)	−1,485.2	$As_2S_2 + 11/2O_2 + 10NaOH \rightarrow 2Na_3AsO_4 + 2Na_2SO_4 + 5H_2O$	−2,868.5
As_2O_3 (arsenic oxide)	−924.9	$As_2S_3 + 7O_2 + 12NaOH \rightarrow 2Na_3AsO_4 + 3Na_2SO_4 + 6H_2O$	−3,657.3
H_2SO_4 (sulfuric acid)	−810.4	$MoS_2 + 6NaOH + 9/2O_2 \rightarrow Na_2MoO_4 + Na_2SO_4 + 3H_2O$	−915.3
SO_2 (sulfur dioxide)	−296.8	$NiS + 2NaOH + 2O_2 \rightarrow Ni(OH)_2 + Na_2SO_4$	−984.0
Na_3AsO_4 (sodium arsenate)	−1,540.0	$V_2S_3 + 8NaOH + 7O_2 \rightarrow NaVO_3 + 3Na_2SO_4 + 4H_2O$	−1,539.9
O_2 (oxygen)	0	**Roasting, 298 K and 1 atm†**	
H_2O (water)	−285.8	$4FeS_2 + 11O_2 \rightarrow 2Fe_2O_3 + 8SO_2$	−834.9
MoO_3 (molybdenum trioxide)	−745.1	$2Fe_7S_8 + 53/2O_2 \rightarrow 7Fe_2O_3 + 16SO_2$	−4,527.3
Na_2MoO_4 (sodium molybdate)	−1,468.1	$2FeAsS + 5O_2 \rightarrow Fe_2O_3 + As_2O_3 + 2SO_2$	−1,130.2
$NaVO_3$ (sodium vanadate)	−1,145.8	$As_2S_2 + 7/2O_2 \rightarrow As_2O_3 + 2SO_2$	−1,376.0
NiO (nickel oxide)	−241.0	$As_2S_3 + 9/2O_2 \rightarrow As_2O_3 + 3SO_2$	−1,646.5
$Ni(OH)_2$ (nickel hydroxide)	−542.6	$MoS_2 + 7/2O_2 \rightarrow MoO_3 + 2SO_2$	−1,103.6
V_2O_5 (vanadium pentoxide)	−1,557.4	$NiS + 3/2O_2 \rightarrow NiO + SO_2$	−449.9
NaOH (caustic)	−426.1	$V_2S_3 + 11/2O_2 \rightarrow V_2O_5 + 3SO_2$	−963.0
Na_2SO_4 (sodium sulfate)	−1,381.5		

* ΔH_f = heat of formation; ΔH_r = heat of reaction.

† Example, pyrite acidic oxidation reaction:

$$\frac{[-2583.0 + (-825.5) + 5 \times (-810.5)] - [4 \times (-171.5) + 15 \times 0 + 5 \times (-285.8)]}{4 \text{ moles}}$$

FeS_2 = 1,336.4 kJ/g·mol FeS_2.

Source: Green and Perry 2008; Speight 2017

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2003; Simmons et al. 1999). Sources of chloride contamination include additives used in core drilling muds, hydrogen chloride (HCl) acid used in the field to detect carbonates, and plant water to name a few.

It has been speculated that gold solubilized in the presence of chlorides is subsequently reduced by the organic carbon. A three-step process has been proposed (Simmons et al. 1999; Bogorodsky et al. 2010; Tsyplakov et al. 2012):

1. Oxidation of metallic gold occurs in the presence of chloride ion and Fe(III) compounds with the formation of soluble chloraurate(III) complexes; shown in the following reaction.

$$4Au^{\circ} + 16Cl^{-} + 12H^{+}\ 3O_2 \rightarrow 4AuCl_4^{-} + 6H_2O,\ \Delta G \text{ at } 220^{\circ}C = -423 \text{ kJ/mole} \quad \textbf{(EQ 20.87)}$$

2. Chloraurate(III) ion is adsorbed on the natural carbon adsorbents.
3. Chloraurate (III) ion is reduced to metallic gold on the carbon adsorbent surface, accompanied by chloride ion regeneration as shown in the following reactions.

$$4AuCl_4^{-} + 3C + 6H_2O \rightarrow 4Au^{\circ} + 16Cl^{-} + 3CO_2 + 12H^{+},\ \Delta G \text{ at } 220^{\circ}C = -218 \text{ kJ/mole} \quad \textbf{(EQ 20.88)}$$

$$4H^{+} + 4Cl^{-} + O_2 \rightarrow 2H_2O + 2Cl_2,\ \Delta G \text{ at } 220^{\circ}C = -42 \text{ kJ/mole} \quad \textbf{(EQ 20.89)}$$

Subsequent cyanidation of the mixture, containing carbon adsorbent with precipitated gold on it, does not lead to gold reporting to solution and results in gold loss with the tailings.

It is surmised that one possible cause of the Au-robbing problem may be high acid levels in the autoclave because of low carbonate content in the feed. This theory was tested (Simmons et al. 1999) by adding limestone and/or dolomite to the problematic ores. It was observed that the gold-robbing effect might be significantly reduced at 20–28 g/L residual sulfuric acid concentration in the autoclave discharge. Optimal blending of ores containing different levels of sulfide and carbonate content may be targeted to achieve lower acid levels in the autoclave.

It was also established that the gold-robbing effect might be lessened if a 450–500 mV redox potential of filtered solution (293 K, saturated calomel electrode [SCE] reference) was maintained at the autoclave exit (Tsyplakov et al. 2012). This lays credence to the observation that the electrical potential of the oxidized discharge slurry is a measure of the driving force for the reaction and provides a good indication of the extent of oxidation. A solution potential of 480 mV is equivalent to a Fe^{+3}/Fe^{+2} ratio of 10:1 and denotes a high degree of oxidation (Marsden and House 2006).

GENERAL INFORMATION by Paul D. Chamberlin, P.E.

The following miscellaneous section is meant to include various information germane to the mining, mineral processing, and environmental engineer.

$$\text{percent solids of a pulp} = \frac{100(sg_p - 1)sg_s}{(sg_s - 1)sg_p} \quad \textbf{(EQ 20.90)}$$

where

sg_p = specific gravity of pulp
sg_s = specific gravity of solids

- Fans raise pressure by approximately 3%; blowers raise pressure up to about 40 psig; compressors are for higher pressures.
- Vacuum pumps: Reciprocating piston types are used down to 1 torr (1 mm Hg; 760 mm Hg per 14.7 psi); rotary piston down to 0.001 torr; two-lobe rotary down to 0.0001 torr; steam-jet ejectors—one-stage down to 100 torr, three-stage to 1 torr, five-stage to 0.05 torr.
- In-leakage of air to evacuated equipment

$$1 \leq P < 10 \text{ torr}; W = 0.026P^{0.34}V^{0.6} \quad \textbf{(EQ 20.91)}$$

$$10 \leq P < 100 \text{ torr}; W = 0.32P^{0.26}V^{0.6} \quad \textbf{(EQ 20.92)}$$

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TABLE 20.49 Feeder capacities and effect of material characteristics

Type of Feeder	Capacity Range, stph	Material Characteristics									Path of Travel		
		Size			Flowability			Abrasiveness					
		Fine	Granular	Large Lumps	Very Free	Free	Sluggish	Nonabrasive	Mildly Abrasive	Very Abrasive	Horizontal	Inclined	Combined
Apron (pan)	57–688		•	•	•	•	•	•	•	•	•	•	•
Belt	40–1,200	•	•		•	•		•	•	•	•	•	
Flight	36–182	•	•	•	•	•	•	•	•		•	•	•
Screw	10–187	•	•		•	•	•	•	•		•	•	
Reciprocating	82–450	•	•	•		•	•	•	•	•	•		
Vibratory, electrical	25–1,000	•	•	•	•	•	•	•	•	•	•		
Vibratory, mechanical	35–1,550	•	•		•	•		•	•	•	•		
Rotary table, stationary plow	12–57	•	•		•	•	•	•	•	•	•		
Rotary plow, stationary table	300–3,500		•	•			•	•	•	•	•		
Rotary (vane)	1.5–117	•	•		•	•		•	•		•		

Source: Anon. 1979

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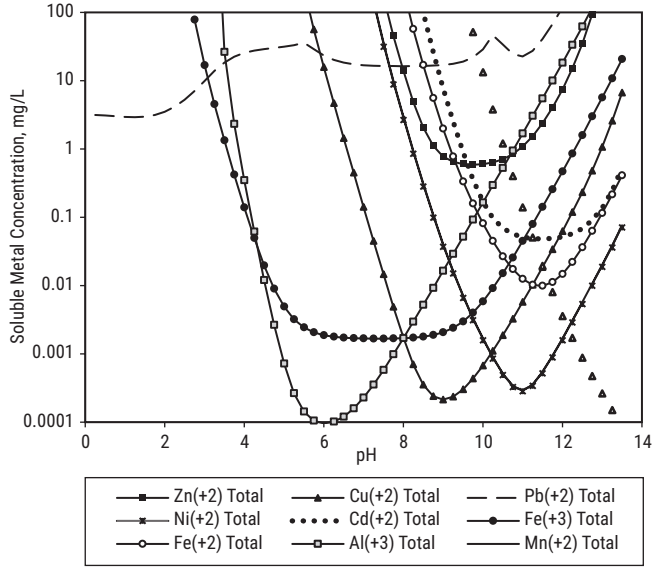

Source: Lewis 2010

FIGURE 20.29 Effect of pH on solubility of metal hydroxide ions

$$100 \leq P < 760 \text{ torr}; W = 0.106V^{0.6} \quad \textbf{(EQ 20.93)}$$

where

P = pressure in equipment, torr
V = volume, ft^3
W = equipment leakage, lb/h (does not include seals)

- Double W to account for static seals; $2W$ plus 5 lb/h for each rotary seal; $2W$ plus 2 lb/h for each mechanical seal and O-ring.
- Better fragmentation in mine blasting usually gives many benefits in processing (e.g., better sampling, fewer boulders to rehandle at the crushing plant, more fractures and a faster leaching rate, and lower crushing and grinding costs). Therefore, insert cone-shaped plastic plugs in the blastholes so that stemming is not blown out of the hole, and the blast energy is retained in the rock.
- Locate conveyor belt scales 6–9 m from the loading point. Avoid locating the scale near the drive pulley (belt tension is too high). Locate the scale >12 m from tangent points on the belt curves. Align (all in same plane) at least two and maybe four idlers on either side of the scale. The following four calibration methods can be used:
  1. Pass the known weight of the material over the scale. The material test is considered the best test.
  2. Hang static weights from the scale.
  3. Use test chains that roll on the moving conveyor.
  4. Test only the electronics and do not weigh anything. The electronics test is used only when the scale cannot be easily accessed.
- One ton (2,000 lb) of refrigeration equals the removal of 12,000 Btu/h of heat. The following heat transfer coefficients are approximate:
  - Water to liquid, 150 Btu/(h)(ft^2)(°F)
  - Condensers, 150 Btu/(h)(ft^2)(°F)
  - Liquid to liquid, 50 Btu/(h)(ft^2)(°F)

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  - Liquid to gas, 5 Btu/(h)(ft^2)(°F)
  - Gas to gas, 5 Btu/(h)(ft^2)(°F)
- Feeders and bins (Table 20.49): The size of the bin opening must be smaller than the feeder to ensure flow. For belt feeders, consider (1) if the bin is behind the feeder rather than on top of it and (2) whether the downstream bin opening is wider than the upstream opening. For screw feeders, increasing the pitch of the screw gives more uniform withdrawal. For rotary valves, add short pipe between the bin bottom and the feeder to eliminate stagnant zone in the bin. Bin discharge width should be greater than two to three times the largest particle size (if there are few large particles). The downstream bin wall should be vertical; other bin walls should be angled at greater than 50 degrees.

Overall Circuit Recovery

In general, percent recovery in a mineral processing circuit can be expressed two ways: one for metallic ore and one for nonmetallic ore. The recovery metallic concentrate is the percentage of the total metal contained in the ore that is recovered from the concentrate; a recovery of 90% means that 90% of the metal in the ore is recovered in the concentrate and 10% is lost in the tailings. The recovery for nonmetallic ores is the percentage of the total mineral contained in the ore that is recovered into the concentrate; that is, recovery is usually expressed in terms of the valuable end product. Figure 20.29 shows the effect of pH on solubility of metal hydroxide ions.

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CHAPTER

21

Tailings and Water Management

Paul W. Ridlen, P.E. and Jeffrey G. Coffin, P.E.

TAILINGS MANAGEMENT

Tailings are the materials left after the extraction of metals or minerals of interest from ore. In most instances, they comprise crushed and ground rock and soil particles, but in some cases, they contain precipitates and/or residual metals or minerals from the ore or the process. Tailings typically form one of the largest volumes of waste from a mining and milling operation, which, considering their physical and chemical characteristics, makes tailings disposal one of the largest elements of cost and risk for a mine. Brown (2002) notes that "the management of the disposal of mill tailings is of critical importance to the success of any mining project. Failures of tailings facilities have resulted in loss of life, devastating environmental damage, the closure of mining operations, dramatic declines in share value and, in some countries, the personal liability of the mine management." In recognition of these high consequences, the mining industry has focused much attention in recent years on tailings management methods and improving the design, operation, surveillance, and closure of tailings facilities through their full life cycle.

Most hard-rock milling operations use conventional tailings management processes that transport tailings in a water-based slurry from the process plant to a tailings impoundment, into which the slurry is discharged in a process called *hydraulic deposition* or *hydraulic filling*. Most impoundments are contained by an embankment constructed of either earth or rock fill, or of the tailings themselves. Typically, these embankments are referred to as a *tailings dam* or *tailings embankment*. In a conventional hydraulically filled impoundment, tailings slurries are discharged either from single-point discharge outlets, header-and-spigot distribution systems, hydrocyclones, or other similar devices. After discharge, the solids settle out of suspension, and the excess water is stored in a pond within the tailings impoundment, from which it is usually decanted and returned to the mill process or discharged into the environment (often after treatment to meet discharge water-quality standards).

Prior to deposition, tailings slurries commonly have solids contents (a measurement of slurry density) ranging from 20% to 60% solids by weight, depending on whether a thickener is included at the process plant. After deposition, the resulting tailings deposits are typically saturated or nearly saturated and exhibit gravimetric moisture contents ranging from approximately 20% to well over 100% by weight (i.e., equivalent to solids contents ranging from more than 80% to less than 50% by weight). In many cases, conventional tailings storage facilities are also used to manage and store water. As a result, the impoundments associated with conventional tailings deposition usually require the design, construction, and maintenance of a tailings dam (or multiple dams) that must achieve minimum requirements of structural integrity in order to safely store and manage the contents (both solids and water) retained in the impoundment.

Although conventional tailings impoundments remain the primary tailings management method for most operating and proposed mines around the world, dewatering technologies have advanced over the past few decades, with thickened-slurry deposition, paste, and filter applications being added to the tailings handling continuum. The development of large-capacity vacuum and pressure filter technology has presented the opportunity for storing tailings in a potentially unsaturated state, using conveyors and stackers for handling the dewatered tailings as a filter *cake*, rather than slurry-based hydraulic transport and deposition. Filtered tailings storage facilities (also referred to as *dry stacks*) typically exhibit gravimetric moisture contents in the range of 15%–20% by weight. For projects where water is scarce, or where site or environmental conditions complicate the use of a conventional hydraulically deposited impoundment, filtered tailings may be an appropriate alternative. As a further advantage, with such a significant reduction in water content, and the avoidance of water storage in the tailings facility, the risk of uncontrolled release of flowable tailings is dramatically reduced.

FACILITY LAYOUT

Although potential facility layouts are practically infinite in variety, they can be placed in general categories, whether for conventional, thickened, or filtered disposal applications. Tailings may also be placed underground, usually as a paste mixed with cement (or other binding admixtures), to backfill mine workings.

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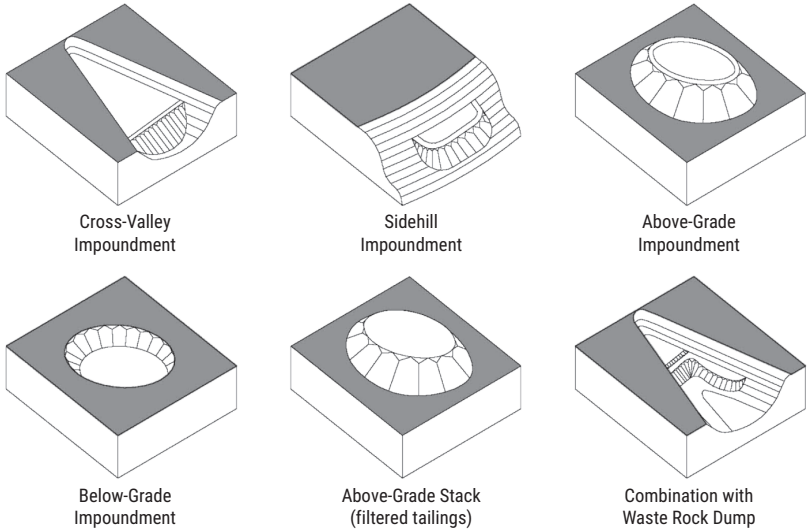

Adapted from Zahl et al. 1992 as appears in Kerr and Ulrich 2011

FIGURE 21.1 Tailings impoundment layouts

This process provides both subsurface stabilization of underground mine workings and a tailings disposal opportunity. Surface facilities may be categorized as follows, and as depicted in Figure 21.1. Table 21.1 shows typical siting considerations for tailings storage facilities.

- **Cross-valley.** The typical layout of a water-storage reservoir is constructed between the natural walls of a valley or other topographic depression. Placing the facility near the head of the drainage reduces inflow from runoff or the need for large diversion structures. Placing the facility further down the drainage path may result in greater storage capacity for a given embankment height or fill volume.
- **Sidehill.** Generally used on slopes with a grade of 10% or less, sidehill facilities use a natural slope as one side of the impoundment with dikes or embankments around the other three sides.
- **Above-grade impoundments (ring dikes or paddocks).** Typically used on flat terrain, all sides of the impoundment are enclosed by embankments or dikes. Above-grade impoundments can be applied to either hydraulically deposited or filtered tailings.
- **Below-grade impoundments.** This type usually describes backfilling an open pit or strip mine or underground paste backfill.
- **Combinations.** Tailings may be disposed of in combination with waste rock storage facilities and heap leach or dump leach facilities, either in a slurried form, as paste, or as a filter cake. Consideration for liner installation, leachate recovery, and delivery systems is important in these layouts.

STAGED DEVELOPMENT

Tailings are generated and progressively deposited over the life of a mine; thus, the development of the storage facility can be sequenced and built in stages, which is one major difference between tailings dams and water-storage dams. These stages may include dam construction, basin raises, or lateral expansions. The advantages of staged development include the following:

- Some of the capital cost is deferred over the life of the mine.
- As experience and knowledge is gained from the early stages, allowances are made for adjustments in the design in the latter stages.

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TABLE 21.1 Factors for siting tailings facilities

Parameter	Effects
Location and elevation relative to mill	Length of tailings/return-water pipelines and/or conveyor
	Capital and operating cost for pumps
Land availability	May constrain available storage volume or potential for expansions
Topography	Embankment layout and fill requirements
	Feasibility of diversions and emergency spillways
Climate, hydrology, catchment area	Require management of greater amounts of water in wet climates; may make dry-stacking impractical
	Risk of failure increased with long-term water accumulation and/or may increase treatment throughput
	Dust management challenges created in dry and windy climates
	Tailings management in winter complicated/limited in cold regions
Geology	Availability of natural borrow types and quantities
	Seepage losses
	Foundation stability
Groundwater	Rate and direction of seepage movement
	Contamination potential
	Moisture content of borrow materials
Social setting	Higher consequence classifications and thus increased design criteria, inspections, monitoring, and emergency response procedures for facilities close to population centers
	May face opposition by existing landowners, possibly needing height limitations for aesthetic and/or risk mitigation reasons
	May require increased seepage and dust control measures
Seismic conditions	Consideration of liquefaction or strength loss potential of any materials in the structural zone(s) and/or foundation of the facility required in modest to high-seismicity areas
Geochemical properties of the tailings	Acid drainage with dissolved metals possible with tailings containing sulfide minerals (e.g., pyrite or chalcopyrite) exposed to air and water

Adapted from Vick 1983

- Allowances for adjustments are made to account for changing conditions, including downstream consequence rating, if the land use around the facility changes.
- Waste rock or drained or cycloned tailings produced over the life of the mine are increased in use.
- A slow rate of loading is needed to maintain stability and staged development at sites where soft or low-permeability soils form the foundation for the embankment.

Staged development also has some disadvantages, such as the following:

- Over time, unless properly managed, some of the original design objectives and supporting information may become lost or unclear as personnel change.
- The quality of the construction may vary from stage to stage if different contractors, materials, and/or quality assurance and control procedures are used.

These disadvantages can be countered with good documentation, the provision of adequate resources at all times through operations, and a commitment by management and the operating staff toward continuous improvement. The staging sequence and schedule through the operation of the facility should be defined at the time of the initial design of the facility. It is usually connected to the mine plan because the mine plan determines the schedule of ore production and, thus by association, the schedule of tailings production. The mine plan also determines the waste rock production over the life of the mine when waste rock is used for construction of the facility. However, most mine plans change frequently, and thus any change that could affect the tailings facility should trigger a review of the staged development and tailings deposition plans.

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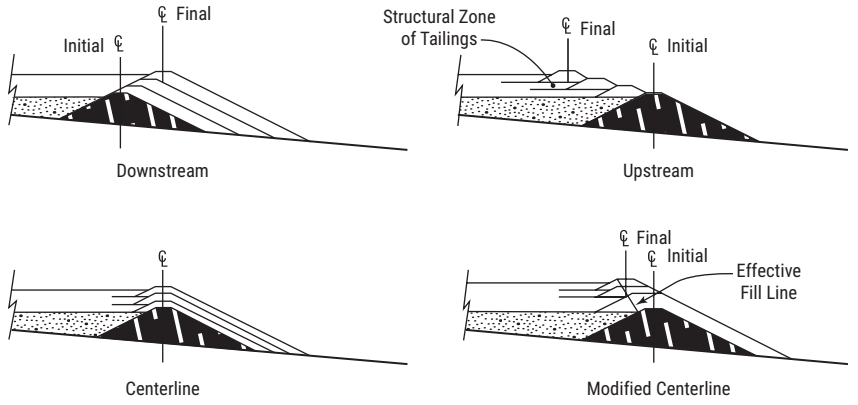

Adapted from Zahl et al. 1992

FIGURE 21.2 Types of embankments

EMBANKMENTS

For all aboveground impoundments, an embankment is required to contain the stored tailings materials. Three basic configurations of embankments are used in tailings facilities, as illustrated in Figure 21.2:

1. Upstream embankment
2. Downstream embankment (including water-retaining-style embankments)
3. Centerline embankment (including modified centerline embankments)

An upstream-method embankment is always constructed as a series of raises (i.e., stages), with each raise enabled by constructing a small berm (often called a *dike*) over the tailings immediately upstream of the previously constructed dike, causing the centerline of the embankment crest to move upstream with each stage. Tailings slurry is spigoted from the upstream face of each dike, which promotes the deposition of the coarsest particles in the tailings slurry adjacent to the upstream face of the dike. The spigoting method consists of numerous, closely spaced, and independently controlled offtake pipes (i.e., spigots) that allow the tailings slurry to be deposited in a low-velocity manner for improved particle segregation. The top surface of this deposit of coarse tailings near the dike is referred to as the tailings *beach*, which, if properly managed over time, forms a relatively well-drained shell on the exterior part of the embankment that functions as a structural zone.

Figure 21.3 illustrates a typical raising scheme for an upstream-method embankment. The upstream method of construction uses the least amount of construction material and is often the lowest-cost option. Each raise dike is typically constructed using coarse tailings excavated from the drained beach, although in some cases, imported borrow is used. Upstream-method embankments entirely rely on the strength of the tailings deposited upstream of the dike for support of subsequently raised dikes. The upstream method of construction is more suited for dry climates and for relatively low annual tailings rates of raise. In Arizona, several copper tailings dams have been successfully constructed and operated to significant heights using the upstream method. However, there have been some major failures of upstream-method tailings embankments. For the most part, such failures have been caused by a lack of drainage in the structural zones and/or an excessively high raise rate (commonly considered as anything greater than 5 m per year). These failures have led to the ban of upstream construction methods in some jurisdictions.

A downstream-method embankment is constructed in stages so that the centerline of the embankment crest moves downstream with each stage, as depicted in stages 1 through 4 in Figure 21.4. This embankment type uses the largest amount of construction material and is therefore often the most expensive option. Its performance is completely independent of the physical properties of the tailings deposit; for this reason, downstream embankments are favored in some jurisdictions (such as Chile), especially where large

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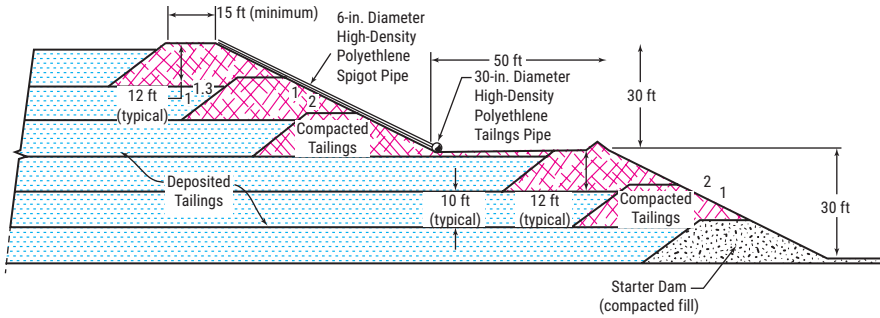
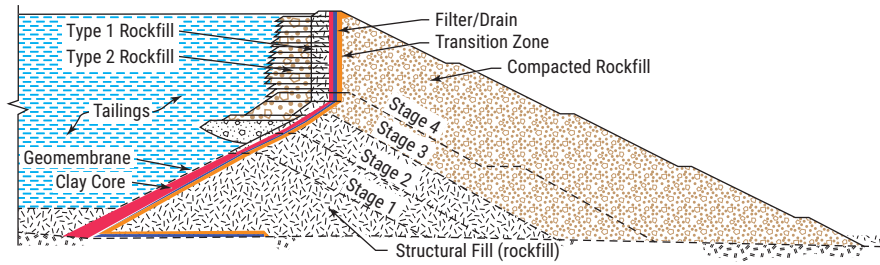


Courtesy of Knight Piesold

FIGURE 21.3 Typical section of upstream-method embankment



Courtesy of Knight Piesold

FIGURE 21.4 Typical cross section of tailings embankment using downstream and centerline methods of construction

earthquakes frequently occur. A water-retaining structure is similar to a downstream-method embankment, because of the isosceles triangle shape. However, a water-retaining dam is typically constructed in a single stage and may be able to incorporate a central core with upstream and downstream shells, such as is often used with water-retention dams. With a true downstream-method embankment, the core (i.e., seepage cutoff) is typically placed on the upstream slope to accommodate the staged construction technique.

Centerline-method embankments are constructed in stages so that the location of the centerline of the embankment crest does not change with each stage. The upstream toe of each centerline embankment raise may be constructed slightly out over the tailings beach, but the majority of each new stage is founded on the previous embankment stage. This method relies on some strength and structural support of the tailings for the upstream slope, but generally does not significantly rely on the strength of the impounded tailings for overall, downstream stability.

Liquefaction of the tailings, as a result of earthquake loading, could result in some localized instability of the upstream slope of the most recent stage, but this does not cause significant damage to the structure. Centerline embankments are frequently constructed out of underflow sand from hydrocyclones, but may be constructed of waste rock or natural borrow materials. The centerline method is a compromise between the higher cost downstream embankment and the higher risk upstream embankment. Figure 21.4 is a design section of a tailings embankment dam constructed of waste rock that uses both downstream and centerline methods of construction.

A variation of the centerline embankment is the modified centerline embankment. In this method, the embankment crest centerline moves slightly upstream, thus reducing the quantity of construction materials required in the downstream shell zone of the embankment. Modern analytical techniques are used to

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determine how far the embankment crest centerline can be moved upstream with staged construction and still keep the embankment independent of the strength of the tailings for overall, downstream stability. These techniques have resulted in significant cost savings without compromising embankment stability. Many modified centerline embankments have been designed, permitted, and constructed in the United States, Canada, and South America.

Embankment Construction Materials

Generally, any soil or rock material can be used for construction of embankments and other fills, as long as the material is clean, properly placed, and compacted. Rock and gravel are generally stable at steeper slopes; however, sands and silts should be used with caution in applications where erosion may be possible. Mine waste rock can make an excellent material for constructing tailings dams, so long as the rock does not contain sulfide mineralization, which can result in acid drainage or is prone to particle crushing or degradation over the life of the facility. Sands and silts exposed to erosive forces should be protected. Silts are also susceptible to frost heave in cold climates.

Coarse tailings may also be used to construct embankments. Hydrocyclones have been successfully used on several large tailings dams to separate out sand (called *underflow*) for dam construction. However, with finer grinds from milling operations, the amount of sand recovery may not be sufficient to construct tailings embankments.

Embankments are generally constructed with downstream slopes in the range of 2.0(H:horizontal) to 1.0(V:Vertical) to 4.0H:1V, depending on the strength of the material and the pore pressure conditions in the embankment and underlying foundation. Flatter slopes offer more stability and provide better access for maintenance and reclamation and may be required for soft soils, especially those subject to contractive, strain-softening behavior and/or for extreme loading conditions. The design of final slopes should be based on seepage, slope stability, and deformation analyses considering the various loading conditions and material parameters of the embankment and foundation materials.

Larger embankments intended for long-term storage of water or other fluids are generally constructed as a zoned fill. Semi-pervious, erosion-resistant materials are used for the downstream portions of the embankment, and potentially for the upstream slope as well. An impervious core of low-permeability material (typically high in clay or silt content) provides resistance to water seepage across the embankment; and drain structures constructed of free-draining materials control seepage within the embankment to maintain stability. Embankments may be constructed as a homogeneous fill for small facilities that do not continuously impound water. A *membrane structure* is sometimes used on the upstream face for small impoundments to act as the hydraulic barrier. This may consist of a homogeneous embankment with a thin membrane of fine-grained soils or a synthetic liner on the upstream face.

Characterization of the tailings materials, embankment construction materials, and foundation soils is crucial to successful construction and operation of a tailings facility. The types of physical and engineering parameters that must be considered include the following:

- Representative particle size distribution of tailings and embankment construction materials
- Atterberg limits, unit weight (dry density), and specific gravity of soil solids
- Consolidation versus effective stress relationships
- Permeability
- Shear strength, both in drained and undrained loading conditions, considering whether tailings and natural soils are, or will be, contractive or dilative in shear
- Liquefaction potential, occurring as *cyclic mobility* and *flow liquefaction*
- Moisture-density relations for compacted soils
- Shear-wave velocity and small-strain shear modulus
- Large-strain modulus and modulus degradation relationship
- Rheology of tailings slurries
- Geochemical characteristics

During project development and design phases of mine development, investigations of future tailings materials, foundations, and future embankment fill materials (typically from borrow sources) are required

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to characterize the physical and engineering properties just identified. Laboratory testing on representative samples of all anticipated materials is needed, and all of these properties are required regardless of whether the tailings facility is developed using conventional slurry deposition, thickened, paste, or filtered methods. After the initial construction phase is complete and during operations, additional field investigations are required to confirm that the design assumptions are being met or, if not, what the actual physical and engineering properties are, especially in the tailings, which may change over time. The cone penetration test (CPT) and shear-wave velocity measurements are especially valuable in situ tests to confirm and characterize the behavior of critical materials.

For filtered tailings facilities, additional testing and characterization include the following:

- Filtration rate and soil moisture content
- Conveyability (if conveyors are used)
- Trafficability

For filtered tailings design, the target moisture content is a key design parameter. If the water content is too high, the filter cake may liquefy on the conveyor; the trafficability may be too poor to run a stacker; the shear strength of the tailings may be lower than the design assumptions, which could lead to slope stability problems; the stack may be susceptible to increased seepage flows, if there is no liner; or other detrimental factors. However, reducing the target water content in the filter process increases filter cycle times, especially if the clay content of the tailings is high. Water contents typically vary significantly during operation, so understanding the range of expected water contents and having contingency plans to handle out-of-specification materials is crucial to designing a successful operation.

Two forms of the gravimetric water content equation exist, which commonly leads to confusion in planning and design (and in extreme cases, can lead to disastrous results at start-up). The difference in these formulas can lead to considerable confusion unless this difference is understood by the process plant designer and the geotechnical engineer designing the filtered tailings facility and this difference is considered in the design. The difference in these two moisture contents may appear to be minor, but if working with a material that may exhibit a plastic behavior, the difference can be very meaningful.

Metallurgists often use the formula

$$w_M = \frac{m_W}{m_T} \times 100 \qquad \textbf{(EQ 21.1)}$$

where

w_M = moisture content as defined by metallurgists, %
m_W = mass of water (in units of mass)
m_T = total mass of solids plus water (in units of mass)

Geotechnical engineers, however, most commonly use the following equation:

$$w_G = \frac{m_W}{m_S} \times 100 \qquad \textbf{(EQ 21.2)}$$

where

w_G = moisture content as defined by geotechnical engineers, %
m_W = mass of water (in units of mass)
m_S = mass of solids (in units of mass)

To adjust one form of moisture content to the other,

$$w_G = \frac{w_M}{100 - w_m} \qquad \textbf{(EQ 21.3)}$$

or

$$w_M = \frac{w_G}{100 + w_G} \qquad \textbf{(EQ 21.4)}$$

FILTERED TAILINGS FACILITIES

The use of filtering technology as part of an overall tailings management program has been growing over the past two to three decades, partly because of the commitments of some mining companies to transition to

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filtered tailings solutions, and partly because of increased scrutiny of the mining industry over its perceived lack of stewardship of the natural environment; beneficial use of available water resources; and safety of the people, facilities, and environment downstream of the impoundments. Several notable failures of conventional impoundments (e.g., the 2015 Samarco failure in Brazil, the 2014 Mt. Polley failure in Canada, and others) have heightened the level of scrutiny of tailings facilities by governmental and nongovernmental organizations. Furthermore, in many locations, the increased scarcity of water has led mining companies to consider technologies that have previously been considered uneconomic.

Conventional tailings facilities typically lose significant amounts of water to entrainment (water that is held in the pore spaces of the settled tailings voids) through seepage into the underlying foundation and through evaporation from the typically large decant ponds, whereas filtered tailings facilities typically result in much higher recovery rates for water in the process. Filtered tailings facilities generally require a smaller footprint for tailings storage because of higher achieved dry density in situ, the removal of the surface pond, often easier reclamation, and lower long-term (closure) liability in terms of potential environmental impact. These stacked facilities typically require much smaller containment structures and should have no associated supernatant water pond.

Experience shows the most applicable projects are those that have one or more of the following attributes:

- The project is located in an arid region, where water conservation is crucial (e.g., Western Australia, Southwest United States, much of Africa, arid regions of South America).
- Processes are used where economic recovery of the commodity or expensive process agents are enhanced by tailings filtration.
- The project is located in an area where very high seismicity significantly increases the cost of some forms of conventional tailings impoundments.
- The project is located in a cold region, where water handling is very difficult in winter.
- The project is located in an area where topographic considerations preclude conventional dam construction and/or viable storage to dam material volume ratios.
- The operating and/or closure liabilities of a conventional tailings impoundment are in excess of the incremental increase to use tailings filtering.
- The project is located in an area where construction materials for conventional dams do not exist or are very expensive to supply.

Filtered tailings are transported either by conveyor or truck and placed, spread, and sometimes compacted to form a dense and stable tailings deposit. Davies and Rice (2001) report on the trafficability of newly placed filtered tailings, providing the following observations:

- Filtered tailings are generally produced at or slightly above their optimum moisture content and as such, trafficability problems can arise especially in wet environments. The tailings surface can quickly become untrafficable, resulting in significant placement problems. Facility designers must take this matter into account to ensure that the operation is not unduly hampered during the wet season.
- The degree of compaction required to achieve trafficability can be higher than the compaction required to achieve slope stability goals. This can work favorably for geotechnical requirements.
- In some designs, there is a requirement to compact the tailings to a higher density in at least the outer reaches of the facility. This requirement increases the need for construction quality control.
- For facilities designed to accommodate materials that are produced during seasonally unfavorable periods, such as rainy seasons and/or cold seasons (usually this requires storage in a portion of the facility that is remote from the outer slopes of the facility), these trafficability issues during unfavorable weather may be greatly reduced or eliminated altogether.
- Several filtered tailings facilities with operating throughput of 20,000 t/d (metric tons per day) or less have been operational for several years, although most operating filtered tailings facilities were developed for mines with a throughput less than 10,000 stpd (short tons per day). The Karara iron ore mine in Western Australia has operated at rates up to approximately 35,000 t/d and is the largest operating filtered tailings operation in the world to date. Several greenfield projects have been considered with production rates in the range of 70,000–120,000 t/d, but the technology has not been proven at those rates. Dewatered tailings systems may have less application for operations where tailings ponds must serve dual roles not only for solids storage but also for water-storage reservoirs. This is particularly true in temperate climates where water balances must be managed to store annual snowmelt runoff or

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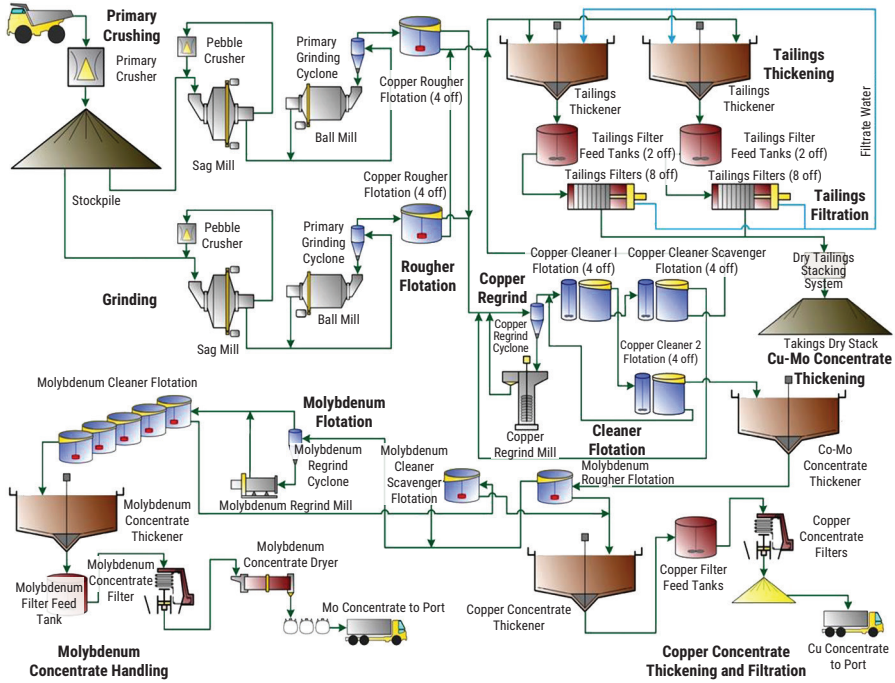
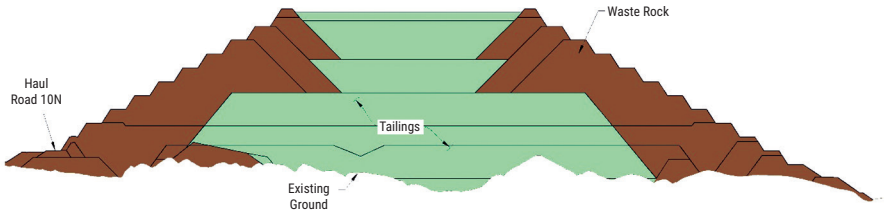


Courtesy of Knight Piesold

FIGURE 21.5 Flow sheet for a copper/molybdenum mine that uses filtered tailings management



Courtesy of Knight Piesold

FIGURE 21.6 Typical cross section of filtered tailings storage facility using waste rock buttresses to form the embankments

a seasonal period of high precipitation to provide water for year-round operation. Figure 21.5 illustrates the flow sheet for a filtered tailings operation proposed in the United States. The filtered tailings at this facility are placed using a radial-arm stacker to a maximum height of 200 m. Figure 21.6 illustrates a cross section of the tailings storage facility, which uses waste rock to create a buttress that avoids the need for compaction of the tailings at the perimeter.

PASTE BACKFILL

The use of tailings as aggregate in cemented paste backfill (CPB) is an important aspect of underground mining. CPB is an engineered mixture of tailings (typically ranging from 75% to 85% solids by weight),

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a cementitious binder (typically portland cement at the rate of approximately 3%–10%, by weight), and mixing water to produce a paste with a percent solids of approximately 60%–85%, by weight. Cemented paste backfilling has direct advantages to underground mining productivity by backfilling mined stopes with a structurally competent material that provides support to pillars and walls, prevents caving and roof falls, and enhances pillar recovery, in addition to reducing the amount of tailings that must be disposed of at the surface. Ercikdi et al. (2017) describe the following advantages of using CPB compared to other forms of backfill:

- No segregation of the paste backfill is needed during its transportation to the mined-out underground voids. Therefore, the use of CPB allows the entire production opening to be filled, thus preventing the collapse of the hanging and side walls and providing a safe work environment.
- About 90% of the processed water can be recovered via thickening and filtration processes prior to the cement backfilling of the tailings. Additionally, this prevents the leakage of pore water outside and the passing of underground water through the paste backfill material owing to its low permeability and high degree of saturation.
- CPB has a low permeability and hence acts as a barrier to prevent underground water seepage. It also reduces the formation of acid mine drainage (AMD) by inhibiting the diffusion of oxygen.
- The transportation of CPB through a pipeline system reduces problems (damage to support systems, traffic problems, etc.) caused by conventional transport systems (conveyors, mobile equipment, etc.).
- Paste backfilling is a relatively fast operation in which the CPB shows a rapid gain of strength for a given binder content. This shortens the mining cycle period compared to hydraulic filling.
- Cement added into CPB provides strength gain and reduces permeability and the formation of AMD by increasing acid neutralization potential.
- Cemented paste backfilling increases the volume of ore extraction through substitution and recovery of the ore from pillars and allows safe working platforms.

Despite the advantages of cemented tailings backfill, cost is a major consideration, and there are several technical challenges that typically must be overcome through sound engineering. Ercikdi et al. (2017) also describe the following general disadvantages of CPB:

- The dewatering of tailings and pumping operations are capital intensive and expensive operations.
- Paste backfilling requires a high pumping pressure to transfer high-density backfill material into the underground voids. Positive displacement pumps are usually required to meet the pumping requirements. This increases the pumping maintenance and energy costs.
- The oxidation of sulfide minerals in the presence of oxygen and water may cause long-term instability problems in the CPB of sulfide-rich tailings. Some metals, such as arsenic, may be mobilized by the high pH conditions resulting from the use of cement binder.
- Transportation problems may occur with changes in the particle size, density, and specific gravity of the tailings and amount of water.
- Paste backfilling requires qualified and accurate engineering work.

Key CPB design components are

- The required design strength, usually specified as a uniaxial compressive strength (UCS) of the cured mix;
- The target rheology of the fresh cement paste for pumping system design; and
- Understanding of the geochemistry of the tailings and mixing water, for both short-term (i.e., mixing, pumping, and curing) and long-term performance (i.e., possible long-term leaching).

Selection of the required design strength of the cured backfill is usually done by numerical modeling, but may be done using limit equilibrium methods. Rheology is typically characterized by laboratory testing at different mix designs, followed by modeling using non-Newtonian fluid mechanics. The primary variables of CPB mix design (Belem and Benzaazoua 2008) are

- Binder content (by dry mass of tailings) and binder type,
- Tailings particle size distribution and mineralogy,
- Mix solids concentration by mass or volume, and
- Mixing water geochemistry.

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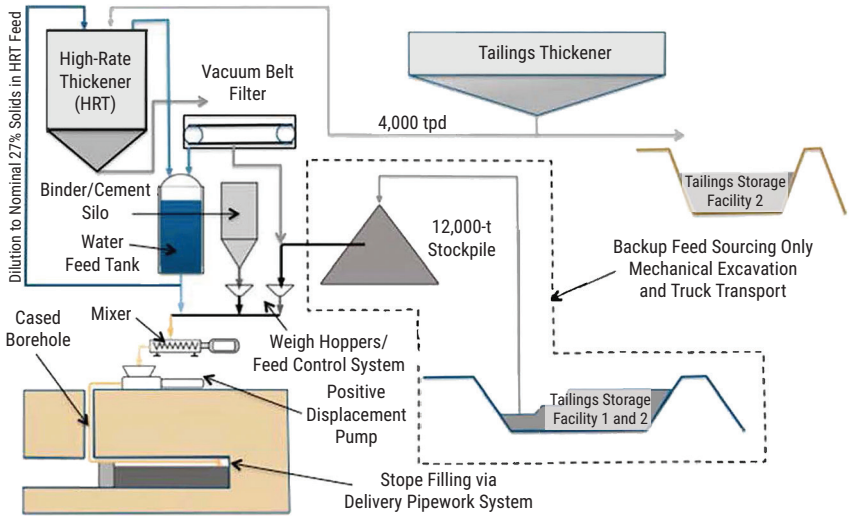

Courtesy of Knight Piesold

FIGURE 21.7 Typical simplified flow sheet for a paste backfill operation

Figure 21.7 provides a flow sheet of an example paste backfill operation, which includes disposal of tailings both in paste backfill and in a conventional hydraulic deposition impoundment. Other variations could consider filtering the entire tailings stream and dry-stacking the non-backfill portion of the tailings, or paste deposition of the non-backfill portion of the tailings in a lined or unlined impoundment.

LINER SYSTEMS

Liner systems are used to control and contain water, process solutions, or other liquids; to collect and conduct leachate; and to protect environmental resources. Liner systems may be as simple as a layer of fine-grained soil placed and compacted for low permeability, or they can include several elements, or layers, such as a geomembrane, geotextile, geocomposite, geonet, or other elements. Liner systems may be used in conjunction with leachate collection and recovery systems, leak detection systems, or various types of drainage systems.

Soil Liners

Compacted soil liners are generally constructed using fine-grained soils such as silts and clays; however, any soil that can provide a sufficiently low permeability may be used. Use Darcy's law to calculate movement of a liquid through a soil as follows:

$$Q = KIA \qquad \text{(EQ 21.5)}$$

where

Q = flow rate
K = coefficient of permeability (depends on soil and liquid properties and is generally determined by laboratory or in situ testing)
I = hydraulic gradient
A = cross-sectional flow area, perpendicular to direction of flow

The preceding equation is valid for any units that are consistent for each parameter. The coefficient of permeability is commonly reported in meters per second, centimeters per second, or feet per second. Values of K for various soil types can be estimated from Figure 21.8.

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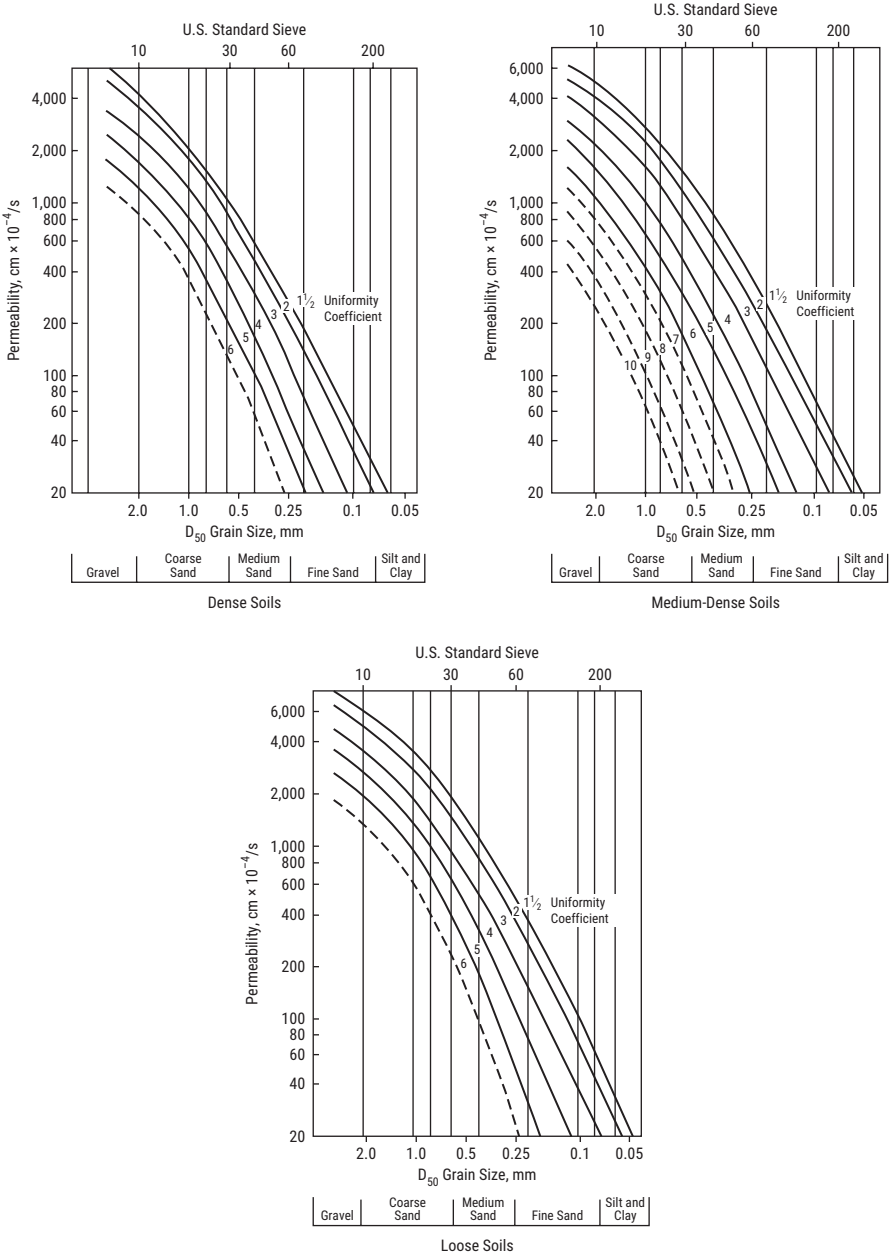

The hydraulic conductivity of materials to be dewatered can be estimated on the basis of particle size, uniformity coefficient, and density when information from pumping tests is not available. In these graphs, the D_{50} grain size represents the diameter of the 50% retained size. To convert permeability in cm × 10^{-4}/s to hydraulic conductivity in gpd/ft^2, multiply by 2.13.

Source: Driscoll 1986

FIGURE 21.8 Hydraulic conductivity for various soils

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Values in Figure 21.8 assume that the fluid is generally clean water. When fluids other than water are used, or temperatures significantly affect the viscosity of the pore fluid, the coefficient of permeability can be separated into two components: intrinsic, which is related to soil properties; and hydraulic, which is related to fluid properties. Relevant relationships are

$$k = CD^2 \quad \text{(EQ 21.6)}$$

$$K = \frac{k\mu}{\gamma} \quad \text{(EQ 21.7)}$$

where

k = intrinsic permeability, typically cm/s
C = constant shape factor that is dependent on the density, grain size distribution, and other soil properties, typically 1/s·cm
D = mean grain size or a defined grain size such as D_{10}, D_{40}, D_{50}, and so forth, cm
K = hydraulic conductivity, cm^2
γ = specific gravity of the fluid, gr/cm^3
μ = viscosity of fluid, $gr{\cdot}s/cm^2$

Other units can be used in these equations, as long as unit consistency is maintained.

General values for the coefficient C in Equation 21.6 have been difficult to determine. A proposed relationship for clean sands is

$$k = 100D_{10}^2 \quad \text{(EQ 21.8)}$$

Research suggests that for a similar relationship based on the $D10$ grain size, C varies.

Calculating the Coefficients of Uniformity and Curvature

Grain diameters are needed to calculate the coefficients of uniformity and curvature. The grain diameter can be found for each percent of the soil passing a particular sieve. For example if 40% of the sample is retained on the No. 200 sieve, then there is 60% passing the No. 200 sieve.

The coefficient of uniformity C_u is a crude shape parameter and is calculated using the following equation:

$$C_u = \frac{D_{60}}{D_{10}} \quad \text{(EQ 21.9)}$$

where D_{60} is the grain diameter at 60% passing, and D_{10} is the grain diameter at 10% passing.

The coefficient of curvature, C_c is a shape parameter and is calculated using the following equation:

$$C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}} \quad \text{(EQ 21.10)}$$

where D_{60} is the grain diameter at 60% passing, D_{30} is the grain diameter at 30% passing, and D_{10} is the grain diameter at 10% passing.

Once the coefficient of uniformity and the coefficient of curvature have been calculated, they must be compared to published gradation criteria (Holtz and Kovacs 1981). See Table 21.2.

Composite Liners

Composite liners, in general, consist of a compacted soil liner covered by a synthetic liner. In some facilities, the liner system may be extended to include additional layers of synthetic or soil liner materials to provide a multilayered system. A composite liner system takes advantage of the low permeability of a synthetic material while the compacted soil liner provides a suitable subgrade and mitigates potential seepage from defects or minor punctures in the synthetic liner.

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TABLE 21.2 Criteria for assigning group symbols and group names using laboratory tests

Major Divisions			Group Symbol	Group Name
Coarse-grained soils more than 50% retained on or above No. 200 (0.075-mm) sieve	Gravel >50% of coarse fraction retained on No. 4 (4.75-mm) sieve	Clean gravel <5% smaller than No. 200 sieve	GW	Well-graded gravel, fine to coarse gravel
			GP	Poorly graded gravel
		Gravel with >12% fines	GM	Silty gravel
			GC	Clayey gravel
	Sand ≥50% of coarse fraction passing No. 4 (4.75-mm) sieve	Clean sand	SW	Well-graded sand, fine to coarse sand
			SP	Poorly graded sand
		Sand with >12% fines	SM	Silty sand
			SC	Clayey sand
Fine-grained soils 50% or more passing the No. 200 (0.075-mm) sieve	Silt and clay liquid limit <50%	Inorganic	ML	Silt
			CL	Clay of low plasticity, lean clay
		Organic	OL	Organic silt, organic clay
	Silt and clay liquid limit ≥50%	Inorganic	MH	Silt of high plasticity, elastic silt
			CH	Clay of high plasticity, fat clay
		Organic	OH	Organic clay, organic silt
Highly organic soils			PT	Peat

Source: ASTM D2487-17

Geomembranes

Geomembranes may be used as primary liner systems or as part of composite liner systems. They are also used as aquitards in embankments, as channel revetment, and in other applications for solution control and containment. These materials are manufactured from synthetic polymers and their properties are variable. Contact the manufacturers to obtain up-to-date properties.

Thickness Determination

Geomembrane liner selection should be based on the material properties of adjacent soils and on potential deformation that can be experienced during the service life of the material. Tables 21.3 through 21.9 show typical geotextile properties; additional detailed properties may be obtained from manufacturers and material-specific laboratory testing. Liner thickness and design result from assessing the static and dynamic forces through the cross section of the system (Figure 21.9).

TABLE 21.3 Typical geotextile permittivity and permeability properties

Geotextile Type	Permittivity, 1/s	Permeability, kg/cm/s
Woven monofilament	1.5–0.05	10–0.001
Woven slit film	1–0.01	0.01–0.001
Nonwoven needle punched	2.5–0.7	1–0.01
Nonwoven heat bonded	0.8–0.2	0.1–0.005

Source: U.S. Bureau of Reclamation 2014

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TABLE 21.4 Polymers in textiles

Polymer	Approximate Specific Gravity*	Coefficient of Thermal Expansion × 10^{-5} per 1°C	Degradation
Polyester (PET)	1.22–1.38 Sinks in water	4–5	Some resistance to ultraviolet exposure Cover within 30 days Can be affected by high pH water Avoid fresh concrete
Polypropylene (PP)	0.91 Floats in water	6	Requires carbon black for sunlight resistance Cover within 14 days Good chemical resistance
Polyethylene High-density polyethylene (HDPE) Low-density polyethylene (LLDPE)	0.90–0.96 Floats in water	13	Requires carbon black for sunlight resistance Cover within 14 days Good chemical resistance
Polyamide (PA) (nylon)	1.05–1.14 Sinks in water	5.5	May be degraded by low pH waters and acids

* Additives cause variations in specific gravity.

Source: U.S. Bureau of Reclamation 2014

TABLE 21.5 Flow capacity

Textile Type	Thickness, mm	Permeability, m/a	Transmissivity, m^2/s
Needle-punched nonwoven	3 to 10	10^{-4} to 10^{-3}	10^{-7} to 10^{-6}
Geonet composites	10 to 20	10^{-1} to 1	10^{-3} to 10^{-2}

Source: U.S. Bureau of Reclamation 2014

TABLE 21.6 Criteria for filtration and drainage

Criteria	Properties
Constructability	Thickness Weight Absorption (wet weight) Flexibility Tensile strength Puncture resistance Cutting resistance Seam strength Flammability Tear strength Ultraviolet stability
Durability	Chemical stability Biological stability Thermal stability
Hydraulics	Soil retention Thickness Permeability Clogging resistance

Source: U.S. Bureau of Reclamation 2014

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TABLE 21.7 Geotextile filtration properties

Geotextile Type	Opening Size (O_{95}), mm ASTM D4751-16	Permittivity, s^{-1} ASTM D4491/ D4491M-17	Comments
Woven monofilament	Common range from 0.15 to 0.85 Can be as low as 0.05 (Ramsey and Narejo 2005)	Common range from 0.05 to 1.5 Can range from 0.01 to 4.0 (Ramsey and Narejo 2005)	Does not transmit flow in the plane of the geotextile. Stiff, direct soil contact is more difficult to achieve. Has lower interface friction strength than nonwovens. Heat-bonding (the fibers are fused together at the weave intersections) provides excellent dimensional stability (maintains apparent opening size [AOS]). Percent open area (POA) is used for permeability design.
Woven slit film	0.35–0.6	0.05–0.6	Large variation in AOS; not recommended for filtration function. Has lower interface friction strength than nonwovens.
Nonwoven needle punched	Common range from 0.15 to 0.5 Can be as low as 0.074 (Hwang et al. 1998)	Common range from 0.7 to 2.5 Can be as low as 0.5 (Ramsey and Narejo 2005) and as high as 4.5 (Hwang et al. 1998)	Flexible and conforms well to soil surfaces and has higher interface friction than woven or heat-bonded geotextiles. Provides higher flow rates than heat-bonded geotextiles and also transmits flow in the plane of the fabric. Increasing depth of burial reduces permeability and AOS, which can cause clogging. Thicker fabrics have great strength, but may clog.
Nonwoven heat bonded	0.1–0.3	0.2–0.8	Thin and stiffer than needle-punched nonwovens, hydrophobic and may require a driving head for flow to occur. Heat-fused fibers have excellent dimensional stability to retain AOS. Wick drains use this geotextile to filter and dewater fine clay and silt.
Knitted	0.6	–	Principal application is a polyester socklike wrapping around a corrugated perforated drainage pipe. Tensile stress causes changes in AOS. Only for shallow burial in a sand filter.

Source: U.S. Bureau of Reclamation 2014

TABLE 21.8 Suggested geotextile strength requirements per AASHTO* (1996)

Application	Contact Stress	Grab Strength, lb ASTM D4632/ D4632M-15a	Elongation, %	Sewn Seam Strength, lb	Puncture Strength, lb ASTM D4833/ D4833M-07e1	Burst Strength, lb	Trapezoid Tear, lb ASTM D4533/ D4533M-15
Subsurface drainage	High stress—angular drainage media, heavy compaction, high confining stress	180	–	160	80	290	50
	Low stress—rounded drainage media, light compaction, low confining stress	80	–	70	25	130	25
Armored erosion control	Direct stone placement	200	15	180	80	320	50
	Sand cushion, low drop height	990	15	80	40	140	30

* AASHTO = American Association of State Highway and Transportation Officials. Source: U.S. Bureau of Reclamation 2014

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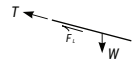
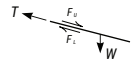
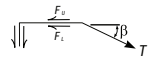
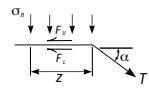
TABLE 21.9 Example of required geotextile physical properties

Property	Test Method	Required Values
Mass per unit area, nominal	ASTM D5261-10	4 oz/yd^2
Grab tensile	ASTM D4632/D4632M-15A	90 lb
Elongation at break	ASTM D4632/D4632M-15A	50%
Trapezoidal tear	ASTM D4533/D4533M-15	40 lb
Puncture strength	ASTM D4833/D4833M-07e1	50 lb
Burst strength	ASTM D3786-18	140 lb/in.^{2}
Permittivity	ASTM D4491/D4491M-17	1.5 s^{-1}
Apparent opening size (minimum U.S. sieve number/ maximum opening size)	ASTM D4751-16	70 U.S. sieve
UV resistance—tensile strength retained at 500 hours, minimum	ASTM D4355/D4355M-14	70%

Source: U.S. Bureau of Reclamation 2014

Problem	Liner Stress	Free Body Diagram	Required Properties: Geomembrane	Required Properties: Landfill	Typical Factor of Safety
1. Liner self-weight	Tensile		G, t, σ_{allow}, δ_L	β, H	10 to 100
2. Weight of filling	Tensile		t, σ_{allow}, δ_U, δ_L	β, h, γ, H	0.5 to 10
3. Impact during construction	Impact		I	d, W	0.1 to 5
4. Weight of landfill	Compression		σ_{allow}	γ, H	10 to 50
5. Puncture	Puncture		σ_p	γ, H, P, A_p	0.5 to 10
6. Anchorage	Tensile		t, σ_{allow}, δ_U, δ_L	β, γ, ϕ	0.7 to 5
7. Settlement of landfill	Shear		τ, δ_U	β, γ, H	10 to 100
8. Subsidence under landfill	Tensile		t, σ_{allow}, δ_U, δ_L, z	α, γ, H	0.3 to 10

Legend

sg = specific gravity
T = tensile force
t = thickness
σ_{allow} = allowable strength
τ = shear strength
I = impact resistance
σ_p = puncture strength
δ_U = friction coefficient with material above
δ_L = friction coefficient with material below
F_U = friction force upper
F_L = friction force lower
β = slope angle
H = landfill height
γ = unit weight
h = lift height
α = subsidence angle
ϕ = friction angle
d = drop height
W = weight
P = puncture force
A_p = puncture area
z = mobilization distance

Source: MSHA 2010

FIGURE 21.9 Stability considerations in liner design

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Anchor Trench Design

The most common method for anchoring synthetic liner materials is the use of an anchor trench or liner runouts. In either case, the liner stability with respect to lateral movement equates the frictional force generated between the liner or liner systems and the rock and soil with the force(s) acted on the liner by gravity and seismic systems. Anchor tranches have the advantage of increasing the contact length thus increasing the friction force opposing movement.

Geotextiles

Geotextiles are sometimes used as cushion layers in ponds and dams to improve subgrades under liner systems. Geotextiles are also sometimes used as filters, to prevent migration of fine-grained soils into drainage systems. Because of solar degradation exposure, caution should be exercised when using a geotextile in lieu of a graded aggregate or sand filter. Also the system interface shear strength needs to be analyzed when geotextiles are used adjacent to geomembranes. Because geotextiles are compressible, the thickness under a given load is taken into consideration. Parameters used in fluid flow are defined as permittivity for cross-plane flow, and transmissivity for in-plane flow.

$$\Psi = \frac{k_n}{t} \quad \text{(EQ 21.11)}$$

$$\theta = k_p t \quad \text{(EQ 21.12)}$$

where

Ψ = permittivity
θ = transmissivity
k_n = cross-plane permeability coefficient for geotextile
k_p = in-plane permeability coefficient for geotextile
t = thickness of the geotextile under given normal load

Units for these equations can be any combination as long as consistency is maintained. Permittivity is generally in units of 1 per second, and transmissivity in generally in units of cubic foot per foot minute or cubic meter per meter minute.

Geonet

Geonet is often used to provide a flow path as a part of a liner system (typically in the leak detection and recovery system). Planar flow is a function of the manufactured characteristics of the geonet. The concept of transmissivity is applicable and compression under load should be considered in design calculations. Applicable equations are

$$q = kiA \quad \text{(EQ 21.13)}$$

$$\theta = \frac{q}{iW} \quad \text{(EQ 21.14)}$$

where

q = volumetric flow rate
k = coefficient of permeability
i = hydraulic gradient
W = width of flow area
A = flow cross-sectional area (= $W \cdot t$ where t is thickness under load)
θ = transmissivity

Units for transmissivity in the previous equation are similar to those earlier for geotextiles. The range of geonet properties is large, and properties often change as manufacturing methods and materials change; consult the manufacturer for properties.

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Interface Friction

Table 21.10 provides preliminary frictional values for the interfaces of soils to geomembranes, geomembranes to geotextiles, and soils to geotextiles, which can be considered during preliminary design phases. Because of expected variability, it is recommended that material-specific testing be conducted to evaluate the interface shear strength.

TABLE 21.10 Interface frictional values

	Geomembrane-to-Soil Type Friction Angle		
Geomembrane	Concrete Sand ($\phi = 30°$)*	Ottawa Sand ($\phi = 28°$)	Mica Schist Sand ($\phi = 26°$)
EPDM-R†	24° (0.77)	20° (0.68)	24° (0.91)
Polyvinyl chloride (PVC)			
Rough	27° (0.88)	–	25° (0.96)
Smooth	25° (0.81)	–	21° (0.79)
CSPE-R‡	25° (0.81)	21° (0.72)	23° (0.87)
High-density polyethylene (HDPE)	18° (0.56)	18° (0.61)	17° (0.63)

	Geomembrane-to-Geotextile Friction Angles				
		PVC			
Geotextile	EPDM-R	Rough	Smooth	CSPE-R	HDPE
Nonwoven, needle punched	23°	23°	21°	15°	8°
Nonwoven, heat bonded	18°	20°	18°	21°	11°
Woven, monofilament	17°	11°	10°	9°	6°
Woven, slit film	21°	28°	24°	13°	10°

	Soil Types		
Geotextile	Concrete Sand ($\phi = 30°$)	Ottawa Sand ($\phi = 28°$)	Mica Schist Sand ($\phi = 26°$)
Nonwoven, needle punched	30° (1.00)	26° (0.92)	25° (0.96)
Nonwoven, heat bonded	26° (0.84)	–	–
Woven, monofilament	26° (0.84)	–	–
Woven, slit film	24° (0.77)	24° (0.84)	23° (0.87)

* Efficiency values in parentheses are based on the relationship $E = (\tan \delta)/(\tan \phi)$.

† EPDM-R = synthetic elastomer based on ethylene, propylene, and nonconjugated diene—reinforced.

‡ CSPE-R = chlorosulfonated polyethylene—reinforced.

Source: Martin et al. 1984. Reprinted with permission from Industrial Fabrics International Association, Roseville, MN. For more information visit www.ifai.com.

Liner Leakage

In practice, there is no such thing as a *perfect* liner (e.g., perfect installation and performance based on the properties of the liner materials); however, with modern materials and techniques, and with a high level of quality control during installation, potential leakage through a liner system can be reduced. Leakage rates may be required for the design of leak detection and secondary leachate collection and recovery systems. Estimates of leakage rates for typical liners at various levels of overall quality control are shown in Table 21.11.

PIPES

Pipes of various sizes, types, and materials are used to convey liquid and slurry to and from ponds, leach pads, tailings storage facilities and other impoundments, and as part of leachate collection, leak detection, and drainage systems. Following are the physical design considerations.

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TABLE 21.11 Leakage rates for various liners

Type of Liner	Overall Quality of Liner	Assumed Values of Key Parameters	Rate of Flow, gal/acre/day*
Compacted soil	Poor	$k_s = 1 \times 10^{-6}$ cm/s	1,200
Geomembrane	Poor	30 holes/acre; $a = 0.1$ cm^2	10,000
Composite	Poor	$k_s = 1 \times 10^{-6}$ cm/s 30 holes/acre; $a = 0.1$ cm^2	100
Compacted soil	Good	$k_s = 1 \times 10^{-7}$ cm/s	120
Geomembrane	Good	1 hole/acre; $a = 1$ cm^2	3,300
Composite	Good	$k_s = 1 \times 10^{-7}$ cm/s 1 hole/acre; $a = 1$ cm^2	0.8
Compacted soil	Excellent	$k_s = 1 \times 10^{-8}$ cm/s	12
Geomembrane	Excellent	1 hole/acre; $a = 0.1$ cm^2	330
Composite	Excellent	$k_s = 1 \times 10^{-8}$ cm/s 1 hole/acre; $a = 0.1$ cm^2	0.1

* L = gal × 3.785. Source: EPA 1991

Pipe Thickness

For pipe made of plastic or other synthetic materials, the standard dimension ratio (SDR) is used as a measure of internal and exterior strength and other pipe properties.

$$\text{SDR} = \frac{D}{t} \quad \textbf{(EQ 21.15)}$$

where

SDR = standard dimension ratio, dimensionless
D = outside pipe diameter (units of length)
t = minimum pipe wall thickness (units of length same as for D)

Wall thickness requirements for pipe may be determined by

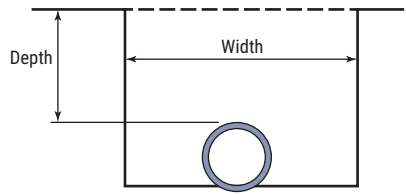
$$t = \frac{pD}{2f_a} \quad \textbf{(EQ 21.16)}$$

where

t = pipe wall thickness (units of length)
D = outside diameter of pipe (units of length consistent with those for t)
p = internal pressure in pipe (force per unit area in units consistent with units for t)
f_a = allowable stress in pipe material (force per unit area in units consistent with units for t)

Earth Pressure on Buried Pipe

In applications such as leachate collection systems in heap leach pads, drainage systems in tailings facilities, and embankment drains, pipe deflection should be considered in selecting pipe size, material, and wall thickness. The following formula (Koerner 1994) for buried pipe in a ditch are offered following Figure 21.10.



Adapted from Merritt and Gardner 1983

FIGURE 21.10 Buried pipe

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For a buried pipe, the load on a ridged pipe is computed from

$$W = C_D whb \quad \text{(EQ 21.17)}$$

where

W = load on the pipe due to earth pressure, lb/linear ft
C_D = ditch conduit load coefficient
w = unit weight of fill, lb/ft^3
h = height of fill above top of pipe, ft
b = width of trench at top of pipe, ft
D = outside diameter of pipe, ft

C_D is calculated from the equilibrium of vertical forces acting on the backfill above the pipe as

$$C_D = \left(1 - \frac{e^{-}kh}{b}\right) \div k\left(\frac{b}{h}\right) \quad \text{(EQ 21.18)}$$

where

$k = 2K_a \tan \theta$
K_a = coefficient of active earth pressure [= (1 − sin ϕ) / (1 + sin ϕ)]
θ = angle of friction between the fill and the adjacent soil (generally equal to or less than the angle of internal friction [ϕ] of the fill)
$e = 2.1718$

See Chapter 2, which covers material properties, for typical strength characteristics of soils and rocks. Any system of units may be used for Equations 21.19 through 21.24 as long as dimensional consistency is maintained. Deflection resulting from earth pressure can be found from

$$\Delta X = \left(\frac{D_L KW}{\frac{EI}{r^3} + 0.061E_S}\right) \quad \text{(EQ 21.19)}$$

where

ΔX = vertical deflection of pipe, in.
D_L = deflection lag factor, varies from 1.0 to 1.5; typically taken as 1.5 in the absence of specific data
K = bedding constant, varies from 0.0 to 0.8; typically taken as 0.1 in the absence of specific data
W = earth pressure due to backfill, psi
E = modulus of elasticity of pipe material, psi
I = moment of inertia of pipe wall, in.^{4} per in. (in.^{3} = t^3/12; where t = average wall thickness of pipe [in.])
r = mean radius of pipe, in.
E_S = modulus of soil reaction, psi

See Table 2.7 in Chapter 2 for typical values of modulus of soil reaction.

An alternative formula encompassing pressure on a buried pipe form subsidence is (Peng 1992):

$$\sum \sigma_x = \frac{\mu PD_1}{2W} + E\alpha\Delta t + \frac{ED_2}{2p} + \frac{s\gamma hfl}{2A} \quad \text{(EQ 21.20)}$$

where

σ_x = total axial pipeline stress
μ = Poisson's ratio for soil
P = internal working pressure, psi
D_1 = inner diameter of pipeline, in.
W = pipe wall thickness, in.
E = Young's modulus of the pipe steel
A = pipe cross-sectional area, in.^{2}
α = thermal coefficient of expansion, in./in. °F
Δt = temperature differential of pipe steel, °F
D_2 = outer diameter of pipeline, in.
p = subsidence basin surface curvature, ft
s = pipeline rubbing surface area, ft^2
γ = soil density, lb/ft^3
h = depth pipeline is buried measured to center of pipe, ft
f = coefficient of friction between pipeline and soil
l = length of pipe in stress

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HYDROLOGY AND HYDRAULICS

See the fluid mechanics section in Chapter 4, which covers physical science and engineering, for more information.

Water Balance

A water balance is generally completed for any impoundment structure that contains solutions as part of mine operations. Water balances for site water management and tailings or heap leach facilities are usually performed using probabilistic methods. (A full treatment of water balances for mining projects is outside the scope of this chapter.) In its simplest form, the water balance may be calculated from

$$S = I - Q - E \quad \text{(EQ 21.21)}$$

where

S = change in storage for the facility
I = inflow into the facility from all sources (operational inputs, direct precipitation, runoff, secondary inputs, etc.)
Q = outflow (removal of solutions for process, seepage, recycle, treatment, other uses, etc.)
E = evaporation from the facility (free-water surface and wet tailings or other materials)

Storm Water Runoff

Precipitation events are important to the overall water balance and for operational considerations. Runoff data is required to design the size of diversion channels and storage ponds. For minor hydraulic structures and small drainage basins (up to 5 mi^2), the peak discharge may be calculated using the rational equation:

$$Q = CIA \quad \text{(EQ 21.22)}$$

where

Q = peak discharge, ft^3/s
C = runoff coefficient based on percentage of precipitation that is direct runoff
I = rainfall intensity, in./h
A = size of the drainage area, acres

The runoff coefficient is dependent on local soil conditions, type, and amount of vegetative cover and the topographic slopes within the drainage basin. Table 21.12 gives some common runoff coefficients.

The rainfall intensity is a function of storm duration or time of concentration and storm frequency. A common method used to calculate intensity is the Steel formula:

$$I = \frac{K}{t + b} \quad \text{(EQ 21.23)}$$

where

I = rainfall intensity, in./h
t = duration of storm, min (time of concentration)
K and b = factors dependent on storm frequency and region

Use Figure 21.11 and Table 21.13 to obtain values for the Steel formula for the United States.

For large drainage areas, there are several methods for determining runoff. Generally, these methods use a unit hydrograph for a given basin, and runoff is found by multiplying the unit hydrograph by the rainfall amount from the design storm. For basins with no existing unit hydrograph, an estimate may be made using the Soil Conservation Service's (U.S. Natural Resources Conservation Service 1972) synthetic unit hydrograph. The following equations are used to estimate key parameters:

$$t_r = 0.133 t_c \quad \text{(EQ 21.24)}$$

$$t_p = 0.5 t_r + t_l \quad \text{(EQ 21.25)}$$

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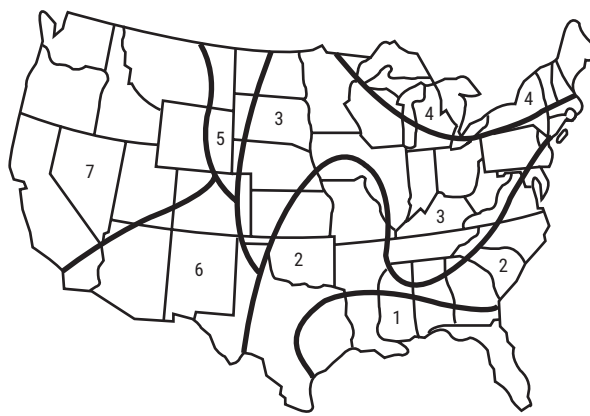
TABLE 21.12 Runoff coefficients for the rational method

Description of Area	Runoff Coefficients
Residential	
Single-family areas	0.30–0.50
Suburban	0.25–0.40
Industrial	
Light areas	0.50–0.80
Heavy areas	0.60–0.90
Unimproved areas	0.10–0.30
Lawns; sandy soil	
Flat, ≤2%	0.05–0.10
Average, 2%–7%	0.10–0.15
Steep, >7%	0.15–0.20
Rural areas (clay and silt loam)	
Woodland	
Flat 0%–5%	0.30
Rolling 5%–10%	0.35
Hilly 10%–30%	0.50
Pasture	
Flat 0%–5%	0.30
Rolling 5%–10%	0.36
Hilly 10%–30%	0.42
Cultivated	
Flat 0%–5%	0.50
Rolling 5%–10%	0.60
Hilly 10%–30%	0.72

Source: Warner 1992



Courtesy of Engineers Edge

FIGURE 21.11 Regions for use with the Steel formula

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TABLE 21.13 Coefficients for use in the Steel formula*

Return Period	Equation Constants	U.S. Region						
		1	2	3	4	5	6	7
2	a	209	140	102	70	70	68	32
	b	30	21	17	13	16	14	11
5	a	247	190	131	97	81	75	48
	b	29	25	19	16	13	12	12
10	a	300	230	170	111	111	122	60
	b	36	29	23	16	17	23	13
25	a	327	260	230	170	130	155	67
	b	33	32	30	27	17	26	10
50	a	315	350	250	187	187	160	65
	b	28	38	27	24	25	21	8
100	a	367	375	290	240	240	210	77
	b	33	36	31	28	29	26	10

* Steel equation constants for U.S. regions, i = in./h

Courtesy of Engineers Edge

$$Q_p = \frac{0.756 A_d}{t_p} \quad \text{(EQ 21.26)}$$

where

t_r = storm duration, h
t_c = time of concentration, h
t_p = time to peak flow, h
t_l = lag time from the centroid of the distribution to peak discharge, may be estimated as $t_l = 0.6\, t_c$
Q_p = peak discharge, cfs
A_d = drainage area, acres

Q_p and t_p provide one point on a hydrograph. Additional points may be found using Figure 21.12. Using time as the independent variable, arbitrary values of time are selected, and the ratio t/t_p is computed. The figure is then used to find values of Q_t/Q_p, and a unit hydrograph is constructed.

Sediment Ponds

A common form of sedimentation control is sediment or settling ponds. Sediment removal is accomplished by reducing the flow velocity to a point where sediment particles settle. Sediment ponds are often sized by experience and common sense, such as 0.5 acre-in. of storage per acre of disturbed area. Ponds that use this type of empirical design generally are effective at removing the coarse sediment faction. However, more detailed design and consideration of many factors are required to provide more complete settling and sediment removal.

The overflow velocity of a sediment pond may be calculated from

$$V_0 = \frac{Q_0}{A} \quad \text{(EQ 21.27)}$$

where

V_0 = overflow velocity
Q_0 = pond outflow
A = surface area of the pond

If the critical settling velocity of any size sediment particle is greater than the overflow velocity, that particle and particles of a larger size settle out in the pond. Settling velocity of an ideal particle in a solution of low solids concentration is given by one of the following three equations, depending on the Reynolds number.

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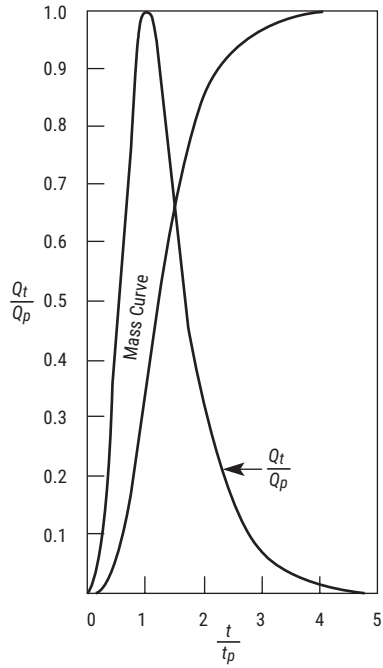

Source: Lindeburg 1992

FIGURE 21.12 Graph for use in estimating a unit hydrograph

Stokes' law:

$$V_s = \frac{g(\rho_s - \rho)D^2}{18\mu} \qquad \text{Re} < 1 \qquad \textbf{(EQ 21.28)}$$

Transition region:

$$V_s = 2.32(\rho_s - \rho)D^{(1.6v - 0.6)}0.714 \qquad 1 < \text{Re} < 1{,}000 \qquad \textbf{(EQ 21.29)}$$

Newton's law:

$$V_s = 1.82g\sqrt{(\rho_s - \rho)D} \qquad 1{,}000 < \text{Re} < 25{,}000 \qquad \textbf{(EQ 21.30)}$$

where

V_s = critical settling velocity, cm/s
g = acceleration due to gravity, 981 cm/s^2
D = diameter of a spherical (ideal) particle, cm
ρ_s = density of particle, g/cm^3
ρ = density of water, g/cm^3
μ = dynamic viscosity of water, g/cm·s
Re = Reynolds number

Sediment pond surface area and free depth (depth above accumulated sediments) is determined from the size of sediment particle to be settled. The total depth of the pond is based on the amount of sediment to be stored and frequency of removing captured sediment over the life of the facility. Design should allow for equipment access so accumulated sediments can be removed periodically.

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Dam

Upstream Face
- Slope Protection ☐
- Erosion-Beaching ☐
- Vegetative Growth ☐
- Settlement ☐
- Debris ☐
- Burrows or Burrowing Animals ☐
- Unusual Conditions ☐

Downstream Face
- Signs of Movement ☐
- Seepage or Wet Areas ☐
- Vegetative Growth ☐
- Channelization ☐
- Condition of Slope Protection ☐
- Burrows or Burrowing Animals ☐
- Unusual Conditions ☐

Abutments ☐
- Seepage ☐
- Cracks, Joints, and Bedding Planes ☐
- Channelization ☐
- Slides ☐
- Vegetation ☐
- Signs of Movement ☐

Crest
- Surface Cracking ☐
- Durability ☐
- Settlement ☐
- Lateral Movement (alignment) ☐
- Camber ☐

Seepage and Drainage Summation
- Location(s) ☐
- Estimated Flow(s) ☐
- Color (staining) ☐
- Erosion of Outfall ☐
- Toe Drain and Relief Wells ☐

Measurement
- Method ☐
- Amount ☐
- Change in Flow ☐
- Clearness of Flow ☐
  - Color ☐
  - Fines ☐
  - Condition of Measurement Devices ☐
  - Records ☐

Other

_____ _____

_____ _____

Performance Instruments
- Piezometer Well ☐
  - Well ☐
  - Frostfloor ☐
  - Ventilation ☐
  - Gauges ☐
  - Piping ☐
  - Security ☐
- Surface Settlement Points ☐
- Crossarm Devices (deviation, station, and offset) ☐
- Reservoir-Level Gauge ☐
- Ice-Prevention System ☐

Other

_____ _____

_____ _____

Spillway

Approach Channel
- Vegetation (trees, willow, etc.) ☐
- Debris ☐
- Slides Above Channel ☐
- Channel Side Slope Stability ☐
- Log Boom ☐
- Slope Protection ☐

Control Structures (observed operation)

Apron
- Surface Condition ☐
- General Condition of Concrete ☐
- Movement ☐
- Settlement ☐
- Joints ☐
- Cracks ☐

Crest
- Surface Condition ☐
- General Condition of Concrete ☐
- Cracks or Areas of Distress ☐
- Signs of Movement ☐

Walls
- Surface Condition ☐
- General Condition of Concrete ☐
- Movement (offsets) ☐
- Cracks or Areas of Distress ☐
- Settlement ☐
- Joints ☐
- Drains ☐
- Backfill ☐

Gates
- Condition ☐
- Hoist Equipment ☐
- Control Equipment ☐

Source: U.S. Department of the Interior 1980

FIGURE 21.13 Dam inspection checklist

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Emergency Spillways

Design of reservoirs and other impoundments should include a spillway to pass flows from extreme storms or other events and to protect the structure from damage from these flows. Generally, spillways are designed as overflow spillways in natural ground along the rim of the impoundment, away from the crest of the dam or embankment, and flow is calculated from

$$Q = C_s L H^{3/2} \quad \textbf{(EQ 21.31)}$$

where

Q = flow, ft^3/s
C_s = spillway coefficient
L = width of the spillway crest, ft
H = total head at the spillway crest, ft (generally the static head; if the velocity head at the spillway is significant, then $H = H_s + V^2/2g$)
H_s = static head at the spillway, ft
V = velocity of flow at the spillway, ft/s
g = acceleration of gravity, ft/s^2

For a sharp crested (ogee) spillway, the coefficient C_s is generally between 3 and 4. A value of 3.97 may be used in the absence of site-specific data.

In some cases, chute spillways may be cut into the abutment of a dam or embankment, or an overflow spillway may be constructed alongside the facility. These can be designed using open-channel flow theory.

INSPECTION AND MAINTENANCE

Any earthen or rock structure that involves cut or fill earth slopes, liner systems, pipe systems, or channels should be inspected on a regular basis by a qualified professional. Damage or irregularities identified should be fully investigated and corrected in a timely manner. A checklist for inspection of an earth-fill dam is given in Figure 21.13.

ACKNOWLEDGMENTS

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CHAPTER

22

Maintenance

Marcus A. Wiley, P.E.

MAINTENANCE THEORY

Equipment availability is key to an optimum return on mine investment. The proper role of maintenance is to provide the lowest cost in maintenance labor and materials and to minimize production losses. Maintenance is a service function and must be coordinated with management and operations personnel. As shown in Figure 22.1, increased maintenance beyond a certain point may ultimately increase overall mine costs. The goal of maintenance should be to achieve the lowest overall cost.

Mines are unique working places, and maintenance often is performed outside a shop and where working conditions are not favorable. Proper maintenance at mines involves judiciously combining the management of people, equipment, and resources with methods for anticipating problems before they occur (Bise 1992).

The goal of mine maintenance is to present equipment that will operate 100% of scheduled time. By analyzing maintenance records and performing statistical analysis on equipment, the correct component replacement episode is identified. The correct episode is when the component risk of failure by further operation is minimized to an acceptable level and at an acceptable cost of planned maintenance. Planned equipment downtime is less costly than unplanned equipment downtime. The primary statistic measured is average runtime to failure.

DEFINITIONS

The following terminology is used in mine maintenance (adapted from Bise 1992; Schissler 2018).

- **Area maintenance.** This is the responsibility of a frontline supervisor for all maintenance within a given geographical area.
- **Backlog.** The total employee-hours required to complete maintenance work compose the backlog. A backlog index is used to determine the effectiveness of the maintenance department.

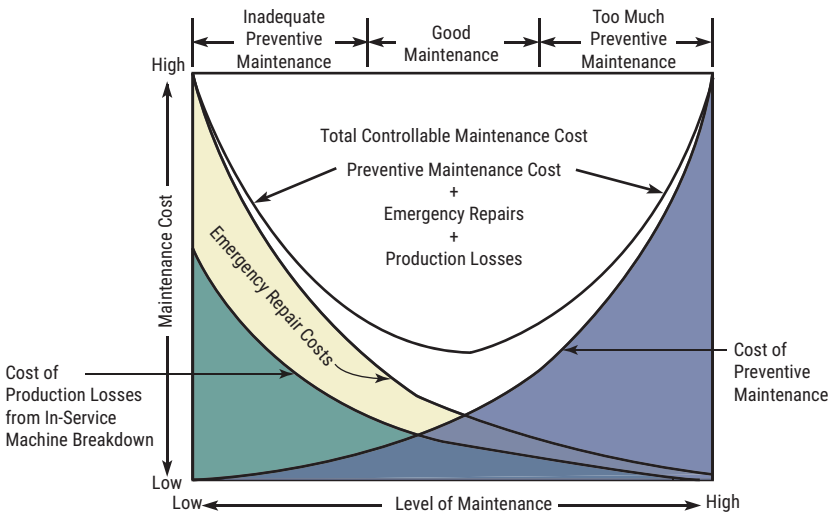

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FIGURE 22.1 Maintenance cost versus level of maintenance

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- **Maintenance availability.** The definition varies by user, but it generally refers to historical machine runtime as a percentage of scheduled time, not including operating delays.
- **Operating delays.** The delays to the production process not caused by machine downtime are considered operating delays. Examples include lunch and travel time. Many mines designate setting bits on a continuous miner or longwall shearer as an operational delay, and lubrication as a maintenance delay.
- **Overhaul.** Restorative maintenance taken before equipment fails is considered overhaul.
- **Predictive maintenance.** A special preventive maintenance, predictive maintenance involves nondestructive testing techniques to predict wear rate, state of deterioration, or imminent equipment failure.
- **Preventive maintenance.** Performed to prolong the life of equipment, preventive maintenance (PM) helps to avoid premature failure. Equipment inspection, lubrication, adjustment, and cleaning are all included in PM.
- **Repair.** Restorative maintenance performed after equipment has failed is called *repair*.
- **Scheduled maintenance.** Planned in advance, scheduled maintenance prevents premature equipment failure.
- **Unscheduled maintenance.** Before it can be used, equipment fails and requires repair.
- **Utilization.** The percentage of time a machine is scheduled to run during the entire year. For example, the only time an electrical power plant shuts down is for major maintenance, but the plant is scheduled to run through all holidays, weekends, and so forth. Hence the utilization of the power plant asset if the plant is down for 1 week is 98% (51 weeks divided by 52 weeks).
- **Utilization delays.** These are delays to scheduled operating time caused by nonproduction times, such as holidays, miners' vacations, unplanned maintenance, and annual shutdown, which usually has major maintenance involved.
- **Work order.** This is the formal document used to communicate maintenance requirements for planning, scheduling, and controlling work to be performed.

MINE MAINTENANCE ORGANIZATION

The 11 steps that follow can build an effective maintenance system (Herbaty 1983):

1. Establish goals and objectives, policies, and procedures.
2. Establish permissible variance from these guidelines.
3. Measure performance to establish guidelines.
4. Compare performance measurement information to guidelines.
5. Isolate and identify deviations beyond tolerances.
6. Determine basic causes for deviations.
7. Determine corrective action.
8. Plan methods of implementing corrective action.
9. Schedule plans for implementing corrective action.
10. Implement corrective action.
11. Follow up to ensure completion of corrective action and to prevent overswing.

The following are maintenance department work categories (Sneddon 1973):

- Daily and routine maintenance
- Planned PM
- Standing work orders (SWOs)
- Cost-improvement orders
- Safety work
- Capitalized construction
- Production work done by maintenance personnel
- Contract work for out-of-plant customers
- Manufacturing items for plant stock

A list of all equipment to be maintained should be assembled. The list should show equipment most critical to the operation (Table 22.1). A small percentage of the equipment is generally responsible for a large percentage of production losses from downtime. A mine maintenance program that focuses on the few critical items should result in substantial savings (Bise 1992).

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TABLE 22.1 Criticality determination

Item Number	Machine Number	Cumulative Maintenance Cost		Annual Cumulative	
		Dollar Cost	Percent	Cost, %	Number of Items, %
1	001	xx.xx	30.0	30.0	0.1
2	008	xx.xx	29.0	59.0	0.2
3	109	xx.xx	10.0	69.0	0.3
4	110	xx.xx	5.0	74.0	0.4
5	005	xx.xx	3.0	77.0	0.5
6	854	xx.xx	1.0	78.0	0.6
7	562	xx.xx	1.0	79.0	0.7
8	687	xx.xx	0.4	79.4	0.8
9	295	xx.xx	0.3	79.7	0.9
10	346	xx.xx	0.3	80.0	1.0
…	…	…	…	…	…
1,000	272	xx.xx	0.1	100.0	100.0

Source: Herbaty 1983

Equipment Management Program

Tomlingson (2010) recommends the equipment management program shown in Figure 22.2, which "is typical of the detail that a large plant would consider in the initial development of their maintenance program." This particular model can be used to illustratively "explain the maintenance program to personnel from all departments." The following list, adapted from Tomlingson (2010) details the numerical sequence used in the figure:

1. Operators perform periodic equipment checks to establish whether equipment is running properly. Although modern equipment design often provides monitoring systems or protected systems with alarms and signals, additional personal attention from operators often finds the unusual problems that might be missed. Operators submit reports of equipment condition to operating supervisors.
2. Operations supervisors, in turn, review the operators' reports as a potential source of future work.
3. Operations supervisors then determine whether new work is required and specify the unit, problem, and urgency of the job, and then identify themselves as work requesters using the standard work request procedure.
4. If the work required is an emergency, the operations supervisor can make a verbal request to expedite repair.
5. Upon receipt of work requests, the maintenance supervisor initially classifies the work. If the work requested meets the criteria for being planned, it is forwarded to the planner. In the case of emergency repairs, the maintenance supervisor immediately assigns work. Work requests for an emergency facilitate reporting of resource use such as labor and materials by maintenance crew members.
6. The maintenance supervisor advises the planner of all work that requires planning based on an assessment of the planning criteria and advises the planner of any special requirements that the work entails.
7. Maintenance planners utilize the information system to determine which PM services are due the following week. The information system identifies PM services due based on fixed time intervals such as weeks or variable intervals like operating hours. Ordinarily, the PM module of the information system is updated once a week, and at that time, services due the following week are identified and placed on the preliminary maintenance plan. Some PM services are dynamic and can be carried out while the equipment is running. These PM services are assigned directly to the maintenance supervisor so they can be fitted into the week's work at the first opportunity. Other PM services due are static services, requiring that the equipment be shut down in order to perform the work. These services are handled by the planner. They are fitted into the weekly plan along with major jobs.

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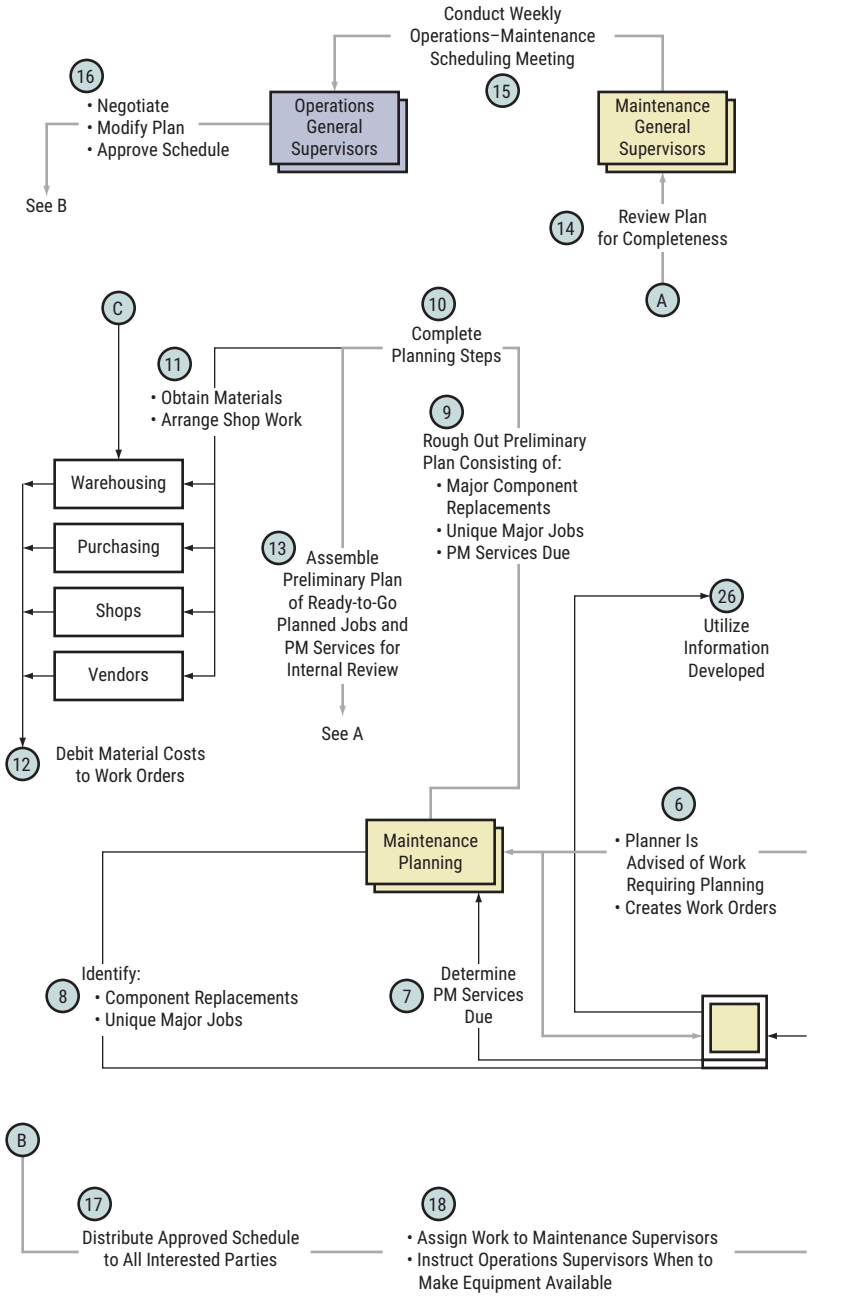

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FIGURE 22.2 Maintenance management system

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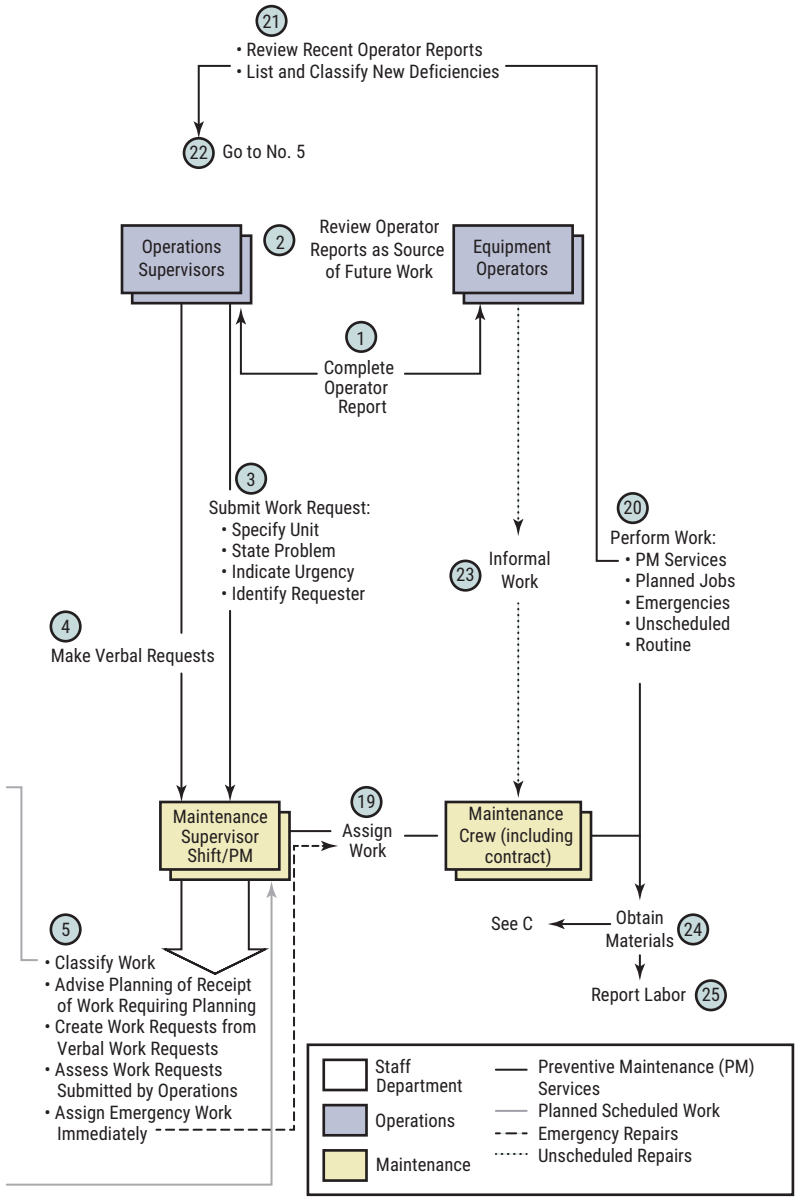

Source: Tomlingson 2010

FIGURE 22.2 Maintenance management system (continued)

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8. The information system should be able to forecast the approximate timing of the replacement of major components, such as engines and drive motors. In addition, the approximate timing of major unique jobs, such as overhauls, is also forecasted. These forecasted actions are normally displayed week by week so that the planner can query the information system and determine what actions are due the following week. After these are identified, the planner determines the actual condition of the equipment, and, after conferring with maintenance supervisors, may either place the activity on the proposed plan for the following week or defer the work based on current repair history and the actual operating condition of the equipment.
9. After the planner has assembled all of the potential work for the following week, a preliminary plan is compiled to include major component replacements, unique major jobs such as overhauls, and static PM services. The plan does not become a schedule until it has been presented to operations personnel and they concur that the plan should be carried out.
10. In carrying out the planning steps, planners address the following issues:
    - Identify the work required.
    - Investigate in the field to determine exact needs.
    - Seek advice from the supervisor who will do the work.
    - Determine whether any standards apply.
    - Confirm the job scope.
    - Make a job plan.
    - Set up the preliminary work order.
    - Determine the needed resources.
    - Establish the labor requirements by technical skill (craft).
    - Identify materials needed and the sources.
    - Specify any shop work.
    - Estimate job cost.
    - Establish the job priority with operations.
    - Get the job approved: cost and preliminary timing.
    - Identify the possible future time for doing the work.
    - Open the work order, making it an "official" accounting document.
    - Order materials through purchasing.
    - Reserve the stocked parts.
    - Order the shop work.
    - Await the receipt of materials and completion of shop work.
    - As materials are ready, confer with operations on job timing.
    - Develop a preliminary weekly plan.
    - Review the plan internally before the scheduling meeting with operations.

    Information on the repeated conduct of work, such as the replacement of major components, is used to develop standards for such jobs, which include task lists, bills of materials, and tool lists. Timing of component replacements is based on the historic life-span of the components.

    With the advent of more sophisticated condition-monitoring techniques, forecasting has become a dependable source of information for maintenance actions. It simplifies planning by providing proven standards for repetitive tasks. As unique major jobs are being planned, the maintenance engineer establishes the job scope, which specifies the necessary work to restore the equipment to an as-designed condition. The maintenance engineer prescribes steps that ensure the reliability of equipment and the application of techniques that ensure its maintainability. In addition, maintenance engineers verify the application of PM techniques, the use of modern technology, and the effectiveness of information use.
11. Planners reserve stock materials required for major jobs to be performed in the following week. In addition, they instruct the purchasing agent to procure direct charge materials. Planners use the information system's purchase order tracking system to monitor the status of these purchase orders. They also order the shop support, confer with the shop planner, and provide sub–work orders as required. They monitor shop work progress using the information system's work order status report.

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12. The cost of materials is debited to the appropriate work orders so the total cost of each job along with the cost of labor can be established. When the work is completed, the total cost of each job can be determined.
13. As materials and shop work become available, the planner assembles a preliminary plan of all candidate work orders and works with maintenance supervisors to ensure that all are feasible and ready to go. See A.
14. The preliminary maintenance plan for the following week is reviewed by maintenance supervisors to determine whether it is feasible and complete or if certain jobs might be deferred or canceled.
15. The approved maintenance plan is then presented to operations by the maintenance supervisor at the weekly operations–maintenance scheduling meeting. The supervisor is ultimately responsible for carrying out the plan. The planner provides a supporting role during the scheduling meeting in the event that job priorities must be changed or resources rearranged. The weekly scheduling meeting with operations is the opportunity for maintenance to present its plan for accomplishing major jobs and key static PM services for the following week. Based on the production plan, operations will consider the needs for equipment and the "mix" they must have against the requirement to perform maintenance at the recommended times. Similarly, the shutdown of fixed equipment, lines, or areas is considered. Necessary negotiations are carried out by the principals with the coordinating planner acting as an advisor. Scheduling meetings should be chaired by the decision makers who will be responsible for making equipment available and accomplishing the work. Purchasing agents and warehouse supervisors should attend scheduling meetings to ensure that their responsibilities are being met. The scheduling meeting should conclude with an approved schedule. Thereafter, operations should make equipment available according to the schedule, and maintenance make its resources available to perform the work. The objective is to accomplish the work with the least interruption to operations and the best use of maintenance resources. Many plant managers use the weekly scheduling meeting as an opportunity to assess progress against their overall improvement goals such as
    - Compliance with the PM program and last week's schedule;
    - Reductions in backlog, overtime, and emergency repairs; and
    - On-site parts delivery performance by the warehouse.
16. Because of the negotiations carried out between maintenance and operations during a scheduling meeting, the plan may be modified to better meet the needs of operations and to utilize maintenance resources more effectively. The scheduling meeting is followed by daily coordination meetings between operations and maintenance to adjust the schedule in the event of delays.
17. The approved schedule is now distributed to all interested parties.
18. Maintenance supervisors are now assigned the specified work on the approved schedule. This step suggests that each supervisor has the opportunity to coordinate directly with purchasing, shops, or the warehouse for on-site material delivery. In addition, transportation and service departments should help coordinate their support. The supporting planner might be asked to assist in coordinating for the supervisor who will be on the jobsite controlling the work. Supervisors assign work on the schedule to individual maintenance personnel. They provide appropriate work orders and necessary instructions for carrying out the work. In addition, operating supervisors are advised when they should make equipment available so that maintenance work can be carried out.
19. Using the weekly schedule as a guide, maintenance supervisors give work assignments to maintenance crew members on a daily basis. As necessary, supervisors consult the information system to determine whether there are any pending small jobs on the same equipment scheduled for shutdown and whether these smaller jobs might be fitted in.
20. Maintenance crew members perform PM services on planned jobs, respond to emergencies, perform unscheduled repairs, and perform routine services.
21. Technicians and operators confer during the inspection of equipment by maintenance crew members. Operators can explain unusual equipment behavior while technicians can show operators simple, helpful checks or adjustments. With mobile equipment, operators are often not present. Therefore, comments are provided based on operator check sheets. With personal inspection, operator input, and results of the operator's checks, the technician can make an accurate, complete list of equipment deficiencies, identified to the equipment component level, and their relative seriousness. Generally, there are three types of deficiencies:
    - Emergency repairs. Deficiencies found just in time to avert an injury, equipment failure, or serious loss of product or production time are usually emergency repairs. These jobs should be acted on as

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soon as possible and, in most instances, key people advised in the event that other actions, like the use of alternate equipment, are necessary.

- Unscheduled minor repairs. The most numerous deficiencies found are those smaller repairs that do not have to be done immediately. These are unscheduled jobs. But because they can develop into more serious problems, they must be recorded and fitted in at the first opportunity. (Maintenance supervisors should instruct technicians to always finish the whole PM service before making any lengthy repairs. This procedure should be followed because the PM service must be performed within a specific time limit. If too many small, incidental repairs are made "as you go," there is a good chance that the total service might not be completed. Thus, there is a danger that problems not found are more serious than those repaired. Conversely, it is a good idea to add a few minutes to the specified time to make simple adjustments that require only tools or limited materials just to save a return trip.)
- Major repairs. A limited number of deficiencies will indicate a need to make significant repairs. These are potentially planned jobs. But if PM services are conducted faithfully, these jobs will always be found far enough in advance to be able to plan the work. Technicians should make a special effort to establish the exact nature of the problem so it can be accurately described to the maintenance supervisor and, in turn, to the planner. Technicians should advise production supervisors directly on the deficiencies they have found and how serious they are, which not only gives a current condition report, but it lets supervisors know that the PM program is alive and well. Moreover, when the operations supervisor learns that each deficiency found will be taken care of, operations sees a solid partnership developing with maintenance people. Operations is also delighted to know that operators have contributed with their checks. This action spreads good will and cooperation.

22. Go to No. 5. Technicians meet with the maintenance supervisor and together they decide on the actions to be taken on the deficiencies reported. Technicians supply firsthand information on actual equipment condition, and supervisors provide knowledge of the bigger picture of what to do, when to do it, and how. Specific decisions are now made on each type of deficiency relative to equipment condition, the urgency of corrective actions, the current plant situation, and the availability of maintenance resources. First, the emergency repairs are acted on. Generally, verbal orders are used to assign work to available technicians with follow-up work orders after the repairs are attended to. Next, those deficiencies requiring planning (No. 6) are discussed with the planner. In this instance, the prudent maintenance organization will already have provided sensible criteria of what work is to be planned. Thus, the planner, supervisor, and technician are in full accord. The technician performing the PM services should enter all of the unscheduled new jobs into the work order system. Remember that the technicians performing the work are the best source of information. Always use them.
23. As maintenance services are performed, technicians may also be called on while near or on the equipment, as with mobile equipment, to provide informal help. Generally, supervisors should have provided criteria to the technicians to determine whether any of the informal repairs made should be entered into repair history.
24. While doing work, maintenance crew members may require additional stock materials. See C.
25. Upon completion of work, maintenance crew members report the labor used. This data is subsequently verified by the maintenance supervisor. Labor and material data recorded on work orders, along with other information such as failure codes, is processed by the information system to provide information reports for managing maintenance, such as labor utilization, cost reports, repair history, work order status, and backlog.

PREVENTIVE MAINTENANCE

Preventive maintenance includes equipment inspection, lubrication, adjustments, and cleaning. The goal is to keep equipment running effectively and to avoid downtime. Success of a PM program can be evaluated by

- Increased equipment operating time,
- Increased capability for more planned maintenance,
- Increased product output, and
- Decreased breakdown maintenance.

Figure 22.3 indicates the important steps in organizing and operating a PM program. Figure 22.4 shows the relationship between equipment deterioration over time with cost and downtime.

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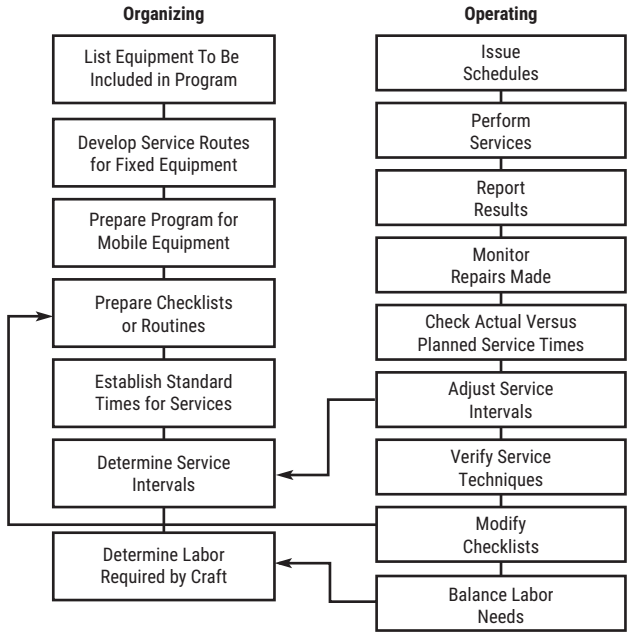
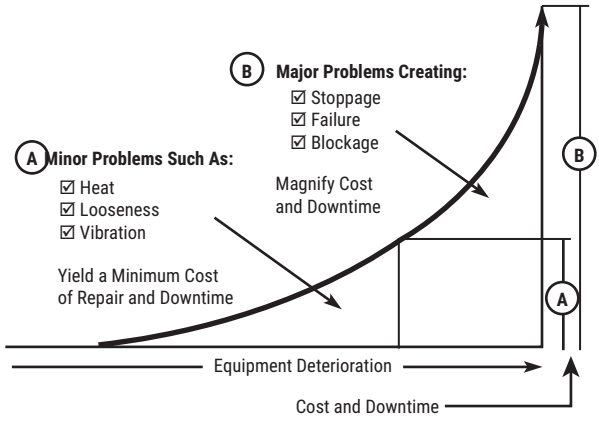


Source: Tomlingson 2010

FIGURE 22.3 Preventive maintenance program



Source: Tomlingson 2010

FIGURE 22.4 Equipment deterioration versus cost and downtime

PREDICTIVE MAINTENANCE

Predictive maintenance is specialized, and it strengthens a PM. Figure 22.5 shows preventive and predictive maintenance techniques. Figure 22.6 illustrates how to start a predictive maintenance program.

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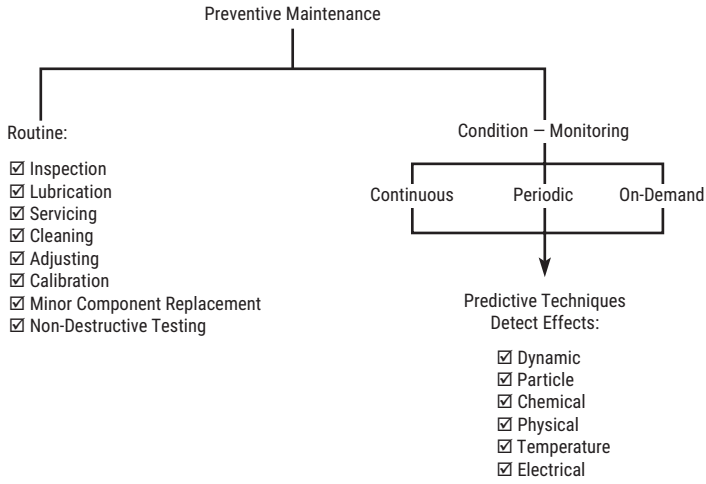
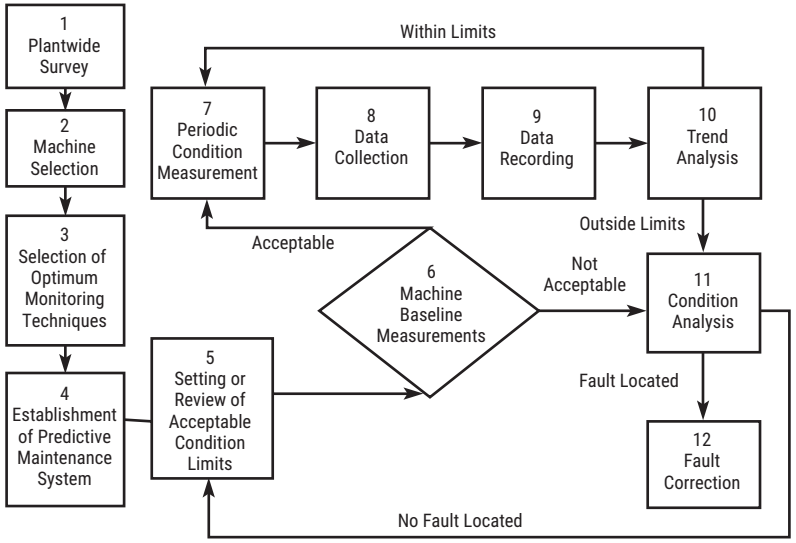


Source: Tomlingson 2010

FIGURE 22.5 Preventive and predictive maintenance techniques



Source: Nolden 1987

FIGURE 22.6 Essential steps in building a predictive maintenance program

LUBRICATION AND EQUIPMENT ANALYSIS

Oil and grease are used to lubricate, cool, and to keep machinery parts clean thereby reducing heat buildup and wear from excessive friction between moving parts. A sample of engine oil can be analyzed for wear metals, contaminants, and condition of the oil in comparison to new oil. This helps determine the condition of the equipment and helps evaluate the success of the maintenance program. Table 22.2 is a typical oil sampling schedule.

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TABLE 22.2 Typical oil sampling intervals

Monthly or at 500 Hours	Bimonthly	Beginning, Middle, and End of Season
Air compressors	Gears and bearings	Air-conditioning compressors
Diesel engines	Hydraulic systems	
Gas turbines	Steam turbines	
Natural gas engines		

Source: Dunn 1987, CFE Media, LLC

Other diagnostic techniques are used to aid in determining when equipment needs maintenance prior to a breakdown, thus preventing unscheduled repairs. Some of these methods are as follows (Bise 1992; Schissler 2018):

- **Vibration analysis.** The measurement of machinery vibration at various intervals is compared to baseline data taken when placing equipment in service. Increased vibration is an indication of deterioration of components that can be replaced during a scheduled maintenance procedure rather than waiting for failure.
- **Shock-pulse analysis.** Lubrication and mechanical condition in ball and roller bearings are measured. When damaged, abnormal high-frequency energy is generated.
- **Ferrography.** Also known as wear-particle analysis, this technique analyzes metal particulates after separation from oil samples. Information on particle shape, composition, size, and quantities can provide data on types of wear within a piece of equipment.
- **Thermography.** Infrared inspection uses a handheld scanner to measure the temperatures of various machinery components to predict potential failure through overheating.
- **Die penetrant analysis (DPA).** A liquid is immersed into metal, which leaves evidence of an anomaly in the form of a crack or porosity. The tool is useful in detecting metal fatigue and possible failure, thus indicating rebuild or replace.
- **Volumetric analysis.** Applied to hydraulic and other fluid circuitry, the quantity and pressure of flow is measured to identify deviation from component specifications.
- **Filtration analysis.** This type of analysis is applied to liquid and air filters of engines and components to identify efficiency of contaminant capture, capture type to indicate source, and assess filter replacement interval.

RECORDKEEPING

Gathering and managing information on equipment and personnel utilization, costs, inventory control, repair history, and equipment specifications is critical to the success of a maintenance program.

Reporting

Records and reports that should be maintained and distributed to individuals having a need for the information are as follows (Sneddon 1973; Schissler 2018):

- Equipment component replacement interval expressed in operated hours
- Equipment data sheet
- Work-order form
- Daily maintenance work schedule
- Employee's daily time card
- Supply requisition
- SWOs
- Monthly equipment cost report
- Backlog report
- Personnel performance report
- Equipment history records

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WORK ORDER | W. O. Number

Priority	Equipment Name	Plant Charge	Acct. No.	Acct. Sub	Equipment No.

Category	Location	Available	Required	Requested By	Date

Description of and Reason for Work

Review After Estimate ☐ Yes ☐ No | Authorized By | Date

Job Seq.	Skill Code	Type of Work	Description of Work	Estimated Personnel and Hours

Materials

Quan.	Description	Loc.	Maj.	Min.	Sub	Price	Unit	Total Cost

Estimated Job Cost →	Personnel-Hours	Labor $	Material $	Other $	Total $

Estimated By | Date | Ckd.

Source: Sneddon 1973

FIGURE 22.7 Sample work-order form

A sample work-order form and instructions for filling it out are provided in Figure 22.7 and Table 22.3, respectively. Also included is Figure 22.8 that offers an example of a completed work order where results have been memorialized to a records system. Work that may require an SWO includes lubrication, PM, janitor services, and delivery of parts and materials.

INVENTORY

A key component of a maintenance department is having the proper inventory of spare parts on hand to perform required maintenance work. Inventory control may need as much management attention as the actual servicing of equipment itself.

An inventory control system maintains inventory, purchases spare parts, establishes maximum/minimum stock or spares levels by part, and arranges for spare part manufacture and component rebuilding to ensure that parts will be in stock when needed. Inventory management is important in managing working capital at a mine. An inventory control system should provide information on the following (Tomlingson 1994):

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TABLE 22.3 Instructions for preparing a work-order form

1. Priority—urgency of work to be done:
   "E"—emergency work, work around the clock until done
   "1"—high-priority work, may require overtime
   "2"—work that should be completed within 3 days
   "3"—routine work
   "P"—preventive maintenance
2. Equipment name—name of piece of equipment or building to be worked on
3. Plant charge—three-digit code number designating the appropriate cost center
4. Account number—two-digit account number within the cost center
5. Account sub—sequential series of numbers for each piece of numbered equipment; enter "00" if the work does not pertain to a numbered item
6. Equipment number—number assigned to equipment to be worked on or a sequential series of numbers if not a numbered item
7. Category—category of work as shown in preceding text
8. Location—location within plant
9. Available—date equipment is available for work to begin
10. Required—date by which work must be completed
11. Requested by—signature of requester
12. Date—date work request was made
13. Description of and reason for work—a complete description of work to be done, including dimensions, etc., and the reasons or need. The information in this space should be complete enough to enable the work to be planned and accomplished without unnecessary delays to determine what is to be done. Drawings and any other pertinent data should be listed, or sketches attached.
14. Review after estimate—the person responsible for costs can designate whether he or she wishes to have the work order fully estimated before granting approval
15. Authorized by—signature of person responsible for costs, indicating approval to have work accomplished
16. Date—the date authority to proceed was granted
17. Job sequence—the chronological order of steps required to complete work
18. Skill code—the craft group code of the particular craft required for each sequence of work
19. Type of work—the name of the craft required (e.g., "mach" for machinist)
20. Description of work—a short description of the work to be done in each sequence
21. Estimated personnel and hours—estimated time required to complete sequence (shown as 2 × 8 for 2 individuals, 8 hours)
22. Materials—all material not readily available is listed to ensure its availability when required
23. Estimated job cost—shows the total estimated labor dollars and material dollars and, when completed, is the budget for that particular work order.

Source: Sneddon 1973

MAINTENANCE WORK ORDER

MWO#	Sec	Unit	Comp	CT	Unit Name	Comp Name
426820	024	CM01	100	S	Cont. Miner	Cutting

Description of Work: RPL CUTTER HEAD MOTOR +

St	Opn	Stt	Cls	Pr	Wa	FC	RH	SN/In	SN/Out
S	0610	0612	0612	90	No	8	X	128875	122889

Work Order Notes

	EST	ACT
Mechanics	12	12
See Material List Electricians		
LABOR	300	300
MATERIAL	25600	25720
TOTAL	25900	26020

Requested:	Approved:	Completed:
B. Easton	J. Southworth	T. Washburn
04/03/09	04/16/09	04/18/09

Source: Tomlingson 2010

FIGURE 22.8 Sample work-order results

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- Stocked parts and substitutes
- Stocked parts related to types of units and components
- Quantities on hand and quantities set aside for identified work
- Number on order and reorder points and order quantities
- Unit costs
- Resupply sources
- Costs of issued parts
- Turnover rate
- Number of issue days on hand for critical parts
- Reconciliation of records and physical counts
- Stocking cost
- Value of stock by code and user area
- Status actions for restocking

An excessive amount of inventory unnecessarily ties up capital resources; however, an insufficient level will not allow an operation to continue efficiently and without interruption. A minimum stock level is the average usage of an item times the lead time necessary to receive it when ordered. This does not take into account the cost to purchase or store an item; nor does it provide a safety factor quantity. Once minimum stock level is reached, a new order should be placed. The optimum number of items to be ordered can be determined from the following equation (adapted from Brown 1973 and Moore and Jaedicke 1967):

$$Q_o = \sqrt{\frac{2 \cdot C_i \cdot U_i}{C_s}} \qquad \textbf{(EQ 22.1)}$$

where

Q_o = optimum number of items to order
C_i = cost of item
U_i = usage or frequency of item that needs replacing
C_s = cost to store or warehouse item

ACKNOWLEDGMENTS

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CHAPTER

23

Health and Safety

Kelvin K. Wu, P.E. and William J. Francart, P.E.

LEGISLATIVE HISTORY

1910—Public Law 61-179

The Bureau of Mines is created in the U.S. Department of the Interior. A federal safety and health role is established for research and investigation of accidents.

1941—Public Law 77-49

Right of entry is given to federal inspectors to conduct inspections and investigations in coal mines to obtain information. No safety or health regulations are mandated.

1947—Public Law 80-328

Federal safety standards are adopted for bituminous coal and lignite mines. Provisions are made for federal inspectors to notify mine operators and state mine agencies of violations. No enforcement provisions are mandated, with the law expiring after 1 year.

1952—Public Law 82-552

The Federal Coal Mine Safety Act is enacted with an emphasis on preventing major coal mine disasters. Under the law, annual inspections become required at underground coal mines. Mandatory safety standards are established for underground coal mines, with more stringent standards for "gassy" mines. Federal inspectors are given authority to issue notices of violation, as well as orders of withdrawal in situations of imminent danger. In addition, orders of withdrawal are mandated where less serious violations were not properly corrected. Enforcement of federal standards by state inspectors are allowed under a state plan system. Anthracite mines are covered, but all surface coal mines and any mine employing fewer than 15 people underground are exempted.

1961—Public Law 87-300

The law authorizes the study of causes and prevention of injuries and health hazards in metal and nonmetal mines. Federal officials are given the right of entry to collect information.

1966—Public Law 89-376

This law extends the coverage of the 1952 law to small underground coal mines. It provides for orders of withdrawal in cases of repeated unwarrantable failures to comply with safety standards. Education and training programs are expanded.

1966—Public Law 89-577

The Federal Metal and Nonmetallic Mine Safety Act of 1966 is enacted, which sets up procedures for developing safety and health standards for metal and nonmetal mines. Standards can be advisory or mandatory. Annual inspections are required for underground mines. Federal inspectors are given authority to issue notices of violation and orders of withdrawal. Enforcement of federal standards by state inspectors is allowed under the state plan system. Education and training programs are expanded.

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1969—Public Law 91-173

The Federal Coal Mine Health and Safety Act of 1969 is enacted, which vastly increases enforcement powers in coal mines. Surface mines are covered in this Act. Four annual inspections are required for each underground coal mine. Stricter standards for gassy mines are abolished, but additional inspections are required in these mines. Miners are given the right to request a federal inspection. State enforcement plans are discontinued. Mandatory fines can be assessed for all violations. Criminal penalties are established for knowing and willful violations. Safety standards for all coal mines are strengthened and health standards are adopted. Procedures are incorporated for developing new health and safety standards. A training grant program is instituted. Benefits are provided to miners disabled by black lung disease.

1973—Secretarial Order 2953

The Mining Enforcement and Safety Administration (MESA) is created as a new U.S. Department of the Interior agency by administrative action. The new agency assumes safety and health enforcement functions formerly carried out by the Bureau of Mines.

1977—Public Law 95-164

The Federal Mine Safety and Health Amendments Act of 1977 is enacted, which places coal, metal, and nonmetal mines under a single law with enforcement provisions similar to the 1969 Act. (Separate safety and health standards are retained.) The Act moves the enforcement agency to the U.S. Department of Labor and renames it the *Mine Safety and Health Administration* (MSHA). The requirement for four annual inspections at all underground mines and two annual inspections at all surface mines is established. Advisory standards for metal and nonmetal mines are eliminated. State enforcement plans in metal and nonmetal sector are discontinued. The Act contains provisions for mandatory miner training. Mine rescue teams are required for all underground mines. The Act calls for increased involvement of miners and their representatives in health and safety activities.

2006—Public Law 109-236

The Mine Improvement and New Emergency Response Act (MINER Act) is enacted, which requires operators of underground coal mines to improve accident preparedness. The legislation requires mining companies to develop an emergency response plan specific to each mine they operate, and requires that every mine have at least two rescue teams located within 1 hour of the mine. The law also limits the legal liability of rescue team members and the companies that employ them. It increases both civil and criminal penalties for violations of federal mining safety standards and gives MSHA the ability to temporarily close a mine that fails to pay penalties or fines. In addition, the Act calls for several studies into ways to enhance mine safety, as well as the establishment of a new office within the National Institute for Occupational Safety and Health (NIOSH) devoted to improving mine safety. The Secretary of Labor is directed to require two-way communications and an electronic system within 3 years of the law's establishment, permitting those on the surface to locate persons trapped underground. Finally, the legislation establishes new scholarship and grant programs devoted to training individuals with respect to mine safety.

HEALTH AND SAFETY REGULATIONS

Table 23.1 lists all the health and safety topics contained in Title 30 (CFR 1–99, 2019).

MINE SAFETY AND HEALTH ADMINISTRATION'S MISSION

MSHA in the U.S. Department of Labor works to prevent death, illness, and injury from mining and promote safe and healthful workplaces for U.S. miners. MSHA carries out the provisions of the Federal Mine Safety and Health Act of 1977 (Mine Act) as amended by the MINER Act of 2006.

The agency develops and enforces safety and health rules for all U.S. mines regardless of size, number of employees, commodity mined, or method of extraction. MSHA also provides technical, educational, and other types of assistance to mine operators. MSHA works cooperatively with industry, labor, and other federal and state agencies to improve safety and health conditions for all miners in the United States. MSHA primarily is a law enforcement agency.

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TABLE 23.1 Health and safety topics under Title 30

Part	Subject
Subchapter B—Testing, Evaluation, and Approval of Mining Products	
1	Mine Safety and Health Administration (MSHA): establishment and use of official emblem
3	Office of Management and Budget control numbers under the Paperwork Reduction Act
5	Fees for testing, evaluation, and approval of mining products
6	Testing and evaluation by independent laboratories and non-MSHA product safety standards
7	Testing by applicant or third party
14	Requirements for the approval of flame-resistant conveyor belts
15	Requirements for approval of explosives and sheathed explosive units
18	Electric motor-driven mine equipment and accessories
19	Electric cap lamps
20	Electric mine lamps other than standard cap lamps
22	Portable methane detectors
23	Telephones and signaling devices
27	Methane-monitoring systems
28	Fuses for use with direct current in providing short-circuit protection for trailing cables in coal mines
33	Dust collectors for use in connection with rock drilling in coal mines
35	Fire-resistant hydraulic fluids
36	Approval requirements for permissible mobile diesel-powered transportation equipment
Subchapter G—Filing and Other Administrative Requirements	
40	Representative of miners
41	Notification of legal identity
42	National Mine Health and Safety Academy
43	Procedures for processing hazardous conditions complaints
44	Rules of practice for petitions for modification of mandatory safety standards
45	Independent contractors
Subchapter H—Education and Training	
46	Training and retraining of miners engaged in shell dredging or employed at sand, gravel, surface stone, surface clay, colloidal phosphate, or surface limestone mines
47	Hazard communication (HazCom)
48	Training and retraining of miners
49	Mine rescue teams
Subchapter I—Accidents, Injuries, Illnesses, Employment, and Production in Mines	
50	Notification, investigation, reports and records of accidents, injuries, illnesses, employment, and coal production in mines
Subchapter K—Metal and Nonmetal Mine Safety and Health	
56	Safety and health standards—surface metal and nonmetal mines
57	Safety and health standards—underground metal and nonmetal mines
58	Health standards for metal and nonmetal mines
Subchapter M—Uniform Mine Health Regulations	
62	Occupational noise exposure
Subchapter O—Coal Mine Safety and Health	
70	Mandatory health standards—underground coal mines
71	Mandatory health standards—surface coal mines and surface work areas of underground coal mines
72	Health standards for coal mines
74	Coal mine dust personal sampler units
75	Mandatory safety standards—underground coal mines
77	Mandatory safety standards—surface coal mines and surface work areas of underground coal mines
90	Mandatory health standards—coal miners who have evidence of the development of pneumoconiosis
Subchapter P—Civil Penalties for Violations of the Federal Mine Safety and Health Act of 1977	
100	Criteria and procedures for proposed assessment of civil penalties
Subchapter Q—Pattern of Violations	
104	Pattern of violations

Source: 30 CFR Parts 1–199

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STATUTORY FUNCTIONS

The Mine Act provides that MSHA inspectors shall inspect each surface mine at least two times a year and each underground mine at least four times a year (seasonal or intermittent operations are inspected less frequently) to determine whether an imminent danger exists and whether there is compliance with health and safety standards or with any citation, order, or decision issued under the Mine Act.

MSHA activities include the following:

- Investigating mine accidents, complaints of retaliatory discrimination filed by miners, hazardous condition complaints, knowing or willful (criminal) violations committed by agents or mine operators, and petitions for modification of mandatory safety standards
- Developing improved mandatory safety and health standards
- Assessing and collecting civil monetary penalties for violations of mine safety and health standards
- Reviewing for approval mining plans and education and training programs from mine operators
- Maintaining the National Mine Health and Safety Academy to train inspectors, technical support personnel, and mining industry personnel
- Approving and certifying the design of certain mining products
- Providing technical assistance to operators in meeting the requirements of the Mine Act
- Providing assistance to mine operators in improving their education and training programs
- Cooperating with states in the development of mine safety and health programs
- Making grants to states in which mining takes place
- Overseeing rescue and recovery operations

ORGANIZATIONAL STRUCTURE

MSHA was created in 1978, when the 1977 Mine Act transferred the federal mine safety program from the U.S. Department of the Interior to the U.S. Department of Labor.

MSHA is headed by an Assistant Secretary of Labor who administers a broad regulatory program to reduce injuries and illnesses in mining. Enforcement of safety and health rules and other responsibilities are carried out by two functional entities:

1. The Coal Mine Safety and Health activity conducts its mine inspection, investigation, and training programs through 11 district offices and a system of subordinate offices in the nation's coal mining regions.
2. The Metal and Nonmetal Mine Safety and Health activity administers its programs for all non-coal mines through six district offices in mining areas throughout the United States.

Other entities that have important roles include the following:

- The Office of Standards, Regulations, and Variances coordinates the development and issuance of safety and health rules and revision of existing rules, continually involving the public in the process.
- The Office of Assessments administers civil penalty assessments against mine operators for failing to comply with health or safety requirements.
- The Technical Support Directorate supplies engineering and technical aid, approves equipment and materials for safe mining use, and assists in mine emergencies and accident investigations. Technical Support operates major facilities in Pennsylvania and West Virginia.
- The Educational Policy and Development Office administers the agency's training programs. From the National Mine Health and Safety Academy, training is conducted on a variety of mine health and safety topics for specialists from government, industry, and labor. Through the Educational Field Services Division, training-related assistance is offered to mine operators throughout the country.
- The Office of Program Evaluation Information Resources (PEIR) conducts internal reviews, evaluates the effectiveness of agency programs, and conducts follow-up reviews to ensure that appropriate corrective actions have been taken. Another PEIR function is to collect, analyze, and publish data obtained from mine operators on the prevalence of work-related injuries and illnesses in the mining industry. PEIR is also responsible for support and training for all MSHA automated information

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systems, data communications networks, and automated data processing equipment. National mine injury and illness data is compiled, analyzed, and distributed to the mining community and public by specialists of the Division of Mining Information Systems (DMIS).

ENFORCEMENT

MSHA has many important tools for reducing injuries and illnesses in the nation's mines. Among them are various enforcement actions that MSHA can use to help ensure that dangerous conditions or practices are prevented or corrected. These are codified under 30 CFR Parts 1 to 199.

Civil Penalties Imposed for Violations

MSHA inspectors must issue a citation or order for each violation of a health or safety standard they encounter. Each issuance entails a civil penalty. These fines may range up to $70,000 per violation. MSHA's Office of Assessments sets the penalties. Most non-serious violations that are corrected promptly are assessed a flat $112 penalty, except that mining operations found to have an excessive history of safety and health violations are not eligible for the $112 penalty. Most other violations are assessed according to a formula that considers six factors:

1. History of previous violations
2. Size of the operator's business
3. Any negligence by the operator
4. Gravity of the violation
5. The operator's good faith in trying to correct the violation promptly
6. The effect of the penalty on the operator's ability to stay in business

These factors are determined from the inspector's findings, MSHA records, and information supplied by the operator. In some cases (often involving fatalities or serious injuries), the formula does not yield an appropriate penalty. In these cases, MSHA may waive the formula and make a special assessment. Civil penalties are assessed against the mine operator. However, agents of corporate operators may individually be fined for violations they knowingly caused or permitted. Individual miners can be fined for violating smoking prohibitions.

"Significant and Substantial"

Several provisions of the Act concern significant and substantial (S&S) violations. An S&S violation is one that is reasonably likely to result in a reasonably serious injury or illness. In writing each citation, the MSHA inspector determines whether the violation is S&S. These violations are not eligible for the flat penalty.

Orders of Withdrawal

In several situations, the law provides that MSHA may order miners withdrawn from a mine or part of a mine. Some of the most frequent reasons for orders of withdrawal are (1) imminent danger to the miners; (2) failure to correct a violation within the time allowed; and (3) failure to secure an area during an accident investigation.

Unwarrantable Failures

If an MSHA inspector finds an S&S violation resulting from an "unwarrantable failure" by the operator to comply with a standard, the inspector incorporates that finding into the citation. If another violation, also resulting from an unwarrantable failure, is found within 90 days, MSHA issues a withdrawal order until it is corrected. Thereafter, any violation similar to the one that led to this withdrawal order will trigger another withdrawal order. This applies until an inspection of the mine discloses no similar violations.

Pattern of Violations

If MSHA determines that a mine has a pattern of S&S violations, the law and regulations provide that the agency shall notify the operator, who is then given an opportunity to improve compliance. Thereafter, if

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a mine is notified that it has a pattern of violations, any S&S violation found within 90 days automatically triggers a withdrawal order. Each additional S&S violation indicates another withdrawal order until the mine has a "clean" inspection with no S&S violations.

Discrimination Protection

The law prohibits discrimination against miners, their representatives, or job applicants for exercising their safety and health rights. MSHA investigates all complaints of discrimination. If evidence of discrimination is found, the U.S. Department of Labor can take the miner's case before the independent Federal Mine Safety and Health Review Commission. In some cases, miners who have been fired can temporarily get their jobs back while a discrimination complaint is being adjudicated.

Criminal Penalties

The Mine Act provides for criminal sanctions against mine operators who willfully violate safety and health standards. MSHA initially investigates possible willful violations; if evidence of such a violation is found, the agency turns its findings over to the Department of Justice for prosecution.

Appeals

Before any citation or order is assessed, the operator or miners' representative can confer with an MSHA supervisor about any disagreement with the inspector's findings. If the disagreement cannot be resolved on this level, the operator is entitled to a hearing before an administrative law judge with the Federal Mine Safety and Health Review Commission. An operator or miners' representative who disagrees with any other enforcement action by MSHA also is entitled to a hearing. The administrative law judge's decision can be appealed to the Commissioners and thereafter to the U.S. Court of Appeals.

TECHNICAL ASSISTANCE

MSHA gives direct assistance for mines around the country from its Technology Center in Pittsburgh, Pennsylvania. Specialists work with mining companies and local MSHA inspectors to gather information on safety and health problems and to offer engineering or other types of solutions. MSHA provides mining companies with help in overcoming health hazards, such as harmful dusts, liquids, vapors, and gases, or physical agents such as noise, ionizing radiation, and heat stress. Specialists also help with ventilation and electrical systems, roof support and ground control methods, mine waste facilities, equipment use, and many other aspects of the mining environment.

In addition, the staff of the Technology Center works to keep the mining industry current with state-of-the-art information on mine safety and health issues. MSHA regularly sponsors lectures, seminars, and training classes, and publishes reports on the latest scientific and technical information.

The following are ways to contact MSHA for more information on MSHA technical support programs.

- MSHA line to report emergencies: (800) 746-1553
- MSHA's Office of the Assistant Secretary: (202) 693-9414
- MSHA's Directorate of Technical Support: (202) 693-9470
- MSHA's Pittsburgh Safety and Health Technology Center: (412) 386-6902
- MSHA's Approval and Certification Center: (304) 547-0400

For state mining agencies, see Chapter 26 or check the MSHA website (MSHA, n.d.).

TESTING PRODUCTS USED IN MINING

MSHA investigates and tests a wide range of mining equipment, components, instruments, and materials to ensure that they meet government standards for safe design and construction. This work helps to ensure that the various products do not contribute to an explosion, fire, electrical failure, vehicle crash, or other kind of accident.

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The extensive list of mining products that MSHA must approve includes equipment such as multi-ton loading scoops, electrical cable and splice kits, panic bar designs, fire-resistant hydraulic fluids, vehicle braking systems, conveyor belts, diverse kinds of electrical equipment, hoses, explosives, mine illumination systems, and monitoring devices such as methane gas detectors. These products are used in coal, metal, and nonmetal mines.

MSHA conducts these tests and investigations at the Approval and Certification Center near Wheeling, West Virginia. Laboratories, a test track, explosion galleries, and offices for administrative work and records keeping make up the center.

MANDATORY TRAINING

The Federal Mine Safety and Health Amendments Act of 1977 recognizes training as an important tool for preventing accidents and avoiding unsafe and unhealthy working conditions. The Act authorizes MSHA to "expand programs of education and training of operators…and miners…."

MSHA requires that each U.S. mine operator have an approved plan for miner training. This plan must include the following:

- Forty hours of basic safety and health training for new miners who have no underground mining experience before they begin work underground
- Twenty-four hours of basic safety and health training for new miners who have no surface mining experience before they begin work at surface mining operations
- Eight hours of refresher safety and health training for all miners each year
- Safety-related task training for miners assigned to new jobs.

For more information on MSHA training programs, contact

- Director, Educational Policy and Development, MSHA Headquarters, Arlington, Virginia: (202) 693-9570; and/or
- Superintendent, National Mine Health and Safety Academy, Beckley, West Virginia, (304) 256-3241.

HISTORICAL DATA ON MINE DISASTERS IN THE UNITED STATES

Table 23.2 shows accidents in the United States with five or more fatalities since 1970.

U.S. MINE SAFETY SUCCESS

The mining industry has achieved success in reducing mining fatalities in the last 120 years. In 1911, 3,539 miners lost their lives; in 2017, 28 miners lost their lives. Figure 23.1 gives a visual of the descending trend.

The NAICS code stands for *North American Industry Classification System* and is the standard used by federal statistical agencies in classifying business establishments for collecting, analyzing, and publishing statistical data related to the U.S. business economy. According to 2016 statistics, mining is the safest industrial occupation except for oil and gas extraction. Table 23.3 shows the U.S. Bureau of Labor industry safety statistics.

Incidence rates are calculated by the Bureau of Labor as the number of injuries and illnesses per 100 full-time workers:

$$\frac{N}{EH} \times 200{,}000 \qquad \textbf{(EQ 23.1)}$$

where

N = number of injuries and illnesses
EH = total hours worked by all employees during the calendar year
200,000 = base for 100 equivalent full-time workers, working 40 hours per week, 50 weeks per year

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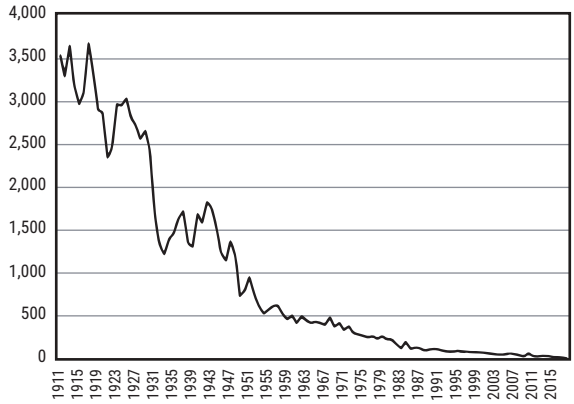
TABLE 23.2 Accidents in the United States*

Year	Month and Day	Mine	Location	Type	Deaths
2010	April 5	Upper Big Branch	Montcoal, WV	Explosion	29
2007	August 6	Crandall Canyon	Huntington, UT	Cave-in	9
2006	January 2	Sago	Tallmansville, WV	Explosion	12
2006	May 20	Darby Miner No. 1	Holmes Mill, KY	Explosion	5
2001	September 23	JWR No. 5	Brookwood, AL	Explosion	13
1992	December 7	South Mountain No. 3	Norton, VA	Explosion	8
1989	September 13	William Station	Wheatcroft, KY	Explosion	10
1986	February 6	Loveridge No. 22	Fairview, WV	Suffocation (surface stockpile)	5
1984	December 19	Wilberg	Orangeville, UT	Fire	27
1983	June 21	McClure No. 1	McClure, VA	Explosion	7
1982	January 20	No. 1	Craynor, KY	Explosion	7
1981	December 8	No. 21	Whitwell, TN	Explosion	13
1981	December 7	No. 11	Kite, KY	Explosion	8
1981	April 15	Dutch Creek No. 1	Redstone, CO	Explosion	15
1980	November 7	Ferrell No. 17	Uneeda, WV	Explosion	5
1979	June 08	Belle Isle (salt)	Franklin, LA	Explosion	5
1978	April 4	Moss No. 3 Portal A	Duty, VA	Suffocation (oxygen deficiency)	5
1977	March 1	Porter Tunnel	Tower City, PA	Inundation	9
1976	March 9	11 Scotia	Oven Fork, KY	Explosion	26
1972	December 16	Itmann No. 3	Itmann, WV	Explosion	5
1972	July 22	Blacksville No. 1	Blacksville, WV	Fire	9
1972	May 2	Sunshine (silver)	Kellogg, ID	Fire	91
1971	April 12	Barnett Complex (fluorspar)	Rosiclare, IL	Hydrogen sulfide gas	7
1970	December 30	Nos. 15 and 16	Hyden, KY	Explosion	38

* Accidents with five or more fatalities since 1970 in reverse chronological order.

Source: Compiled from records and documents of
U.S. Bureau of Mines, Mining Enforcement and Safety Administration (MESA),
Mine Safety and Health Administration (MSHA), and the U.S. Mine Rescue Association



Source: MSHA, n.d.

FIGURE 23.1 Total U.S. mine fatalities

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TABLE 23.3 Occupational safety statistics in ascending order of incidence rates*

Industry	NAICS Sub-Code	Total for Lost Time, Job Transfer, or Restriction
Finance and insurance	52	0.2
Professional, scientific, and technical services	54	0.3
Oil and gas extraction	211	0.4
Management of companies and enterprises	55	0.4
Finance, insurance, and real estate	—	0.5
Information	51	0.7
Utilities	22	1.2
Other services (except public administration)	81	1.2
Administrative and support and waste management and remediation	56	1.4
Real estate and rental and leasing	53	1.4
Accommodation and food services	72	1.4
Service-providing	—	1.5
Mining (except oil and gas)	**212**	**1.6**
All industries including private, state and local government	—	1.7
Educational and health services	—	1.8
Construction	23	1.9
State government	—	1.9
Health care and social assistance	62	1.9
Retail trade	44–45	2.0
Manufacturing	31–33	2.1
Trade, transportation, and utilities	—	2.2
Arts, entertainment, and recreation	71	2.2
Local government	—	2.3
Transportation and warehousing	48–49	3.3
Agriculture, forestry, fishing and hunting	11	3.6

* Ascending order of occupational injuries and illnesses incidence rates. Source: U.S. Bureau of Labor Statistics 2017

This method is also used by MSHA and the U.S. mining industry. Mines have achieved safety success by treating safety as a management system, emphasizing safety as a core value, and implementing risk-based safety programs. A mine may embrace any or all of the following programs.

- Workplace inspection program by the miner, including risk analysis
- Risk analysis and management
- Workplace inspection program by others
- Law enforcement
- Mine engineering, ventilation, roof control
- Safe work instruction program
- Task analysis and training
- Cross-training
- Competency verification
- Safe work incentives and awards
- Personal protective equipment with continuous improvement
- Incident investigation analysis
- Accident investigation
- Miss and near-miss prioritization, loss control
- Safety contact program
- Behavioral-based safety
- Executive management assigning value
- Responsibility and accountability program (miner, supervisor, engineer)
- Equipment design with regard to safety
- Occupational health
- Occupational hygiene
- Emergency and crisis management, preparation, planning, and response
- Regulatory compliance, minimum standard
- Safety data analysis
- Ethics

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An excellent training resource is found at CoreSafety (n.d.). This is a thorough mine safety training program compiled by the National Mining Association.

GENERAL SAFETY

Safety is defined as the condition of being safe from undergoing or causing hurt, injury, or loss. Safety is protecting people from threats or failures that could cause harm to their physical, emotional, or financial well-being. Achieving safety involves the control of known threats to attain some level of acceptable risk.

Hazard

A hazard is any capacity to cause harm to a person or an object. A hazard is an inherent characteristic of a material, condition, machine, or process.

Risk

Risk is the chance or probability that a person or entity will be harmed by a particular hazard. In safety-related analyses, risk is expressed as

$$\text{risk} = \text{hazard} \times \text{probability} \quad \textbf{(EQ 23.2)}$$

In health-related analyses, risk is typically expressed as

$$\text{risk} = \text{hazard} \times \text{exposure} \quad \textbf{(EQ 23.3)}$$

Risk Assessment

Risk assessment is a method to evaluate the relative consequences posed by a specific hazard. Figure 23.2 shows a typical risk assessment matrix. For any given hazard, the consequences can range from insignificant with no injuries, financial loss, or disruptions to catastrophic with fatalities, massive financial losses, or business closure. The likelihood that the specific hazard can occur ranges from rare to near certainty. The level of risk posed by this hazard depends on the product of likelihood and consequence. Low-likelihood, low-consequence events pose little risk as shown in green, and high-likelihood, high-consequence events pose high risk shown in red. The likelihood and consequence scales are subjective and vary with each hazard considered.

Job Safety Analysis

Job safety analysis (also called *activity hazard analysis* or *job hazard analysis*) is a method to help integrate safety and health principles and practices into a particular job. Each basic step in a job or task is reviewed in detail commensurate with the hazards associated with the job or task. All potential hazards are identified and procedures are developed and documented to describe the safest, lowest-risk method to complete the job or task. Following are typical hazards that the job safety analysis might consider:

- Can the person make injurious contact with the object, causing possible traumatic injury?
- Can the person become entangled in an object or objects?
- Can the person strain muscles through exertion caused by lifting, pulling, or pushing?
- Is the person exposed to harmful gases, vapors, dust, electricity, or radiation?

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Likelihood	Consequences				
	Insignificant (1) No injuries No monetary loss No disruptions	Minor (2) First aid treatment Low monetary loss Some disruption	Moderate (3) Medical treatment Moderate monetary loss Moderate disruption	Major (4) Hospitalization High monetary loss Severe disruption	Catastrophic (5) Death(s) Massive monetary loss Massive disruption
Near Certain (5) Occurs often Once a week >90% chance	Moderate (5)	High (10)	High (15)	Catastrophic (20)	Catastrophic (25)
Likely (4) Occurs less often Once a month 50%–90% chance	Moderate (4)	Moderate (8)	High (12)	Catastrophic (16)	Catastrophic (20)
Moderate (3) Could happen Once a year 10%–50% chance	Low (3)	Moderate (6)	Moderate (9)	High (12)	High (15)
Unlikely (2) Has not happened Once in 10 years 1%–10% chance	Low (2)	Moderate (4)	Moderate (6)	Moderate (8)	High (10)
Rare (1) Conceivable Once in 100 years <1% chance	Low (1)	Low (2)	Low (3)	Moderate (4)	Moderate (5)

Source: FEMA, n.d.

FIGURE 23.2 Risk assessment matrix

Incidence Rates and Incidence Variable Values

Two concepts can be important when completing Occupational Safety and Health Administration (OSHA) forms. These concepts are incidence rates (IRs) and severity rates. On occasion, it is necessary to calculate the total injury/illness IR of an organization in order to complete OSHA forms. This calculation must include fatalities and all injuries requiring medical treatment beyond mere first aid. The formula for determining the total injury/illness IR is as follows:

$$IR = N \times 200{,}000 \div T \quad \text{(EQ 23.4)}$$

where

IR = total injury/illness incidence rate
N = number of injuries, illnesses, and fatalities
T = total hours worked by all employees during the period in question

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The number 200,000 in the formula represents the number of hours 100 employees work in a year (40 hours per week × 50 weeks = 2,000 hours per year per employee). Using the same basic formula with only minor substitutions, safety managers can calculate the following types of IRs:

- Injury rate
- Illness rate
- Fatality rate
- Lost workday cases rate
- Number of lost workdays rate
- Specific hazard rate
- Lost workday injuries rate

Safety Data Sheets

The Safety Data Sheet (SDS) provides comprehensive information for use in workplace chemical management. Employers and workers use the SDS as a source of information about hazards and to obtain advice on safety precautions. The SDS is product related and usually is not able to provide information that is specific for any given workplace where the product may be used. However, the SDS information enables the employer to develop an active program of worker protection measures, including training, which is specific to the individual workplace, and to consider any measures that may be necessary to protect the environment. Information in an SDS also provides a source of information for those involved with the transport of dangerous goods, emergency responders, poison centers, those involved with the professional use of pesticides, and consumers.

The Globally Harmonized System of Classification and Labeling of Chemicals (GHS) and SDS have 16 sections in a set order, and minimum information is prescribed. The Hazard Communication Standard (HCS) requires chemical manufacturers, distributors, or importers to provide an SDS to communicate the hazards of hazardous chemical products. Currently, HCS requires new SDSs to be in a uniform format, and include the section numbers, headings, and associated information under the headings as shown in Table 23.4.

TABLE 23.4 Mandated Safety Data Sheet section titles

Section 1	Identification: Includes product identifier; manufacturer or distributor name, address, phone number; emergency phone number; recommended use; restrictions on use
Section 2	Hazard(s) identification: Includes all hazards regarding the chemical; required label elements
Section 3	Composition/information on ingredients: Includes information on chemical ingredients; trade secret claims
Section 4	First aid measures: Includes important symptoms/effects, acute, delayed; required treatment
Section 5	Fire-fighting measures: Lists suitable extinguishing techniques, equipment; chemical hazards from fire
Section 6	Accidental release measures: Lists emergency procedures; protective equipment; proper methods of containment and cleanup
Section 7	Handling and storage: Lists precautions for safe handling and storage, including incompatibilities
Section 8	Exposure controls/personal protection: Lists the Occupational Safety and Health Administration (OSHA)'s permissible exposure limits (PELs); threshold limit values (TLVs); appropriate engineering controls; personal protective equipment (PPE)
Section 9	Physical and chemical properties: Lists the chemical's characteristics
Section 10	Stability and reactivity: Lists chemical stability and possibility of hazardous reactions
Section 11	Toxicological information: Includes routes of exposure; related symptoms, acute and chronic effects; numerical measures of toxicity
Section 12	Ecological information*
Section 13	Disposal considerations*
Section 14	Transport information*
Section 15	Regulatory information*
Section 16	Other information: Includes the date of preparation or last revision

* Because other agencies regulate this information, OSHA will not be enforcing Sections 12 through 15 (29 CFR 1910.1200(g)(2)).

Source: OSHA 2005

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Signal Words

The signal word found on every product's label is based on test results from various oral, dermal, and inhalation toxicity tests, as well as skin and eye corrosion assays in some cases. Signal words are placed on labels to convey a level of care that should be taken (especially personal protection) when handling and using a product, from purchase to disposal of the empty container, as demonstrated by the Table 23.5.

TABLE 23.5 Pesticide toxicity categories

Signal Word on Label	Toxicity Category	Acute-Oral LD_{50} for Rats*	Amount Needed to Kill an Average-Size Adult	Notes/Labels
Danger–Poison	Highly toxic	50 or less	Taste to a teaspoon	Skull and crossbones; Keep Out of Reach of Children
Warning	Moderately toxic	50–500	1–6 teaspoons	Keep Out of Reach of Children
Caution	Slightly toxic	500–5,000	1 ounce to a pint	Keep Out of Reach of Children
Caution	Relatively nontoxic	>5,000	More than a pint	Keep Out of Reach of Children

* LD_{50}: see the "Risk Assessment and Toxicology" section of this chapter. Source: EPA, n.d.

Flammability

Flammable describes any solid, liquid, vapor, or gas that will ignite easily and burn rapidly. A flammable liquid is defined by the National Fire Protection Association (NFPA) and the U.S. Department of Transportation (USDOT) as a liquid with a flash point below 38°C (100°F). Flammability is further defined with lower and upper limits:

LFL = lower flammability limit (volume percent in air)
UFL = upper flammability limit (volume percent in air)

The LFL is also known as the *lower explosive limit* (LEL). The UFL is also referred to as the *upper explosive limit* (UEL). There is no difference between the terms flammable and explosive as applied to the lower and upper limits of flammability.

A vapor–air mixture will only ignite and burn over the range of concentrations between LFL and UFL. Examples are shown in Table 23.6.

Predicting Lower Flammable Limits of Mixtures of Flammable Gases (Le Chatelier's Rule)

Based on an empirical rule developed by Le Chatelier, the lower flammable limit of mixtures of multiple flammable gases in air can be determined. A generalization of Le Chatelier's rule is

$$\sum_{i=1}^{n} \frac{C_i}{LFL_i} \geq 1 \qquad \text{(EQ 23.5)}$$

where C_i is the volume percent of fuel gas i in the fuel/air mixture, and LFL_i is the volume percent of fuel gas i at its lower flammable limit in air alone. If the indicated sum is greater than unity, the mixture is above the lower flammable limit. This can be restated in terms of the lower flammable limit concentration of the fuel mixture, LFL_m, as shown in the following equation and Figure 23.3:

$$LFL_m = \frac{100}{\sum_{i=1}^{n} \frac{C_i}{LFL_i}} \qquad \text{(EQ 23.6)}$$

where C_{fi} is the volume percent of fuel gas i in the fuel gas mixture.

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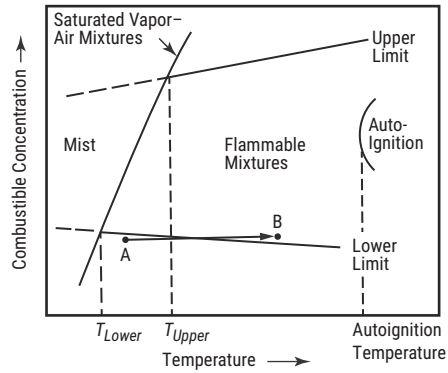
TABLE 23.6 Flammability limits

Gas/Compound	Lower Flammability Limit (LFL)	Upper Flammability Limit (UFL)
Acetone	2.6	13.0
Acetylene	2.5	100.0
Ammonia	15.0	28.0
n-Butane	1.8	8.4
Carbon disulfide	1.3	50.0
Carbon monoxide	12.5	74.0
Cycloheptane	1.1	6.7
Cyclohexane	1.3	7.8
Cyclopropane	2.4	10.4
Diethyl ether	1.9	36.0
Ethane	3.0	12.4
Ethyl acetate	2.2	11.0
Ethyl alcohol	3.3	19.0
Ethyl ether	1.9	36.0
Ethyl nitrite	3.0	50.0
Ethylene	2.7	36.0
Gasoline 100/130	1.3	7.1
Gasoline 115/145	1.2	7.1
Hydrazine	4.7	100.0
Hydrogen	4.0	75.0
Hydrogen sulfide	4.0	44.0
Isobutane	1.8	8.4
Methane	5.0	15.0
Propane	2.1	9.5

Source: Dinenno et al. 2008



Source: SFPE/NFPA 1988

FIGURE 23.3 Effect of temperature on limits of flammability of a combustible vapor in air at constant initial pressure

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Granular Storage and Process Safety

Some materials that are not inherently hazardous can become hazardous during storage or processing. An example is the handling of grain in grain bins. Grain bins should not be entered when the grain is being removed because grains flow to the center of the emptying bin and create suffocation hazards. Bridging may occur at the top surface because of condensation and resulting spoilage creating a crust.

Organic vapors and dusts associated with grain handling often contain toxic yeasts or molds and have low oxygen contents. These organic vapors and dusts can also be explosive.

Confined Space Safety

Many workplaces contain spaces that are considered *confined* because their configurations hinder the activities of employees who must enter, work in, and exit them. A confined space has limited or restricted means for entry or exit and is not designed for continuous employee occupancy. Confined spaces include, but are not limited to, underground vaults, tanks, storage bins, manholes, pits, silos, process vessels, and pipelines.

OSHA uses the term "permit-required confined spaces" (permit space) to describe a confined space that has one or more of the following characteristics:

- Contains or has the potential to contain a hazardous atmosphere
- Contains a material that has the potential to engulf an entrant
- Has walls that converge inward or floors that slope downward and taper into a smaller area that could trap or asphyxiate an entrant
- Contains any other recognized safety or health hazard such as unguarded machinery, exposed live wires, or heat stress

OSHA has developed standards, directives (instructions for compliance officers), standard interpretations (official letters of interpretation of the standards), and national consensus standards related to confined spaces. The following gases are often present in confined spaces:

- **Ammonia.** Irritating at 50 ppm and deadly above 1,000 ppm with a sharp, cutting odor
- **Hydrogen sulfide.** Irritating at 10 ppm and deadly at 500 ppm; accumulates at lower levels and in corners where circulation is minimal with a rotten egg odor
- **Methane.** Explosive at levels above 50,000 ppm, lighter than air, odorless
- **Carbon dioxide.** Heavier than air, accumulates at lower levels and in corners where circulation is minimal, displaces air leading to asphyxiation

RISK ASSESSMENT AND TOXICOLOGY

Dose is expressed as the following:

$$\frac{\text{mass of chemical}}{\text{body weight} \times \text{exposure time}} \qquad \textbf{(EQ 23.7)}$$

Electrical Safety

Table 23.7 lists the probable effect on the human body of different electrical current levels.

Dose–Response Curves

The dose–response curve relates toxic response (i.e., percentage of test population exhibiting a specified symptom or dying) to the logarithm of the dosage (i.e., mg/kg·day ingested). A typical dose–response curve is shown in Figure 23.4.

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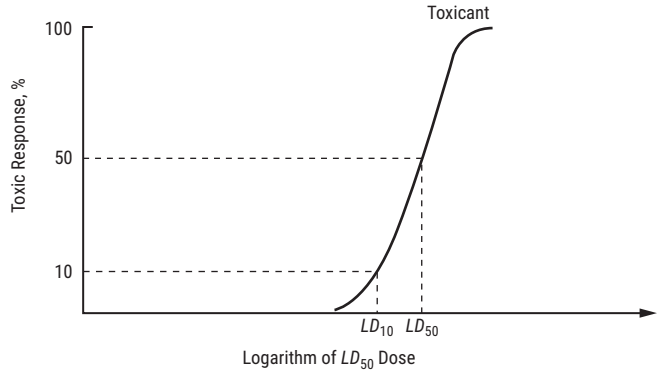
TABLE 23.7 Electrical safety

Current Level, mA	Probable Effect on Human Body
1	Perception level; slight tingling sensation; still dangerous under certain conditions
5	Slight shock felt; not painful but disturbing. Average individual can let go. However, strong involuntary reactions to shocks in this range may lead to injuries.
6–16	Painful shock, begin to lose muscular control; commonly referred to as the freezing current or "let-go" range
17–99	Extreme pain, respiratory arrest, severe muscular contractions. Individual cannot let go. Death is possible.
100–2,000	Ventricular fibrillation (uneven, uncoordinated pumping of the heart). Muscular contraction and nerve damage begin to occur. Death is likely.
>2,000	Cardiac arrest (stop in effective blood circulation), internal organ damage, and severe burns. Death is probable.

Source: NIOSH 1998; Greenwald 1991



Source: NCEES 2013

FIGURE 23.4 Dose–response curve

LC_{50}

LC_{50} represents the median lethal concentration in air that, based on laboratory tests, is expected to kill 50% of a group of test animals when administered as a single exposure over 1 or 4 hours.

LD_{50}

LD_{50} represents the median lethal single dose, based on laboratory tests, expected to kill 50% of a group of test animals, usually by oral or skin exposure. Similar definitions exist for LC_{10} and LD_{10}, where the corresponding percentages are 10%.

Table 23.8 gives toxic chemical lethal doses, Table 23.9 shows some chemical interaction effects, and Table 23.10 gives the exposure limits for selected compounds.

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TABLE 23.8 Comparative acutely lethal doses

Actual Ranking Number	LD_{50}, mg/kg	Toxic Chemical
1	15,000	Polychlorinated biphenyls (PCBs)
2	10,000	Alcohol (ethanol)
3	4,000	Table salt—sodium chloride
4	1,500	Ferrous sulfate—an iron supplement
5	1,375	Malathion—pesticide
6	900	Morphine
7	150	Phenobarbital—a sedative
8	142	Tylenol (acetaminophen)
9	2	Strychnine—a rat poison
10	1	Nicotine
11	0.5	Curare—an arrow poison
12	0.001	2,3,7,8-TCDD (dioxin)
13	0.00001	Botulinum toxin (food poison)

Adapted from Loomis and Hayes 1996

TABLE 23.9 Selected chemical interaction effects

Effect	Relative Toxicity (hypothetical)	Example
Additive	2 + 3 = 5	Organophosphate pesticides
Synergistic	2 + 3 = 20	Cigarette smoking + asbestos
Antagonistic	6 + 6 = 8	Toluene + benzene or caffeine + alcohol

Adapted from Williams et al. 2000

TABLE 23.10 Exposure limits for selected compounds

N	Allowable Workplace Exposure Level, mg/m^3	Chemical (use)
1	0.1	Iodine
2	5	Aspirin
3	10	Vegetable oil mists (cooking oil)
4	55	1,1,2-Trichloroethane (solvent/degreaser)
5	188	Perchloroethylene (dry-cleaning fluid)
6	170	Toluene (organic solvent)
7	269	Trichloroethylene (solvent/degreaser)
8	590	Tetrahydrofuran (organic solvent)
9	890	Gasoline (fuel)
10	1,590	Naphtha (rubber solvent)
11	1,910	1,1,1-Trichloroethane (solvent/degreaser)

Source: NIOSH 2007

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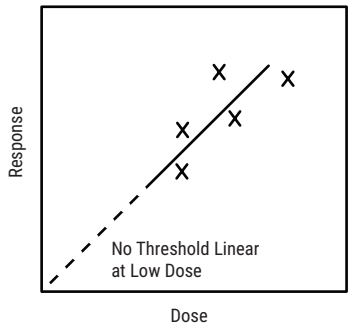
Carcinogens

For carcinogens, the U.S. Environmental Protection Agency (EPA) considers an acceptable risk to be within the range of 10^{-4} to 10^{-6}. The added risk of cancer is calculated as follows:

$$\text{risk} = \text{dose} \times \text{toxicity} = CDI \times CSF \quad \textbf{(EQ 23.8)}$$

where *CDI* is chronic daily intake, and *CSF* is the cancer slope factor. Figure 23.5 shows the slope of the dose–response curve for carcinogenic materials.



Source: NCEES 2013

FIGURE 23.5 Carcinogenic dose–response curve slope

Threshold Limit Value

The threshold limit value (TLV) is the highest dose (parts per million by volume in the atmosphere) that the body is able to detoxify without any detectable effects. Examples are shown in Table 23.11.

Noncarcinogens

For noncarcinogens, a hazard index (*HI*) is used to characterize risk from all pathways and exposure routes. EPA considers an *HI* > 1.0 as representing the possibility of an adverse effect occurring (Figure 23.6).

$$HI = \frac{CDI_{\text{noncarcinogen}}}{RfD} \quad \textbf{(EQ 23.9)}$$

where $CDI_{\text{noncarcinogen}}$ is the chronic daily intake of a noncarcinogenic compound, and *RfD* represents the reference dose.

Reference Dose

Reference dose (*RfD*) is determined from the noncarcinogenic dose–response curve using no observable adverse effect level (NOAEL), the dose below which there are no harmful effects.

$$RfD = \frac{NOAEL}{UF} \quad \textbf{(EQ 23.10)}$$

and

$$SHD = RfD \times W = \frac{NOAEL \times W}{UF} \quad \textbf{(EQ 23.11)}$$

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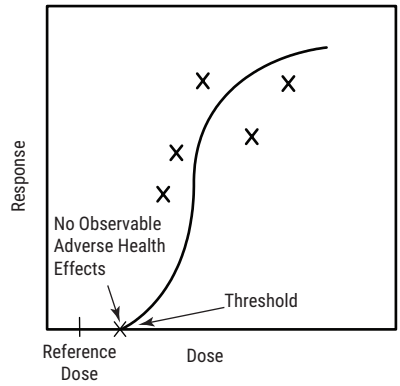
TABLE 23.11 Threshold limit value

Compound	Threshold Limit Value (TLV), ppm
Ammonia	25
Chlorine	0.5
Ethyl chloride	1,000
Ethyl ether	400

Source: NCEES 2013



Source: NCEES 2013

FIGURE 23.6 Noncarcinogenic dose–response curve

where

RfD = lifetime (i.e., chronic) dose that a healthy person can be exposed to daily without adverse effects
SHD = safe human dose, mg/day
NOAEL = threshold dose per kg of test animal from the dose–response curve, mg/kg·day
UF = total uncertainty factor, depending on nature and reliability of the animal test data
W = weight of the adult male, kg (lb)

Exposure

The following equations are used to find the residential exposure levels for various pathways (EPA 1989).

Ingestion in Drinking Water

$$CDI = \frac{(CW)(IR)(EF)(ED)}{(BW)(AT)} \quad \textbf{(EQ 23.12)}$$

Ingestion While Swimming

$$CDI = \frac{(CW)(CR)(ET)(EF)(ED)}{(BW)(AT)} \quad \textbf{(EQ 23.13)}$$

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Dermal Contact with Water

$$AD = \frac{(CW)(SA)(PC)(ET)(EF)(ED)(CF)}{(BW)(AT)} \quad \text{(EQ 23.14)}$$

Ingestion of Chemicals in Soil

$$CDI = \frac{(CS)(IR)(CF)(FI)(EF)(ED)}{(BW)(AT)} \quad \text{(EQ 23.15)}$$

Dermal Contact with Soil

$$AD = \frac{(CS)(CF)(SA)(AF)(ABS)(EF)(ED)}{(BW)(AT)} \quad \text{(EQ 23.16)}$$

Inhalation of Airborne (Vapor Phase) Chemicals

$$CDI = \frac{(CA)(IR)(ET)(EF)(ED)}{(BW)(AT)} \quad \text{(EQ 23.17)}$$

Ingestion of Contaminated Fruits, Vegetables, Fish, and Shellfish

$$CDI = \frac{(CF)(IR)(FI)(EF)(ED)}{(BW)(AT)} \quad \text{(EQ 23.18)}$$

where

ABS = absorption factor for soil contaminant, unitless
AD = absorbed dose, mg/kg·day
AF = soil-to-skin adherence factor, mg/cm^2
AT = averaging time, days
BW = body weight, kg
CA = contaminant concentration in air, mg/m^3
CDI = chronic daily intake, mg/kg·day
CF = volumetric conversion factor for water = 1 L/1,000 cm^3 = conversion factor for soil = 10^{-6} kg/mg
CR = contact rate, L/h
CS = chemical concentration in soil, mg/kg
CW = chemical concentration in water, mg/L
ED = exposure duration, years
EF = exposure frequency, d/yr or events/year
ET = exposure time, h/day or h/event
FI = fraction ingested, unitless
IR = ingestion rate, L/day or mg soil/day or kg/meal = inhalation rate, m^3/h
PC = chemical-specific dermal permeability constant, cm/h
SA = skin surface area available for contact, cm^2

Table 23.12 shows the recommended values for estimating intake per the EPA.

Concentrations of Vaporized Liquids

Vaporization Rate from a Liquid Surface

The vaporization rate, Q_m (mass/time) is determined by

$$Q_m = \frac{MKA_S P^{\text{sat}}}{R_g T_L} \quad \text{(EQ 23.19)}$$

where

M = molecular weight of volatile substance
K = mass transfer coefficient
A_S = area of liquid surface
P^{sat} = saturation vapor pressure of the pure liquid at T_L
R_g = ideal gas constant
T_L = absolute temperature of the liquid

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TABLE 23.12 Intake rates—variable values

Parameter	Standard Value
Average body weight, female adult	65.4 kg
Average body weight, male adult	78 kg
Average body weight, child*	
6–11 mo	9 kg
1–5 year	16 kg
6–12 year	33 kg
Amount of water ingested, adult	2.3 L/day
Amount of water ingested, child	1.5 L/day
Amount of air breathed, female adult	11.3 m^3/day
Amount of air breathed, male adult	15.2 m^3/day
Amount of air breathed, child (3–5 year)	8.3 m^3/day
Amount of fish consumed, adult	6 g/day
Water swallowing rate, while swimming	50 mL/h
Inhalation rates	
Adult (6-h day)	0.98 m^3/h
Adult (2-h day)	1.47 m^3/h
Child	0.46 m^3/h
Skin surface available, adult male	1.94 m^2
Skin surface available, adult female	1.69 m^2
Skin surface available, child	
3–6 year (average for male and female)	0.720 m^2
6–9 year (average for male and female)	0.925 m^2
9–12 year (average for male and female)	1.16 m^2
12–15 year (average for male and female)	1.49 m^2
15–18 year (female)	1.60 m^2
15–18 year (male)	1.75 m^2
Soil ingestion rate, child 1–6 year	>100 mg/day
Soil ingestion rate, persons >6 year	50 mg/day
Skin adherence factor, gardener's hands	0.07 mg/cm^2
Skin adherence factor, wet soil	0.2 mg/cm^2
Exposure duration	
Lifetime (carcinogens, for noncarcinogens use actual exposure duration)	75 year
At one residence, 90th percentile	30 year
National median	5 year
Averaging time	Exposure duration (ED) (365 d/yr)
Exposure frequency (EF)	
Swimming	7 d/yr
Eating fish and shellfish	48 d/yr
Oral ingestion	350 d/yr
Exposure time (ET)	
Shower, 90th percentile	12 min
Shower, 50th percentile	7 min

*Data in this category from Copeland et al. 1994.

Source: EPA 1989

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Mass Flow Rate of Liquid from a Hole in the Wall of a Process Unit

$$Q_m = A_H C_0 \sqrt{2\rho g P_g} \quad \text{(EQ 23.20)}$$

where

A_H = area of hole
C_0 = discharge coefficient
ρ = density of the liquid
g = gravitational constant
P_g = gauge pressure within the process unit

Concentration of Vaporized Liquid in Ventilated Space

The concentration in parts per million (C_{ppm}) is determined using

$$C_{ppm} = \frac{Q_m R_g T \times 10^6}{k Q_V P M} \quad \text{(EQ 23.21)}$$

where

T = absolute ambient temperature
k = non-ideal mixing factor
Q_V = ventilation rate
P = absolute ambient pressure

Sweep-Through Concentration Change in a Vessel

$$Q_V t = V \ln\left(\frac{C_1 - C_0}{C_2 - C_0}\right) \quad \text{(EQ 23.22)}$$

where

Q_V = volumetric flow rate
t = time
V = vessel volume
C_0 = inlet concentration
C_1 = initial concentration
C_2 = final concentration

ERGONOMICS

The following formula from NIOSH is used to determine the recommended weight limit (NCEES 2013):

$$RWL = 51\left(\frac{10}{H}\right)(1 - 0.0075|V - 30|)\left(0.82 + \frac{1.8}{D}\right)(1 - 0.0032A)(FM)(CM) \quad \text{(EQ 23.23)}$$

where

RWL = recommended weight limit, lb
H = horizontal distance of the hand from the midpoint of the line joining the inner ankle bones to a point projected on the floor directly below the load center, in.
V = vertical distance of the hands from the floor, in.
D = vertical travel distance of the hands between the origin and destination of the lift, in.
A = asymmetry angle, in degrees
FM = frequency multiplier (see Table 23.13)
CM = coupling multiplier (see Table 23.14)

Biomechanics of the Human Body

In conjunction with Figure 23.7, the following basic equations are used in ergonomic determination.

Basic Equations

$$H_x + F_x = 0 \quad \text{(EQ 23.24)}$$

$$H_y + F_y = 0 \quad \text{(EQ 23.25)}$$

$$H_z + W + F_z = 0 \quad \text{(EQ 23.26)}$$

$$T_{Hxz} + T_{Wxz} + T_{Fxz} = 0 \quad \text{(EQ 23.27)}$$

$$T_{Hyz} + T_{Wyz} + T_{Fyz} = 0 \quad \text{(EQ 23.28)}$$

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TABLE 23.13 Frequency multiplier table

F, min^{-1}	≤8 h/day		≤2 h/day		≤1 h/day	
	V < 30 in.	V ≥ 30 in.	V < 30 in.	V ≥ 30 in.	V < 30 in.	V ≥ 30 in.
0.2	0.85		0.95		1.00	
0.5	0.81		0.92		0.97	
1	0.75		0.88		0.94	
2	0.65		0.84		0.91	
3	0.55		0.79		0.88	
4	0.45		0.72		0.84	
5	0.35		0.60		0.80	
6	0.27		0.50		0.75	
7	0.22		0.42		0.70	
8	0.18		0.35		0.60	
9	0.00	0.15	0.30		0.52	
10	0.00	0.13	0.26		0.45	
11	0.00	0.00	0.00	0.23	0.41	
12	0.00	0.00	0.00	0.21	0.37	
13	0.00	0.00	0.00	0.00	0.00	0.34
14	0.00	0.00	0.00	0.00	0.00	0.31
15	0.00	0.00	0.00	0.00	0.00	0.28

Source: Waters et al. 1994

TABLE 23.14 Coupling multiplier table*

Container			Loose Part/Irregular Object	
Optimal Design		Not	Comfort Grip	Not
Optimal Handles or Cut-Outs	Not	Poor	Good	
Good	Flex Fingers	90 Degrees	Not	
	Fair		Poor	
Coupling	V < 75 cm (30 in.)		V ≥ 75 cm (30 in.)	
Good	1.00			
Fair	0.95			
Poor	0.90			

* Function of coupling of hands to load.

Source: Waters et al. 1994

$$T_{Hxy} + T_{Fxy} = 0 \quad \textbf{(EQ 23.29)}$$

The coefficient of friction μ and the angle α at which the floor is inclined determine the equations at the foot.

$$F_x = \mu F_z \quad \textbf{(EQ 23.30)}$$

With the slope angle α

$$F_x = \alpha F_z \cos \alpha \quad \textbf{(EQ 23.31)}$$

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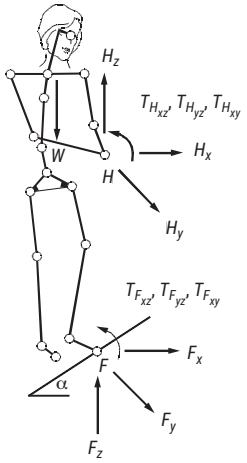

Source: NCEES 2013

FIGURE 23.7 Human body mechanics

Of course, when motion must be considered, dynamic conditions come into play according to Newton's second law. Force transmitted with the hands is counteracted at the foot. Further, the body must also react with internal forces at all points between the hand and the foot.

NOISE POLLUTION

$$SPL\,(\text{dB}) = 10\log_{10}\left(\frac{P_2}{P_0^2}\right) \qquad \textbf{(EQ 23.32)}$$

$$SPL_{\text{total}} = 10\log_{10}\sum 10^{SPL/10} \qquad \textbf{(EQ 23.33)}$$

The following equation is used for point source attenuation.

$$\Delta SPL\,(\text{dB}) = 10\log_{10}\left(\frac{r_1}{r_2}\right)^2 \qquad \textbf{(EQ 23.34)}$$

Line source attenuation is determined by

$$\Delta SPL\,(\text{dB}) = 10\log_{10}\left(\frac{r_1}{r_2}\right) \qquad \textbf{(EQ 23.35)}$$

where

SPL (dB) = sound pressure level, dB
P = sound pressure, Pa
P_0 = reference sound pressure, 2×10^{-5} Pa
SPL_{total} = sum of multiple sources
ΔSPL (dB) = change in sound pressure level with distance, dB
r_1 = distance from source to receptor at point 1
r_2 = distance from source to receptor at point 2

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TABLE 23.15 Noise level over time

Noise Level, dBA	Permissible Time, h
80	32
85	16
90	8
95	4
100	2
105	1
110	0.5
115	0.25
120	0.125
125	0.063
130	0.031

Source: NCEES 2013

Permissible Noise Exposure

OSHA has a permissible noise exposure standard: Noise dose D should not exceed 100% (Table 23.15).

$$D = 100\% \times \sum\left(\frac{C_i}{T_i}\right) \quad \textbf{(EQ 23.36)}$$

where

C_i = time spent at specified sound pressure level, SPL, h
T_i = time permitted at SPL, h
$\sum C_i = 8$ h

If $D > 100\%$, noise abatement is required. If $50\% \le D \le 100\%$, a hearing conservation program is required. $D = 100\%$ is equivalent to 90 dBA time-weighted average (TWA). $D = 50\%$ is equivalent to TWA of 85 dBA. dBA is defined as A-weighted decibels, a relative expression of loudness.

A hearing conservation program requires: (1) testing employee hearing, (2) providing hearing protection at employee's request, and (3) monitoring noise exposure. Exposure to impulsive or impact noise should not exceed 140 dB sound pressure level (SPL).

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CHAPTER

24

Environment and Reclamation

Briana Gunn, P.E.

Every country in the world has its own unique set of laws, acts, and regulations that define how mines facilities are to be developed, operated, and closed. This includes the resources that they impact (land, water, and air) as well as the materials (chemical and other substances) that they utilize for processing ore and concentrates. Although the information provided in this chapter is constantly changing, every effort has been made to furnish the reader with the most current facts.

DOMESTIC LAWS

Environmental laws governing mining have been in place in the United States for more than a hundred years. State and federal statutes apply to mining claims, exploration activities, mining and beneficiation operations, and closed and abandoned mine facilities. Government-approved permits are required for all new and ongoing mining operations, including exploration activities, and are designed to promote responsible mining by providing protections for water and air quality, historic sites, endangered species, and other public resources. Federal laws authorize and provide guidance and responsibilities to federal agencies, such as the U.S. Environmental Protection Agency (EPA) to protect resources on public lands and regulate mining operations. Some of these laws are summarized in Table 24.1.

TABLE 24.1 Federal laws* providing guidance and responsibilities to mining

Act	General Description
National Environmental Policy Act (NEPA)	NEPA, passed in 1969, outlines basic environmental policies as well as a process for evaluation and communicating environmental consequences such as permitting a new mine on federal lands. The process established by NEPA is used to define environmental considerations that should be included in decision making. NEPA applies to mining operations that require a federal approval (when the mines are located on federal land). Operations must prepare environmental impact statements (EISs) on federal actions that significantly affect the quality of the human environment. Federal agencies must also obtain comments and views from other federal, state, and local agencies, along with the public. The EIS is intended to develop, study, and describe alternatives to the recommended decision with clear reasons as to why the preferred alternative was selected.
Clean Air Act (CAA)	CAA, passed in 1970, is the comprehensive federal law that regulates air emissions from area, stationary, and mobile sources. This law authorizes the U.S. Environmental Protection Agency (EPA) to establish National Ambient Air Quality Standards (NAAQS) that protect public health and the environment.
Resource Conservation and Recovery Act (RCRA)	RCRA, passed in 1976, focuses on preventing the release of material classified as hazardous waste into the environment and provides guidance for management of these wastes from generation through disposal. Mining wastes that fall under high-volume, low-hazard wastes, such as milling and smelting wastes, are not regulated under RCRA; they are typically the responsibility of the governing state agencies.

(continues)

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TABLE 24.1 Federal laws* providing guidance and responsibilities to mining (continued)

Act	General Description
Clean Water Act (CWA)	CWA 33 U.S.C. § 1251 et seq (1977) is an amendment to the Federal Water Pollution Control Act (FWPCA) of 1972, which set the basic structure for regulating discharges of pollutants to U.S. waters. The law gave the EPA the authority to set effluent standards on an industry basis (technology based) and continued the requirements to set water quality standards for all contaminants in surface waters. The CWA makes it unlawful for any person to discharge any pollutant from a point source into navigable waters unless a permit (National Pollutant Discharge Elimination System [NPDES]) is obtained under the act. CWA allows the EPA to delegate many permitting, administrative, and enforcement aspects of the law to state governments. In states with the authority to implement CWA programs, the EPA retains oversight responsibilities.
	Section 404 In 1972, amendments to the FWPCA added what is commonly called Section 404 authority (33 U.S.C. § 1344) to the program. The secretary of the Army, acting through the chief of engineers, is authorized to issue permits, after notice and opportunity for public hearings, for the discharge of dredged or fill material into U.S. waters at specified disposal sites. Selection of such sites must be in accordance with guidelines developed by the EPA in conjunction with the secretary of the Army; these guidelines are known as the 404(b)(1) guidelines. The discharge of all other pollutants into U.S. waters is regulated under Section 402 of the Act, which supersedes the Section 13 permitting authority.
Toxic Substances Control Act (TSCA)	TSCA, passed in 1977, controls the development and application of new and existing chemical substances. These materials are often used in the processing of ore or concentrates, such as sodium cyanide used in leaching practices.
Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA)	CERCLA, passed in 1980 (42 U.S.C. § 9601), is commonly known as the *Superfund Act*. This law created a tax on the chemical and petroleum industries and provided broad federal authority to respond directly to releases or threatened releases of hazardous substances that may endanger public health or the environment. The Superfund Amendments and Reauthorization Act (SARA) of 1986 amended CERCLA and made several important changes and additions, stressing the importance of permanent remedies and innovative treatment technologies in cleaning up hazardous waste sites. SARA provided new enforcement authorities, increased state involvement, increased focus on human health problems posed by hazardous waste, and encouraged greater citizen participation in deciding how sites should be cleaned up.
Endangered Species Act (ESA)	ESA 7 U.S.C. § 136; 16 U.S.C. § 460 et seq (1973), provides a program for the conservation of threatened and endangered plants and animals and the habitats in which they are found. The U.S. Fish and Wildlife Service (FWS) maintains a list of 632 endangered species (326 are plants) and 190 threatened species (78 are plants). Species include birds, insects, fish, reptiles, mammals, crustaceans, flowers, grasses, and trees. Anyone can petition FWS to include a species on the list. The law prohibits any action, administrative or real, which results in a "taking" of a listed species or adversely affects habitat.
Surface Mining Control and Reclamation Act (SMCRA)	SMCRA of 1977 (30 U.S.C. § 1201 et seq) applies to active and abandoned surface and underground coal mines on private, state, federal, and Native American lands. SMCRA has two major provisions: (1) the regulation of active coal mines and (2) the reclamation of abandoned mines (Abandoned Mine Lands or AML). SMCRA is administered through grants to states under the concept of primacy, except for those states that choose not to exercise primacy and on Native American lands. Coal mines on federal lands are regulated under cooperative agreements with the primacy states. Abandoned mines are reclaimed through the AML Fund, which is based on fees on active coal mining. The AML program also responds to emergencies that result from hazards of past coal mining practices, such as landslides, coal fires, subsidence from underground mines, and unstable highwalls.

* Other laws that impact mining include the Rivers and Harbors Appropriations Act, the General Mining Act of 1872, National Historic Preservation Act, Federal Mine Safety and Health Act, Migratory Bird Treaty Act, Solid Waste Disposal Act, Federal Water Pollution Control Act, Safe Drinking Water Act, National Environmental Policy Act, and Federal Land Policy and Management Act.

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ENVIRONMENTAL REGULATIONS

Environmental regulations and standards that govern the mining industry outside the United States include laws, acts, and other legislations as well as international commissions. Table 24.2 provides an overview of some of the environmental permits and agencies that exist in major mining countries.

TABLE 24.2 Environmental regulations in Canada and select Latin American and Asia-Pacific countries

Country	General Description
Canada	The federal constitution of Canada (Constitution Act of 1867) distributes governmental power between the central federal authority, 10 provincial governments, and 3 territorial governments. Provinces and territories have jurisdiction over mineral resources within their political boundaries. However, the federal government retains responsibility for resources on federal lands and for the territories, with the exception of the Northwest Territories, which is managed under the Mackenzie Valley Resource Management Act (Mackenzie Valley Land and Water Board) and Nunavut, which is managed under the Nunavut Land Claims Agreement (Nunavut Wildlife Management Board, Nunavut Impact Review Board, Nunavut Water Board). All aspects of uranium mining are controlled by the Canadian Nuclear Safety Commission. In addition, the federal government has jurisdiction over sea coasts and inland fisheries, navigable waterways, and "Indians and lands reserved for Indians." Similar to the United States, Canada has regulations of industry standards for potential emissions that can impact the environment, which include the following: • Clean Air Act • Canadian Environmental Assessment Act • Canadian Environmental Protection Act • Fisheries Act – Metal Mining Effluent Regulations • Navigable Waters Protection Act • Migratory Birds Act
Chile	Basis for environmental regulation is found in the National Environmental Framework Law (NEFL) of 1994. NEFL established the National Environment Commission (CONAMA), which is the public entity that develops Chilean environmental regulations. The Ministry of Mining, specializing in the development of mining regulations, contributes to the development of environmental laws that specifically impact the mining industry. Other contributors to the environmental law landscape regarding mining issues are the National Geology and Mining Service, Environmental Assessment Service, and the Superintendence of Environmental Affairs.
Bolivia	All mining projects need an environmental license, granted by the Ministry of Environment and Water.
Mexico	Prospecting and exploration require a Preventive Report. This meets the requirements of Mexican official standard number 120-SEMARNAT-2010, which establishes the specifications for environmental protection for direct mining exploration activities. An active mining operation requires the submission of an environmental impact statement (MIA) and a request for authorization of change of use on forest land (CUSTF), or submission of a unified technical paper (DTU). The MIA or the DTU must contain a risk assessment. It is also necessary to prove compliance with the Official Norm, NOM-141. In case of the leaching of gold, silver, and copper, it is necessary to prove compliance with the NOM 155 and 159.
China	Environmental regulations in mining are continuing to progress. Companies financing mines are subject to environmental guidelines developed by the China Banking Regulatory Commission (CBRC), which encourage China's bank to promote *green* lending, supports the growth of a green economy, and improves management of environmental and social risks. Additionally, some banks have adopted polices for lending to high-polluting industries and for evaluating environmental and social risks prior to investing. Companies listed on the Shanghai Stock Exchange (SSE) and Shenzhen Stock Exchange (SZSE) are subject to disclosure regulations and guidelines, and both exchanges have issued guidelines on corporate social responsibility (CSR) reporting. The Ministry of Environmental Protection (MEP) and financial regulators collaborate on financing, credit, and policies.

(continues)

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TABLE 24.2 Environmental regulations in Canada and select Latin American and Asia-Pacific countries (continued)

Country	General Description
India	The Ministry of Mines is responsible for survey and exploration of all minerals (other than natural gas and petroleum) for mining and metallurgy of nonferrous metals such as aluminum, copper, zinc, lead, gold, nickel, etc., and for administration of the Mines and Minerals (Development and Regulation) Act, 1957 (MMDR Act) in respect of all mines and minerals other than coal and lignite. Environmental issues are primarily managed by five acts and associated rules, notifications, and framework. The regulations include reducing further damage to the environment from mining, conserving the environment, restoration/reclamation, and defining the authorities to administer the polices: 1. Water (Prevention and Control of Pollution) Act, 1974 and its amendments 2. Water (Prevention and Control of Pollution) Act, 1977 3. Water (Prevention and Control of Pollution) Act, 1981 4. Environmental (Protection) Act, 1986 5. Public Liability Insurance Act, 1991
Australia	The Department of Mines and Petroleum (DMP) is responsible for development of Western Australia's (WA) mineral, petroleum, and geothermal resources. Environmental approvals for mining activities are granted in accordance with the Mining Act 1978 and include the following: • Programs of work, which are submitted when a company or prospector wants to explore for minerals, usually on a small scale • Mining proposals, which are submitted by a company for a proposed mining operation and accompanied by a mining lease application • Mine closure plans, which are submitted by a company as part of a mining proposal and covering all aspects of mine decommissioning and rehabilitation
Brazil	The Federal Constitution of Brazil establishes the framework for mining legislation as a federal interest in Brazil and declares that mining is subject to environmental licensing. The Mining Code of Brazil (Decree-law No. 227 of 1967) defines and classifies mineral deposits and mines; sets requirements and conditions for obtaining exploration authorizations, mining concessions, and other licenses, and permits; as well as the rights and responsibilities of holders of exploration authorizations and mining concession. In transition from exploration authorization to mining concession, a mining project must pass through a three-stage environmental licensing process in the state jurisdiction, the stages of which are (1) preliminary license, (2) installation license, and (3) operation license. The environmental licensing process is only executed at the federal level for projects that can impact the federal jurisdiction, such as projects in national environmental conservation units or projects located within two or more states. A mining concession is granted through the Brazilian Ministry of Mines and Energy. The holder of a mining concession must commence development within 180 days of grant, subject to obtaining all required environmental licenses and authorizations.

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TABLE 24.3 Regulatory agencies, guidance, and standards for waste-management facilities*

Region/State/Country	Regulatory Agencies, Guidance, and Standards
Canada	Mining regulations are controlled by the provinces with the exception of uranium, which is regulated by the federal government. The Mining Association of Canada (MAC), as well as the Canadian Dam Association (CDA), provides guidance and regulations on the design, operation, and construction of dams and other impoundments.
Australia	Mining regulations are controlled by the individual states. In Western Australia, this includes the Department of Mines, Industry Regulation and Safety (DMIRS) that implements the Mining Act, Mining Act Regulations, Mines Safety and Inspection Act, and Mines Safety and Inspection Regulations that regulate the safety and environmental impacts of waste disposal. In Victoria, the Mining and Petroleum Division (MPD) is responsible for regulation of mineral, petroleum, and extractive offshore waters. In Queensland, waste management is regulated under the Environmental Protection Act. In Tasmania, the Minerals Resources Development Act, the Assessment Committee for Dam Construction, and the Environmental Management Pollution Control Act provide guidance for waste facilities. In South Australia, there are no specific regulations for tailings impoundments, but regulations may exist for other environmental management.
South Africa	Mining is regulated by the Water Act of 1998, the Mineral Act of 1991, and the Mine Health and Safety Act of 1996. The Department of Minerals and Energy (DME) is responsible for enforcing provisions of the act.
Chile	Chile Servicio de Minas del Estado is responsible for looking at the safety of tailings management. Supreme Court Decree No. 86, "The Construction and Operation of Tailings Impoundments," provides best management requirements.
United States	Mining regulations are controlled by states when mines are on private land and by federal agencies on public land (National Environmental Policy Act [NEPA]). As a consequence, the regulations controlling impoundments are varied. The U.S. Environmental Protection Agency (EPA) has some specific guidance on design, management, and closure of these facilities.
Organizations	International organizations include International Committee on Large Dams (ICOLD), Australian Committee on Large Dams (ANCOLD), and U.S. Committee on Large Dams (USCOLD).

* Facilities include dams, waste rock, and stockpiles.

Documentation exists related to the design, construction operation, and closure of waste management facilities. This varies by state, country, and material deposited. Table 24.3 presents information on mining regulations for Canada, Australia, South Africa, Chile, and the United States.

PERMITS AND APPROVALS

In the United States, permits or operating licenses are required from exploration through closure of a facility. Environmental impact statements (EISs), as described in Table 24.1, are required for mine sites permitted on federal land. Other federal permits are site specific and may include but are not limited to Clean Water Act Section 404 permits, National Pollutant Discharge Elimination System (NPDES) permits, and Title V Air Quality permits. Table 24.4 outlines the environmental permits and approval for mines within the United States.

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TABLE 24.4 Domestic environmental permits and approvals required for projects*

Agency	Permit(s)
Federal	
U.S. Department of the Interior (DOI), Bureau of Land Management (BLM), and/or U.S. Department of Agriculture (USDA) Forest Service	Environmental impact statement Plan of operations Archeological clearance Right-of-way for utility corridors, roadways, etc. Mineral materials sale
DOI Fish and Wildlife Service	Endangered Species Act compliance Bald Eagle Protection Act compliance
Department of Labor, Mine Safety and Health Administration (MSHA)	MSHA mine identification, number Fire evacuation and rescue plans Notice of state of operations Miner training plan
U.S. Environmental Protection Agency (EPA)	Hazardous waste generator identification number Stormwater pollution prevention plan (may be delegated to state) Spill prevention, control, and countermeasures plan National Pollutant Discharge Elimination System (NPDES) permit (may be delegated to state) Clean Air Act Title V permit Toxic release inventory reporting
U.S. Army Corps of Engineers	Clean Water Act (CWA) Section 404 permit
Department of Justice, Bureau of Alcohol, Tobacco, Firearms and Explosives	Permit to purchase and store explosives Explosives inventory and use reports
State†	
Mining authority (i.e., for Colorado Division of Reclamation, Mining and Safety [DRMS])	Plan of operations or reclamation plan Environmental impact report (or equivalent)
Water quality control authority	NPDES permit Section 401 (CWA) certification Groundwater protection permit Waste disposal permit (leach pads, tailings, etc.)
State historic preservation office	Archaeological clearance
State occupation health and safety authority	Notice of mine opening Injury and illness prevention program Hazardous materials communication plan
State engineer	Water rights Dam safety permit
Air pollution control authority	Authority to construct Permit to operate Air toxics emission inventory plan (California only)
State lands commission	Mining lease or permits Water well lease
Fish and game authority	Stream alteration permit State Endangered Species Act compliance Artificial industrial pond permit
Local‡	
Various	Condition use permit Mining reclamation permit Hazardous materials business plan Risk management prevention program Acutely hazardous materials registration Domestic water system permit Sewage disposal permit Water well permits

* Not all permits are required for every project; permits are dependent on location, physical and environmental concerns, project requirements, and design.
† State regulations vary as well as regulatory agencies. Some facilities are not considered as jurisdictional in all states.
‡ Local permits vary, and this list is just an example of general permits that may be required.

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INTERNATIONAL GUIDELINES

International standards are different than specific standards in other countries. This section describes the environmental guidelines that come into play to support operations in underdeveloped countries or where there is otherwise limited governmental capacity to evaporate environmental risk related to proposed operations. Table 24.5 describes a few select organizations that provide international guidance.

TABLE 24.5 Select international guidelines

Guideline	General Description
World Bank/International Finance Corporation (IFC)	The Environmental, Health, and Safety Guidelines (EHS Guidelines) have technical examples of "good international practice" as defined by the IFC Performance Standard 3: Resource Efficiency and Pollution Prevention. General information on (1) environment, (2) occupational health and safety, (3) community health and safety, and (4) construction and decommissioning is included. IFC uses the EHS Guidelines as a technical source of information for project evaluation to identify performance levels and measurements that are acceptable to IFC and that are generally achievable with existing technology. For IFC financers, the EHS Guidelines may include site-specific targets. The environmental assessment process recommends alternative (higher or lower) levels or measures that can become project-specific requirements. When the host countries' regulations differ from EHS Guidelines, then projects are required to achieve more stringent requirements.
Equator Principles	The Equator Principles (EPs) are a risk management framework developed by financial institutions for evaluating and managing environmental and social risks associated with projects and to provide minimum standards to support due diligence and other decision making. EPs focus on social and community standards and responsibilities including indigenous people, labor, and engagement. The EPs have been adopted by 96 financial institutions in 37 countries, covering most international project finance debt within developing and emerging economies. Financing is only granted after the institutions commit to apply the principles for all projects. EPs are not intended to be applied retroactively, but they can be used by financial institutions for expansions or upgrades to existing projects that change the scope or scale to the environmental and social risks and impacts.

Adapted from IFC 2019 and Equator Principles Association 2019

ENVIRONMENTAL IMPACTS OF MINING

This section describes some of the impacts that mining has on the environment as related to waste, air, water, wildlife, and vegetation. These are potential effects that change throughout the life cycle of the mine and should be considered during planning, operation (monitoring), and at closure.

Waste

This section describes the waste, including hazardous waste, which can be developed as part of mining practices, and provides general guidance for definition and supervision. Table 24.6 summarizes characterization, management, and reclamation of waste rock, tailings, and heap leach pads. This information is provided in further detail in other chapters of the handbook.

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TABLE 24.6 Waste characterization and management

Waste	Characterization	Operational Management	Reclamation
Waste rock	Sampling and analysis of representative rock (or tailings) samples, usually from exploration drilling (or process testing in the case of tailings), characterize the potential for waste rock to generate acid or neutral rock drainage that would potentially impact the environment.	Management and closure plans of a waste facility can include selective handling of waste rock depending on its propensity to impact the environment.	Facility closure typically involves regrading; soil cover design, supported by infiltration modeling, often based on site-specific soil (moisture retention and permeability) data; and channels or benches to support stormwater management and erosion and sediment control.
Tailings	Sampling and analysis of representative rock (or tailings) samples, usually from exploration drilling (or process testing in the case of tailings), characterize the potential for material to generate acid or neutral rock drainage that can potentially impact the environment. Rheology of the tailings, as well as other geotechnical characteristics, is also important.	This information often includes emergency response procedures, such as detection, mitigation, and other requirements. Management of the decant/reclaim pond on the surface of the facility and maintaining sufficient freeboard and beach width to meet design requirements are also considered.	This often includes regrading; development of a soil cover design, supported by infiltration monitoring; revegetation; channels (stormwater management); and aspects for erosion and sediment control. Instrumentation is often in place to support closure monitoring requirements.
Heap leach	Similar to the waste rock and tailings, sampling of material is taken to understand the requirements for minimizing leachate requirements while maximizing the return on resources from the rock. The type and placement of material, leach application rate, and process following leaching area all depend on the leached material and the mineral to be extracted.	This involves monitoring of application and ponding of solution, return of leachate solution, instrumentation, and stormwater management. The plan usually identifies operation requirements of a facility.	Heap leach closure typically involves regrading; soil cover design, supported by infiltration modeling, often based on site-specific soil (moisture retention and permeability) data; channels or benches to support stormwater management; and erosion and sediment control.

Hazardous Wastes

Hazardous materials can be used throughout the life cycle of a mine and may include but are not limited to the following: fuels, solvents, reagents (sulfuric acid, flocculants, and lime), chemicals for blasting, and solid waste. Sites often develop a spill prevention, control, and countermeasures (SPCC) plan that provides information on containment and safety measures for storage, transport, and cleanup of hazardous wastes. Hazardous wastes are most often transported to facilities with applicable federal and state regulations that are lined and/or constructed to manage potential release.

The EPA enforces requirements under the Resource Conservation and Recovery Act for the handling, treatment, storage, and disposal of hazardous wastes. Owners and operators of sources producing, processing, and storing substances must identify hazards associated with an accidental release. Designing and maintaining a safe facility includes the preparation of a risk management plan (RMP), which helps minimize consequences of accidental release that can occur.

Air Quality

Reducing or eliminating air emissions is another important aspect of managing the environmental impact of mines. Air quality permits are obtained from the regulatory agencies prior to beginning any active mining for a project. The mining companies are required to complete studies identifying the potential sources of emissions and equipment that will be used for control.

Mining activities can generate dust from mining and mineral processing operations and associated truck traffic; and by releasing particulates, nitrous oxide, sulfur dioxide, and carbon monoxide from the equipment

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TABLE 24.7 Air quality emissions and controls

Area	Dust	Nitrous Oxide	Sulfur Oxide	Carbon Monoxide
Source	Haul trucks, crushers, conveyors, stockpiles, and blasting	Diesel engines in heavy equipment at plants and mills	Diesel engines in heavy equipment at plants and mills	Diesel engines in heavy equipment
Management	Apply water to roads. Use dust collection systems (mist and sprays). Apply chemical surfactants.	Tier 2 diesel engines and low-sulfur diesel fuel Preventive care and maintenance GPS data to manage fuel consumption and maintenance Filters, scrubbers, and other control devices at process facilities		

used to mine and process ore minerals. These emissions can generate smog and other forms of air pollution that may impact local air quality. Often, best management practices are used to control and manage emissions. Table 24.7 summarizes some sources and controls.

Wildlife

Biodiversity is the variety of species, wildlife, and plants that exist on earth. Large- and small-scale studies, often observed and evaluated at both local and regional levels, are used to understand the potential impacts of mining on biodiversity.

- Understand the interface between mining activities and biodiversity by integrating biodiversity management through policy and strategies and identifying best management practices, including key performance indicators during all stages of planning, operation, and closure.
- Assess the likelihood of impact of mining activities on biodiversity by completing a risk and opportunity assessment to prioritize, focus, and identify key company-level risks and opportunities.
- Mitigate potential impacts on biodiversity by ensuring that all site-level processes detail actions that can avoid, minimize, mitigate, and avoid impacts.
- Explore potential contributions to biodiversity conservation by understanding how to protect and conserve biodiversity.

Guidance for general practices on the assessment and management of wildlife is outlined in Table 24.8.

Water Quality

All methods of surface and underground mining can affect surface and groundwater quality. Meteoric water, surface water, and groundwater that comes into contact with mining-related disturbance can all be affected. The contamination of water at a site depends on many factors:

- **Type of ore being mined.** Some ore bodies and associated waste rock are more reactive than others. For example, sulfide ores have the potential to oxidize and generate acidity, resulting in enhanced potential to mobilize metals, metalloids, and salts.
- **Type of mining.** Surface mines, underground mines, and solution mines all have different interactions with the water source.
- **Chemicals used in processing.** Chemicals used to mine and process metal ores include blasting agents, cyanide, sulfuric acid, and other inorganic and organic chemicals. These have the potential to impact the environment if leached into unmanaged contact waters.
- **Climate.** The amount of meteoric and surface water requiring management on mine sites is dependent, in part, on climate. For example, mines in cold climates with permafrost may not have the same concerns as those that are in tropical environments with large rivers and ponded water on the sites.
- **Stage of mining.** During different stages of mining, there are different potential areas of contamination. For example, process chemicals will have less impact following closure, but underground workings can be flooded and dissolved minerals can impact groundwater.

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TABLE 24.8 General practices for assessing and managing wildlife

Evaluation	Description
Biodiversity assessments	Environmental and social impacts assessments (ESIAs), environmental impact statements (EISs), and environmental assessments (EAs) generally include the following: • Screening and scoping • Baseline studies • Impact prediction and assessment • Mitigation and enhancement measures • Monitoring • Follow-up auditing • Requirements for stakeholder engagement
Impact assessments	The ESIA assesses the types and levels of biodiversity and potential mining project impacts: • Cumulative impacts • Ecosystem loss • Habitat isolation • Ecological process alteration • Disturbances • Pollution
Management practices	Environmental management systems provide a framework for management of biodiversity during operations and closure: • Integrating into environmental policies • Documenting and assessing consultation with stakeholders • Identifying and assessing risk • Maintaining a register of legal and other requirements • Planning and developing mitigation measures • Implementing mitigation measures • Monitoring, measuring, and reporting performance • Managing and reviewing procedures and outcomes • Adopting a continual improvement approach

- **Environmental management.** Best management practices, such as surface water controls, proper reservoir design and management, water optimization, conservation, reuse, and keeping impacted water separate from clean waters, can be used to minimize environmental risk.

Some sort of environmental impact evaluation (e.g., EIS in the United States) is required in more than 100 countries, necessitating companies to identify potential sources of water contamination at proposed mine sites and the environmental management strategies that the mining companies intend to use to limit or prevent water contamination. Enforceable effluent regulations and permit conditions are often used to ensure compliance with standards set out in the regulations.

Regulations are often dependent on the location, downstream waters, proximity to drinking water sources, and other factors. These play a part in the requirements for treatment of mine water to meet discharge requirements. In the United States, the Clean Water Act (CWA) regulates discharged water and provides standards for discharge concentrations.

Some of the most common water quality parameters are listed in Table 24.9 with an explanation of why it is generally important to understand the parameter as described.

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TABLE 24.9 Common water quality parameters and measurements

Parameter	Description
Temperature	• A physical property of water that can have an impact on organisms that live or reproduce in the water body • Reduces the solubility of oxygen • Can enhance the sensitivity of toxic substances • Should be measured with a calibrated probe
Turbidity	• A measure of the clarity of water • Measured as a physical property, primarily based on the amount of suspended solids in a column of water • An aesthetic concern as well as an interference of light penetration into water, impacting aquatic and plant life
Bacteria (fecal coliform, *Escherichia coli* and *Enterococcus*)	• Can result in people or animals becoming sick • Can originate from waste, including sewage animal waste or other septic sources • Generally a numeric criterion for limiting bacteria
pH	• The balance been hydro ions, which are acidic and basic; balanced pH = 7 • A pH from 6 to 9 preferred by aquatic organisms • Affects other chemical constituents and biological processes • A numeric criteria that can be measured with a probe
Dissolved oxygen	• Oxygen required for animal and human life • Dissolved oxygen concentration and water body health in direct correlation • Low dissolved oxygen indicative of an excess of nutrients (causing excess vegetation) • Biological oxygen demand (BOD), a measure of oxygen amount needed to decompose all organic matter • BOD often the parameter used as numeric criteria
Ammonia	• Can be toxic to aquatic life • Can be naturally occurring but also a result of treated effluent, such as sewage • Criteria often tied to pH and temperature levels within the water body
Metals	• High concentrations (e.g., arsenic, cadmium, copper, mercury, and lead) harmful and threatening to aquatic life, animals, and humans • Accumulate in both animals and humans • Often measured through the chemical analysis of water or through sediment and fish tissue analysis

Adapted from Idaho Department of Environmental Quality 2018

Acid Rock Drainage–Metal Leaching

Acid rock drainage–metal leaching (ARD-ML), also known as *acid mine drainage* (AMD) or *acid and metalliferous drainage*, is of major concern for water quality in the mining sector. ARD is caused by oxidation of sulfide-bearing minerals on exposure to air (oxygen) and water, resulting in the generation of sulfuric acid. The reaction can be catalyzed by microorganisms.

The sulfide-bearing material is usually present in the form of pyrite, marcasite, and pyrrhotite, and most commonly associated with volcanogenic massive sulfide (VMS), high-sulfidation epithermal porphyry copper, and skarn deposits. In both surface and underground operations, the excavation of rock with sulfide minerals may accelerate the oxidation process by increasing the exposure of sulfide minerals to air, water, and microorganisms. The drainage produced from the oxidation process may be neutral to acidic, may contain dissolved metals, and always contains sulfate. ARD is typically defined as drainage with pH less than 6, with sulfate concentrations exceeding 1,000 mg/L, and with moderate to elevated metals concentrations.

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Sulfide-bearing minerals react with oxygen in the air and with water and are oxidized into ferrous sulfate and sulfuric acid. Flowing water leaches the ferrous sulfate and the sulfuric acid when they are exposed, allowing for further oxidation to form ferric sulfate. The following chemical reactions represent ARD generation from pyrite:

Reaction 1:	$FeS_2 + {}^{7}\!/_{2}O_2 + H_2O \leftrightarrow Fe_2{}^{+} + 2SO_4{}^{2-} + 2H^{+}$	
Reaction 2a:	$Fe^{2+} + {}^{1}\!/_{4}O_2 + {}^{5}\!/_{2}H_2O \leftrightarrow Fe(OH)_3 + 2H^{+}$	(nonacidic conditions)
Reaction 2b:	$Fe^{2+} + {}^{1}\!/_{4}O_2 + H^{+} \leftrightarrow Fe^{3+} + {}^{1}\!/_{2}H_2O$	(acidic conditions)
Reaction 3:	$FeS_2 + 14Fe^{3+} + 8H_2O \leftrightarrow 15Fe^{2+} + 2SO_4{}^{2-} + 16H^{+}$	(acidic conditions)

Reaction 3 is faster than Reaction 1 (two to three orders of magnitude) and generates significantly more acidity (eight times) per mole of pyrite oxidized. However, it is limited to acidic conditions, which are required to convert ferrous iron into ferric iron. Therefore, pyrite oxidation is typically initiated via Reaction 1 and sustained or accelerated via Reaction 3.

The reactions can be catalyzed by microorganisms leading to substantially faster reactions. For example, the rate of Reaction 2b can increased by five to six orders of magnitude with the presence of iron-oxidizing bacteria.

At acidic pH, water has a greater capacity to leach elements from rock, such as metals and salts. These can include typical major rock constituents (calcium, magnesium, potassium, sodium, aluminum, iron, and manganese), as well as trace elements (antimony, arsenic, cadmium, chromium, cobalt, copper, lead, molybdenum, nickel, selenium, and zinc).

The neutralizing potential of host materials may be sufficient to counter the acid potential of the sulfide material. Carbonates (specifically calcite and dolomite) and, to a lesser extent, silicates are the main alkaline compounds that can occur in sufficient concentrations to prevent oxidation from occurring or to neutralize acid formed by oxidation.

The reaction(s) continue until at least one of the reactants is exhausted or no longer available for reaction. For example, a water cover may be maintained on a reactive tailings storage facility to prevent ingress of oxygen, or a waste rock stockpile may be covered with an impermeable cover to prevent ingress of water. As a potential long-term liability, mines must be carefully managed to maximize the prevention and mitigation of ARD-ML. At sites where sulfidic wastes are disposed without the application of ARD-ML prevention and mitigation technology, acid generation will likely persist for hundreds of years.

WATER TREATMENT

Water treatment is utilized to control the quality of mining-influenced water that is to be discharged to the environment or reused. Table 24.10 summarizes the common contaminants in mining-influenced water, along with their common sources and conventional treatment mechanisms. Reverse osmosis, ion exchange, adsorption, and biological treatment are nonconventional treatment processes that are becoming more prevalent in the mining sector.

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TABLE 24.10 Common contaminants of concern in mining-influenced water

Common Contaminant	Common Source	Conventional Treatment
Acidity	Acid rock drainage–metal leaching (ARD-ML) Mineral processing (lixiviants, off-gas handling)	• Neutralization
Alkalinity	Mineral processing (pH modifiers, neutralization)	• Reacidification
Ammonia	Explosives Cyanide Domestic sewage Mineral processing (lixiviants)	• Explosives • Best management practices • Passive natural degradation • Air stripping • Reacidification*
Cyanide	Mineral processing (lixiviants)	• Cyanide destruction • Passive natural degradation
Metals and metalloids: Antimony Aluminum Arsenic Cadmium Chromium Cobalt Copper Iron Lead Manganese Mercury Molybdenum Nickel Selenium Zinc	ARD-ML Mineral processing (reagents and catalysts) Water treatment (reagents and catalysts)	• Oxidation • Precipitation • Coprecipitation
Nitrate/Nitrite	Explosives Cyanide	• Explosives • Best management practices
Organics	Mineral processing (flotation, solvent extraction)	–
Phosphate	Domestic sewage	• Precipitation • Coprecipitation
Radionuclides	Mineral processing (leaching)	• Precipitation • Coprecipitation
Sulfate	Acid and neutral rock drainage–metal leaching Mineral processing (flotation, lixiviants, off-gas handling)	• Precipitation
Thiosalts	Mineral processing (oxidation)	• Oxidation • Passive natural degradation
Total dissolved solids	ARD-ML Mineral processing (leaching, oxidation, ion exchange, pH modifiers, neutralization, off-gas handling, electrowinning, and electrorefining)	• Precipitation
Total suspended solids	Mining Mineral processing (ore size reduction) Waste management	• Solid–liquid separation

*pH adjustment is used to control the proportion of un-ionized ammonia, which is more toxic to aquatic species.

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pH Control and Metal Hydroxide Precipitation

Table 24.11 describes methodologies for pH control and the precipitation of metal hydroxides, including general information on technology, application, control, benefits, and challenges.

TABLE 24.11 pH Control and metal precipitation methodologies

Methodology	Description
Hydrogen precipitation	Used to precipitate dissolved metals as metal hydroxides, the precipitation reactions are carried out at alkaline pH. The target pH depends on the target dissolved metal(s) and desired removal efficiency, as the theoretical minimum solubility for different metal hydroxides occurs at different pH values. The reactions may be carried out in a series of stages to allow for the selective precipitation of multiple metal hydroxides. Afterward, the metal hydroxide precipitates are separated from the treated effluent via solid–liquid separation technologies. The decant stream may require additional polishing or filtration to meet total suspended solids (TSS) discharge limits and reduction of pH/reacidification to meet pH discharge limits. Hydroxide precipitation and solid–liquid separation can be carried out using a wide variety of technologies, including the following: • Pond-based systems involve the mixing of lime with effluent as it flows into a pond where precipitates settle. Decant may flow into a secondary pond for polishing. Pond-based systems require periodic dredging of accumulated low-density sludge (typically <3 wt % solids) to ultimate disposal. • Pit-based systems are similar to pond-based systems, with the exception that an open pit (if available) is utilized for precipitate settling and sedimentation. The pit itself is normally the ultimate disposal destination for the low-density sludge (typically <3 wt % solids), depending on pit capacity. Decant is pumped out of the pit utilizing a floating barge system that does not disturb settled precipitates. • Conventional/low-density sludge (LDS) systems utilize reaction tank(s) for lime and effluent mixing and clarifiers for precipitate settling and sedimentation. Conventional/LDS systems generate LDS, typically <5 wt % solids. • Simple sludge recycle systems are similar to conventional/LDS systems; with the exception that clarifier underflow is recycled to the reaction tank(s). Simple recycle systems may generate sludge with up to 15 wt % solids. • High-density sludge (HDS) systems and other sludge recycle processes (Heath Steele, Geco, staged neutralization) are more complex sludge recycle systems. These systems can generate sludge with up to 30 wt % solids. Neutralization reagents for hydroxide precipitation include limestone ($CaCO_3$), dolomitic limestone ($CaCO_3$ containing magnesium carbonate [$MgCO_3$]), quicklime (CaO), hydrated lime ($Ca(OH)_2$), sodium hydroxide (NaOH), soda ash (Na_2CO_3), and magnesium oxide (MgO). Of these, quicklime and hydrated lime are the most commonly used reagents because of their ability to reach high-target pH, low relative cost to other reagents, and coagulating properties. The calcium in limestone and lime also aids in hardness removal, sulfate removal via precipitation as calcium sulfate (*gypsum*), arsenic (as pentavalent arsenic) removal via precipitation as calcium arsenate, and phosphorus (as phosphate) removal via precipitation as hydroxylapatite. Carbon dioxide injection and sulfuric acid dosing may be used for post-treatment pH adjustment. A wide variety of flocculants are used to facilitate solid–liquid separation. Metal hydroxide sludge is generated by hydroxide precipitation. Sludge is typically disposed of in situ for pond-based and pit-based systems, or in on-site features including tailings storage facilities, engineered dewatering ponds, open pits, underground mine workings, co-disposal with waste rock, etc. Sludge may be mechanically dewatered prior to disposal.

(continues)

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TABLE 24.11 pH Control and metal precipitation methodologies (continued)

Methodology	Description
Sulfide precipitation	Used to precipitate dissolved metals as metal sulfides to very low treated effluent concentrations, the fast reaction is carried out at slightly alkaline pH to promote sulfide ion formation. The metal sulfide precipitates are separated from the treated effluent via solid–liquid separation technologies, most commonly sedimentation in a clarifier/thickener and/or media filtration. Further processing of clarifier/thickener underflow solids, such as dewatering via filter press, may be required where metal sulfides are to be recovered to a smelter. Sulfide reagents include hydrogen sulfide (H_2S) gas, and sodium hydrosulfide (NaHS), sodium sulfide (Na_2S), calcium sulfide (CaS), and ferrous sulfide (FeS). Proprietary polymeric chemicals with anionic sulfur-containing functional groups can act as both sulfide precipitant and coagulant/flocculant, to attempt to overcome the challenge of small particle size/colloidal nature associated with conventional sulfide precipitation. Because of their polymeric nature, the proprietary chemicals are associated with a lower risk of downstream oxidation of residual reagent sulfide and a lower risk of downstream toxicity, as polymers are less bioavailable to aquatic organisms. As sulfide reagents can be expensive to purchase or generate on site, sulfide precipitation is typically only used to target dissolved metals where value recovery is possible from high volumetric flow rate effluent streams. However, sulfide precipitation may be used as a polishing step to treat these and other metals where no value recovery is possible. Sulfide precipitation is effective over a wider pH range than hydroxide precipitation so less pre- and post-treatment pH adjustment may be possible. Moreover, sulfide precipitation is relatively insensitive to the presence of weak chelants (e.g., ammonia up to 45 mg N/L), which can interfere with hydroxide precipitation. Metal sulfide sludge is generated by sulfide precipitation. Metal sulfide sludge is easier to thicken and dewater than metal hydroxide sludge. If the content of valuable metals in the sludge is high, the sludge can be dewatered and recovered to a refinery process for value recovery to offset effluent treatment costs. However, metal sulfide sludge is less stable than metal hydroxide sludge under oxidizing conditions and requires disposal in reducing conditions. Most operations that employ sulfide precipitation technology do so with the intent of recovering saleable metals and do not, in practice, dispose of their residuals.
Ferric or aluminum salt coprecipitation	Iron salts (e.g., ferric sulfate [$Fe_2(SO_4)_3$], ferric chloride [$FeCl_3$]) and aluminum salts (e.g., alum [$Al_2(SO_4)_3$], aluminum chloride [$AlCl_3$], and polyaluminum chloride) are cationic coagulants that can be utilized to precipitate and/or coprecipitate some metals (arsenic, phosphorus, and selenium, among others) at acidic pH. Iron and/or aluminum added as salts reprecipitate as an oxyhydroxide and, through sorption, can strip out oxy-anion metals, i.e., metals with a negative charge. If the pH of the solution is kept acidic (but not too acidic such that the Fe/Al phase dissolves), then the surface charge is positive and sorption of negatively charged species occurs. Alternatively, if the pH goes above about 7 or so, the surface charge switches negative and then only positively charged species will sorb. Ferric iron and aluminum salts are also frequently used as coagulants to aid TSS removal in solid–liquid separation processes. Coagulants neutralize the charges on particles to destabilize small particulate and colloidal solids. Adjustment of pH may be required to achieve the desired coagulation pH. Pentavalent arsenic can be precipitated with ferric iron salts as basic ferric arsenate at slightly acidic pH. The precipitation of stable basic ferric arsenate requires careful control of ferric iron to arsenic molar ratio (i.e., Fe:As > 4). Dissolved phosphorus (as PO_4^{3-}) may be precipitated with alum as aluminum phosphate or with ferric iron as ferric phosphate at slightly acidic pH. Selenium as selenite (SeO_3^{2-}) may be coprecipitated with ferric iron at slightly acidic to circum-neutral pH (pH 4–6). Selenium as selenate may also be coprecipitated with ferric iron; however, sulfate can reduce the effectiveness of coprecipitation. Sludge is typically disposed of on site including in tailings storage facilities, engineered dewatering ponds, open pits, underground mine workings, co-disposal with waste rock, etc. Sludge may be mechanically dewatered prior to disposal. Sludge has the potential to release selenium as ferrihydrite crystallizes to more stable phases. Little is known about the long-term stability of residuals and the potential for selenium remobilization. If residuals are not kept stable through prudent disposal techniques, significant costs associated with residuals stabilization technology or retreatment of residual leachate could be incurred.

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TABLE 24.11 pH Control and metal precipitation methodologies (continued)

Methodology	Description
Barium chloride coprecipitation	The radionuclide radium-226 may be coprecipitated by addition of barium chloride ($BaCl_2$), resulting in a precipitate of barium radium sulfate [$BaRa(SO_4)_2$]. The residual concentration of dissolved radium-226 is controlled by the solubility of barium sulfate, which is very low. The precipitates are separated from the treated effluent via solid–liquid separation technologies. Coagulation/flocculation is typically used to agglomerate the fine slow-settling precipitates that are formed. The decant stream may require additional polishing or filtration to meet TSS discharge limits.
Oxidation	Aeration can be used to convert soluble dissolved metals to minimally soluble metals via oxidation. Most commonly, aeration is used to convert ferrous (II) iron to ferric (III) iron with dissolved oxygen. The ferric hydroxide oxidation product is insoluble and forms precipitate that is subsequently removed by some form of solid–liquid separation. The most common methods of aeration are waterfall aeration, where effluent is sprayed through air as small droplets or a thin film; and air diffusion, where pressurized air is diffused into a vessel containing effluent. Of these methods, air diffusion is the most efficient for mass transfer, and it is also the most common method when aeration is integrated with a hydroxide precipitation process. Surface aeration via floating surface aerators and paddlewheel aerators is typically utilized for aeration of equalization ponds. Subsurface aeration via venturi jet aeration or coarse, medium, and fine bubble aeration is typically utilized for aeration of effluent treatment process vessels, as surface aeration is less efficient. As aeration is typically integrated with hydroxide precipitation, the precipitates typically report to a clarifier/thickener underflow sludge stream, as described earlier. Chemical oxidation with a strong oxidant may be used to convert soluble dissolved metals to minimally soluble metals via oxidation. Such strong oxidants include chlorine (e.g., ClO_2, Cl_2, HOCl), hydrogen peroxide, permanganate (MnO_4), and ozone (O_3). Aeration is generally first utilized to convert easily oxidizable soluble metals to less soluble metals. Strong oxidants are then added to achieve lower efficiency oxidations. Although chlorination has been reported to oxidize manganese, potassium permanganate is a more effective oxidizing agent that can be utilized at neutral pH.
Reacidification	Reacidification may be utilized to neutralize effluents with alkaline pH to meet pH discharge limits. Carbon dioxide (CO_2) and sulfuric acid (H_2SO_4) are the two most commonly used reagents for reacidification. Of these two, carbon dioxide reacidification is most often preferred, as it does not add sulfate to effluent, is easier to control, and safer to handle and store than sulfuric acid. Carbon dioxide will also add alkalinity to the effluent, which may be desirable for some operations. Moreover, sulfuric acid is a regulated toxic substance in some jurisdictions, and therefore may be associated with a higher regulatory burden for operations than carbon dioxide. In a carbon dioxide–based reacidification system, liquid carbon dioxide is first vaporized to gaseous carbon dioxide via heat exchange and then injected into the effluent via spargers/diffusers. Normally, reacidification does not generate residuals unless there is sufficient hydroxide alkalinity and calcium in the effluent.

Solid–Liquid Separation

Solids requiring solid–liquid separation can consist of gravity settleable solids and nonsettleable particulate or colloidal solids. Normally, solid–liquid separation is accomplished by coagulation, flocculation, and settling/sedimentation, in both pond- and clarifier-based systems. However, solid–liquid separation can also be accomplished by filtration and flotation. Coagulation can be accomplished with ferric iron and aluminum salts as well as low-molecular-weight water-soluble cationic polyelectrolytes. Flocculation can be accomplished with high-molecular-weight water-soluble polymers. Stokes' law describes the process. Methodologies for solid and liquid separation are described in Table 24.12, biological treatment methods are described in Table 24.13, and other treatment methodologies are described in Table 24.14.

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TABLE 24.12 Solid and liquid separation methodologies

Method	Description
Pond and pit-based settling	Total suspended solids (TSS) removal is achieved via gravity. Pond- or pit-based settling/sedimentation systems do not require equipment for TSS removal, with the exception of flocculent make-up and dosing systems, where required. However, large surface areas are required to achieve adequate settling, and it may be impossible to remove small particulate and colloidal TSS in pond- or pit-based systems because settled solids can be easily disturbed by inflow and outflow upsets as well as wind and wave action and pond inversions. Periodic dredging is required to counter solids accumulation at low density within the pond.
Conventional clarification	TSS removal is achieved via gravity. Conventional clarifiers are available in a variety of configurations, such as horizontal flow in rectangular, circular, or square basins; upflow; sludge blanket; and solids-contact. Clarifiers may contain tube settlers or plate settlers (*lamella*) to increase surface area, decrease specific overflow rate, and promote settling. Settled solids are removed from the clarifier as sludge to be dewatered, if required, and disposed of in an appropriate location. Clarification can produce a more dense sludge than pond-based settling/sedimentation. Treated effluent quality is a function of contaminant and hydraulic loading.
Dissolved air flotation (DAF)	TSS removal is done via buoyant separation on air bubbles. The air bubbles, diffused into coagulated/flocculated effluent, are adhered or adsorbed to floc surface or entrapped under floc, and then float to the effluent surface. Float sludge is then skimmed or hydraulically flooded off the effluent surface. Skimmed float sludge can be comprised of up to 3–6 wt % solids. DAF is not commonly used in the treatment of mining effluents and is not effective for dense solids.
Media filtrations	TSS are primarily removed via size exclusion, but also through adhesion and adsorption. Pressure media filtration is most commonly used for polishing after upstream TSS removal via other methods but can be used directly for low turbidity effluents. Sand and anthracite are the most common filter media; however, garnet, magnetite, and other media are also used. In multimedia filtration, multiple filter media are used with coarse, less dense media at the top of the filter and finer, denser media at the bottom of the filter. Periodically, the media filters must be taken out of service to undergo backwashing to remove the buildup of suspended solids.
Ultrafiltration (UF) and microfiltration (MF)	Membrane-based technologies that achieve TSS removal via size exclusion, UF and MF are most commonly used for polishing after upstream TSS removal via other methods but can be used directly for low turbidity effluents. They are particularly effective on fine and colloidal TSS. UF and MF technologies are quite similar, but have different pore size ranges and can remove different size fractions of TSS; while UF can remove TSS between 0.001 μm and 0.1 μm, MF can remove TSS between 0.1 μm and 10 μm. Membranes are available in many materials and in different configurations, including tubular, hollow fiber, spiral-wound (UF only), pleated cartridge (MF only), and flat sheet. Hollow fiber configuration is the most common for industrial effluent treatment. Low pressure or vacuum is used to drive effluent through the membranes to the permeate side while TSS greater than a specific size is retained on the reject side. Feed to UF and MF systems must be pretreated through strainers to protect the membranes from clogging and damage. Coagulants may be added prior to the UF or MF system to agglomerate very fine particles.
Enhanced coagulation and settling (ECS) technologies	Variations on high-rate clarification have advantages over traditional clarification, particularly a reduction in footprint. The main competing technologies can be grouped into ballasted flocculation/sedimentation and high-rate solids contact clarification/thickening. ECS processes are typically integrated downstream of chemical precipitation processes. In contrast to conventional clarification technology, which takes long periods to achieve steady-state operation, ECS technology reaches steady-state operation following start-up in less time than conventional clarifiers. ECS technologies operate at much higher loading rates than conventional clarification technology, allowing for significant reductions in required footprint and potentially significant cost savings. This also allows ECS units to be modular in nature resulting in lower installation costs than conventional clarifiers. The recycle of ballast or sludge in ECS processes can be utilized to maintain TSS loading to the system despite variations in feed TSS. ECS technologies offer lower chemical consumption based on the internal recirculation of solids.

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TABLE 24.13 Biological treatment methodologies

Method	Description
Aerobic biological treatment	Used to oxidize compounds such as ammonia and cyanide, aerobic biological oxidation can be accomplished in suspended growth (e.g., activated sludge processes, membrane bioreactors, sequencing batch reactors, and aerated lagoons) and attached growth/fixed film processes (e.g., trickling filters, rotating biological contactors, packed-bed reactors, moving bed biofilm reactors, and submerged attached growth reactors). In suspended growth processes, microorganisms are maintained in suspension in the effluent to be treated via mixing. In attached growth processes, microorganisms are attached as biofilms to inert packing material through which the effluent to be treated flows. Integrated fixed-film activated sludge (IFAS) systems combine suspended growth and attached growth/fixed-film processes. Fixed-film processes are generally more appropriate than suspended growth processes for the treatment of mining effluent as biofilms are less susceptible to variations in contaminant loading (because of variation in flow or quality) and temperature, are more compact, and can have minimal sludge-handling requirements. In rotating biological contactors (RBCs), closely spaced disks that provide large surface area for the establishment of biofilms are partially submerged in effluent and continuously rotated, such that they are alternately exposed to air, which provides oxygen, and effluent, which provides biodegradable matter, to support aerobic biological oxidation. Fixed-film processes generate less residual and have lower sludge management requirements than suspended growth processes. Fixed-film processes do not require the recycle of activated sludge. In fixed-film processes, excess biomass sloughs off from the biofilm carrier and requires downstream solid–liquid separation (e.g., clarifier, dissolved air flotation) and disposal.
Anaerobic biological treatment	A fluidized-bed reactor (FBR) is a completely mixed attached growth process reactor with plug flow characteristics, utilized for the anaerobic or anoxic biological reduction of oxyanions. FBR vessels contain granular solid media that provide large surface area for the establishment of biofilms of naturally occurring heterotrophic facultative bacteria. The granular solid media is typically sand or granular activated carbon (GAC). Untreated effluent and recycled effluent are pumped into the bottom of the FBR in an upward flow direction at a high enough velocity to fluidize the granular solid media. Fluidization increases the active surface area of the granular solid media. Organic carbon electron donor (e.g., methanol, molasses, MicroCg) and nutrient (e.g., phosphorus and micronutrients) reagent addition is required to support biological function. Excess biomass, including particulate and colloidal Se, is sheared from the media by effluent flow and with biomass separation device.

TABLE 24.14 Other treatment methodologies

Method	Description
Cyanide destruction using the INCO SO_2/air process and/or hydrogen peroxide	This process utilizes sulfur dioxide (SO_2) and oxygen (from air) in the presence of copper catalyst to oxidize cyanide to cyanate. The process may be optimized to preferentially treat weak acid dissociable (WAD) or strong acid dissociable (SAD) cyanide depending on equipment configuration, operating pH, and copper catalyst dosage. Cyanate generated from the various cyanide oxidation reactions hydrolyzes to ammonia. Sulfur dioxide can be provided as liquid sulfur dioxide or generated from sodium sulfite (Na_2SO_3) or sodium metabisulfite ($Na_2S_2O_5$). Sulfur may also be provided via sulfur burner using sulfur prill. Hydrogen peroxide may also be used to destroy cyanide via oxidation of WAD cyanide and via precipitation of SAD cyanide. Hydrogen peroxide is provided as a concentrated solution. Oxygen is provided via air sparging into the reaction tank. Copper catalyst is provided as hydrated copper sulfate ($CuSO_4 \cdot H_2O$) if insufficient background copper is available in the effluent. The reaction is carried out at alkaline pH (typically 8.3–10.5). The pH target and neutralization of hydrogen ions is achieved via lime addition. The pH target must balance both cyanide destruction and metals precipitation. The precipitates are separated from the treated effluent via solid–liquid separation technologies. Because of the generation of ammonia via cyanate hydrolysis, downstream ammonia removal may be required.

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TABLE 24.14 Other treatment methodologies (continued)

Method	Description
Air stripping	This can be used to volatilize un-ionized ammonia gas (NH_3) from effluent at alkaline pH. The proportion of total ammonia available to be removed as un-ionized ammonia is pH and temperature dependent, with the most efficient ammonia removal achieved at pH greater than 11.5. The effluent is pH adjusted, typically with lime or sodium hydroxide, and then pumped to the top of a tower(s) and distributed evenly across the tower(s). Typically, atomizing spray nozzles distribute the effluent droplets evenly across the tower(s). As the effluent droplets fall by gravity through a bed of packing, ambient or heated air is blown countercurrent to the effluent flow. The tower(s) contains packing, which provides surface area for mass transfer. Un-ionized ammonia gas is stripped from the effluent to the air. Then, the pH of the effluent is readjusted to meet discharge limits, typically with carbon dioxide, prior to discharge. The process efficiency is highly sensitive to variation in effluent and ambient temperatures. Ammonia solubility decreases and the proportion of total ammonia as un-ionized ammonia increases as feed temperature increases. For example, at 20°C, typical removal efficiency of 90%–95% can be achieved, while at 10°C, removal efficiency falls to 75%. Fogging and icing also impede ammonia removal by decreasing the rate of mass transfer of ammonia from effluent to air. As such, preheating of effluent and/or air, and/or seasonal operation may be required. This process is best suited to total ammonia concentrations in the range of 10–100 mg N/L, but is otherwise relatively insensitive to variations in effluent quality. Pretreatment for total suspended solids may be required to counter plugging of the stripping tower packing. Also, treatment for hardness, iron, manganese, phosphate, and microorganisms may be required to counter scaling/fouling of the stripping tower and packing. Air stripping is a mechanical process that creates no residual stream. Scale may be occasionally removed via hydraulic cleaning processes and disposed of according to chemical composition.
Passive natural degradation	Natural degradation is the most commonly used technology for the passive removal of ammonia, cyanide, and thiosalts from mining effluents. Natural degradation of ammonia primarily occurs through volatilization of un-ionized ammonia gas (NH_3) and biological oxidation. These processes are influenced by pH, temperature, and dissolved oxygen concentration. Passive treatment systems rely on naturally occurring processes and, ideally, require only infrequent monitoring and maintenance. The feasibility and performance of passive systems depend on many conditions including climate, availability of land, substrate requirements for biological systems, effluent flow rate, and effluent chemistry. Passive systems are generally very sensitive to temperature and residence time. In many cases, these systems are used for secondary and tertiary treatment rather than primary treatment. Some of the concerns surrounding these technologies include the availability of good design data and documentation of long-term performance; geographical suitability (e.g., land availability, containment); their application to cold climates, particularly the low temperatures in winter; and their robustness to variability in effluent flow and quality (e.g., such as caused by freshet). Favorable conditions for natural degradation of ammonia can be found at sites with large storage basins and polishing ponds, which allow for long residence time and seasonal discharge. Cyanide may be degraded naturally in on-site impoundments via volatilization of hydrogen cyanide gas (HCN), hydrolysis, photodegradation, dissociation of metal cyanide complexes, chemical oxidation, aerobic biological oxidation, and precipitation. Natural degradation of cyanide can be used as primary treatment in combination with active treatment seasonally. However, it is impeded by ice/snow cover in winter (which limits CO_2 ingress, HCN volatilization, and O_2 ingress) as well as cold temperatures. Cyanidation tailings, although normally alkaline, may approach pH 8 through atmospheric carbon dioxide dissolution. Thiosalts in mining effluents are most frequently controlled through natural degradation to sulfate/sulfuric acid in tailings ponds or holding ponds. An adequate degree of degradation may be achieved with sufficient volume and residence time as well as sufficiently high temperature (degradation rate is higher in warmer waters). Phosphorus may be synergistically removed in biological natural degradation processes, as it is a nutrient required for aerobic biological oxidation. Passive treatment can involve routing a waste stream (usually at a very low flow rate, such as less than 100 gpm) through a series of treatment cells and/or canals, loaded with substrate, such as anoxic limestone drains, and sulfate-reducing bioreactors.

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TABLE 24.14 Other treatment methodologies (continued)

Method	Description
Reverse osmosis and nanofiltration	Reverse osmosis removes total dissolved solids (TDS) (including dissolved metals, selenium, phosphorus, chlorides, ammonia, and cyanide) by the high-pressure application of effluent to selectively permeable membranes that exclude dissolved solids but allow the passage of water molecules. Reverse osmosis membranes exclude dissolved solids by charge and size. The high feed pressure is required to overcome the osmotic pressure that develops across reverse osmosis membranes, pushing the water molecules through the membrane to generate a low TDS permeate stream. Dissolved solids are retained on the feed side of the membrane and exit the pressure vessels in the high TDS concentration reject stream. Prior to discharge, the permeate requires pH adjustment and potentially remineralization (e.g., readdition of Ca^{2+}, Mg^{2+}, etc.) to meet discharge toxicity requirements. Reverse osmosis is generally reported as being able to remove over 95% of TDS from feed effluent. Reverse osmosis systems operate best with stable feed quality and volumes and at indoor ambient temperatures, although they can tolerate variations in these parameters. Nanofiltration is a membrane process that removes multivalent dissolved solids (including aluminum, arsenic, copper, iron, lead, manganese, nickel, zinc, radium-226, and selenium) from effluent via the pressurized application of effluent to a charged, selectively permeable membrane that rejects multivalent ions. Unlike reverse osmosis, however, the membrane allows the passage of water and monovalent ions (e.g., sodium, chlorides, cyanide, ammonia), and the required feed pressure is lower. Nanofiltration has been proposed as a substitute to reverse osmosis in instances where rejection of multivalent ions (e.g., Cu^{2+}, Ni^{2+}) is more important than rejection of monovalent ions (e.g., Na^+, Cl^-, NH_4^+, CN^-) and where lower removal efficiencies are acceptable. Removal of multivalent metals is frequently the objective for treatment of mining effluents, and so nanofiltration is an interesting option. Nanofiltration has been reported to remove up to 95% of targeted multivalent ions in effluent. As solutes rejected by the nanofiltration and reverse osmosis membranes accumulate on the feed side of the membrane, there is a risk of scale formation as low solubility ions reach their solubility limits. This risk is managed by antiscalant dosing into the feed stream and clean-in-place regimes; however, if scaling occurs, it can reduce recoveries and shorten membrane life. Residuals generated by nanofiltration and reverse osmosis are the high TDS reject stream, the spent clean-in-place chemicals, and backwash water. The most significant among these is the high TDS reject stream, as it constitutes a large fraction of the residuals requiring management. Reject management is a critical consideration in the design of reverse osmosis systems for mine and mill effluent treatment and can add significant capital and operating costs.
Ion exchange	Ion exchange (IX) is the adsorption of ions of a given charge (either cation or anion) in a solution onto the surface of a solid material (IX resin) and replacement in the solution by an equivalent quantity of ions of similar charge from the solid material surface. The resin is insoluble in water but hydrated and may be a salt, acid, or base. Resins are formed through a polymerization process and functional groups added in various activation methods. A wide variety of functional groups can be grafted onto these polymers, which are then used to selectively uptake ions, principally metals. The functional groups of the polymers are initially associated with ions such as sodium (Na^+), hydrogen (H^+), chloride (Cl^-), and hydroxide (OH^-). The functional groups and resin polymer dictate what ions can be exchanged by the resin. Many resins are designed to target and selectively remove one or only a few parameters. To achieve the removal of a wide suite of parameters, several IX steps in series, each with a specialized resin to target certain parameters, may be required. As the effluent passes through the column, functional group associated ions are displaced from the resin by the targeted ions due to the higher affinity of the resin for the targeted ions (e.g., Ni^{2+} Cu^{2+} or other multivalent metals, and ammonia depending on the resin employed) than for the functional group associated ions. The targeted parameters are immobilized onto the resin and the functional group associated ions are released into the effluent exiting the column. When resins become saturated with the targeted ions, they are regenerated by passing concentrated solution(s) (*regenerant*) containing the replaced ions (e.g., NaOH, H_2SO_4, NaCl, HCl) through the column. Spent regenerant exits the column with the targeted ions in solution and may require further treatment prior to permanent disposal. The column is then flushed to remove residual regenerant, and effluent treatment can recommence.

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TABLE 24.14 Other treatment methodologies (continued)

Method	Description
Ion exchange (continued)	The spent regenerant solutions, after being used to regenerate the resin, are the main residuals produced by the use of IX in an effluent treatment process. These solutions contain the targeted ions (e.g., metals, ammonia, selenium) at pH and concentrations that are not suitable for release to the environment and thus require further management.
Adsorption	Adsorption may be used to precipitate molecules from a liquid or gas onto a solid substrate. For example, zero valent iron (ZVI) may be used to remove selenium via the reduction and precipitation of oxidized forms of selenium (selenate and selenite) to insoluble elemental selenium and also via coprecipitation of selenite with ferric hydroxide. The process is carried out at slightly acidic pH (pH 4–6) to promote the oxidation of ZVI to ferric iron. ZVI may be supplied as granular ZVI, shredded cast iron, steel wool, iron filings, iron powder, or nanoscale iron. The selection of media will depend on site-specific factors (e.g., media availability, delivery costs, residuals disposal) and required removal efficiency.

RECLAMATION

Reclamation as earlier defined is the process of implementing a portion of the closure plan to restore the land to a pre-mine or regulated post-mine land use. Reclamation, like closure, can and often does begin during planning and extends through operations. Reclamation often includes post-closure monitoring and maintenance of modified areas (Table 24.15).

TABLE 24.15 Reclamation regulations and plan elements

Regulations for Development of Reclamation Plans	Typical Reclamation Plan Elements
The Surface Mining Control and Reclamation Act of 1977 (SMCRA) is a federal regulation of the environmental effects of coal mining in the United States. Under SMCRA, prior to receiving a mining permit, operators must present a detailed and comprehensive plan for reclaiming the land after mining has been completed. Various state and local regulations, including bonding requirements, exist.	• Pre-mining condition and use of the land to be mined • Proposed use of the land after reclamation • Estimated timetable for the reclamation • Actions for complying with the relevant air and water quality laws

Mine Reclamation and Closure Planning

The development of a closure plan requires communication and engagement among stakeholders and is essential for reducing a mining company's long-term liabilities associated with the site (Table 25.16).

TABLE 24.16 Closure plan development and plan elements

Plan Elements	Plan Development Process
• Compliance with regulatory requirements • Understanding and mitigating for the protection of public health and safety during and post-closure • Considering sustainability of the land, water, ecosystem, and community • Understanding and mitigating for social impacts • Developing a basis for estimating and managing costs associated with closure and post-closure operation and maintenance • Optimizing costs for all parties during and post-closure • Achieving land-use conditions • Enhancing corporate responsibility of the mining company and other stakeholders	• Reviewing data, stakeholder objectives, and the current status of plans for closure • Establishing closure plan strategies, criteria, and objectives • Assessing risks and evaluating closure options • Selecting preferred closure plan option • Developing a cost estimate for preferred closure option • Establishing a plan for updates and improvements • Updating a closure plan and documenting changes

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CLIMATE CHANGE

Climate change can directly impact these resources resulting in a potential risk to mining operations, infrastructure, and people. Examples of these potential impacts include, but are not limited to the following:

- Droughts resulting in limited water supply available, or wet periods resulting in excessive water inventory during mining and closure
- Changes in temperatures and evaporation rates, resulting in changes in requirements for dust and fire controls/suppression activities
- Inaccurately designed water storage/conveyance facilities for possibly larger flood volumes and flows, resulting in limited access and/or requirements to release water
- Change in water quality requirements because of changes in baseline levels in discharge streams

Regulating agencies are starting to include requirements for evaluating climate change within permits for mining and closure. The International Council on Mining & Metals (ICMM) framework provides a methodology for evaluating climate change risk and opportunities that includes elements such as climate adaption plans, actions, risk mitigation, remedy plans, monitoring, and contexts for collaboration (ICMM 2013).

For more detail on environment and reclamation, the reader is directed to the following references:

- EPA (2014)
- ICMM (2006)
- International Network on Acid Prevention (INAP 2014)
- Jessup Bingham (2011)
- Pouw et al. (2014)
- Tremblay and Hogan (2001)

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Jessup Bingham, E.L. 2011. Closure planning. In *SME Mining Engineering Handbook*, 3rd ed., vol. 2. Edited by P. Darling. Englewood, CO: SME. pp. 1753–1764.

Pouw, K., Campbell, K., and Babel, L. 2014. *Study to Identify BATEA for the Management and Control of Effluent Quality from Mines*. MEND Report 3.50.1. Ottawa, ON: Mine Environment Neutral Drainage (MEND) Program and Mining Association of Canada.

Tremblay, G.A., and Hogan, C.M., eds. 2001. [Mine Environment Neutral Drainage] *MEND Manual*. MEND 5.4.2. 6 vols. Ottawa, ON: MEND.

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CHAPTER

25

Bonding and Liabilities

Brent C. Bailey, P.E.

BONDING (FINANCIAL ASSURANCE)

Closure is the general term used to describe all the activities involved in decommissioning a mining operation and/or processing facility including reclamation, revegetation, removing equipment and structures, removal of chemicals and reagents, removal of hazardous wastes, remediation of any releases of hazardous substances to the environment, and postclosure monitoring. Closure refers to decommissioning an operation in accordance with its reclamation plan (or closure plan) to meet the prescribed land uses, protect human health and the environment, and minimize the need for further maintenance.

A mine operation generally must guarantee that it will perform closure or reclamation in accordance with its closure plan or reclamation plan before beginning operations. This guarantee is often provided with a bond from a surety company, but many regulating entities allow the use of cash deposits, certificates of deposit, irrevocable letters of credit, trust funds, or other security instruments.

The amount of a bond should equal the cost of successfully completing reclamation of the disturbed lands. The amount can be determined by applying the general principles of cost estimating. The following methodology demonstrates a general approach to developing reclamation cost estimates (California Department of Conservation 2019).

- Describe the tasks to be performed:
  - Reclamation—may include establishing final slopes on all cuts and fills, removal of haul/access roads, constructing drainage/erosion controls, decompacting stockpile areas, topsoil replacement and distribution, finish grading, demolition and disposal of building foundations and other underground structures (i.e., storage tanks and septic tanks).
  - Revegetation—may include soil preparation and amendment, mulching, installation of irrigation systems, collection of custom seeds and plants, nursery services, hydroseeding, seeding and planting, plant protection, and revegetation maintenance.
  - Removal of equipment and structures—may include dismantling and removing of plant equipment and buildings. Although there is a salvage value to the equipment and buildings, separate costs for removal of each major piece of equipment and building should be estimated. Similarly, the salvage value for each major piece of equipment and building should be estimated and deducted from the corresponding cost of removal to provide a net cost (or salvage value).
  - Miscellaneous costs—may include cleanup of boneyard areas, well closures, remediation of any contaminated soil, disposal of chemical or hazardous substances, and establishing access restrictions.
  - In identifying the various reclamation tasks, it is important to include the tasks of reclaiming mineral processing facilities. Such facilities may require additional special considerations such as detoxification of process solutions, pumping costs (electricity), water treatment costs, removal of tanks and lined ponds, and disposal of sludge. For example, reclaiming a cyanide leaching operation requires detoxification of the leached ore. Column detoxification tests can be used to determine the amount of fresh water or detoxifying agent required to adequately detoxify all cyanide in the pond solutions and in the ore that had been processed with cyanide solutions. Consideration must be given to future drain-down solutions and the possibility of reducing these with a cover and applying treatment if they do not meet water quality requirements (Posey et al. 1997).
  - Monitoring
- Identify the equipment, labor, materials, and miscellaneous items necessary to complete the proposed tasks, and determine the unit costs for these items.
- Calculate the necessary quantities required for each of the tasks (i.e., cubic yards of material to be moved, hours of labor). This can be determined from production rates of equipment and labor and from standard seed application rates, plant densities, or hydroseeding application rates.

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TABLE 25.1 Costs covering contingencies

Total Direct Cost	Contingency, %
0–$500,000	10
$500,000–$5 million	7
$5 million–$50 million	4
Greater than $50 million	2

Source: OSMRE 1987

- Determine the total costs for equipment, labor, and materials from the quantities and the unit costs for a particular task.
- Determine the total base cost of reclamation by adding the costs of the individual tasks.
- Add charges for mobilization, supervision, profit and overhead, and contingencies.

Mobilization costs are attributed to moving personnel, equipment, office trailers, and support facilities to the project site. These costs vary depending on the site location and the cost of the reclamation work. They normally vary between 1% and 5% of the total direct cost of reclamation (State of California 1998).

Supervision refers to management of the reclamation programs and can range from 2% to 7% for a $100,000,000 project to a $10,000 project, respectively (State of California 1998).

Many regulating entities require that the reclamation cost estimate be prepared as if the site had been abandoned by an operator and a third-party contractor was performing the reclamation (U.S. Department of Interior 1997). Profit and overhead are provided in the case of a third-party contractor performing the reclamation. They can range from 3% to 14% for a $100,000,000 project to a $250,000 project, respectively (State of California 1998). Contingencies are supplied to cover uncertainties in the cost estimate. Table 25.1 provides a sliding scale.

Principles of Bond Release

After a mining company has met all the reclamation requirements of the approved permit and regulatory program, the mine as a standard practice endeavors to seek release of the reclamation bond(s) associated with the extraction of minerals. The permittee may apply for release of the bond on all or part of the permit area or increment as reclamation is completed, which is called *concurrent reclamation*. Some agencies use the term *contemporaneous reclamation*. Contemporaneous reclamation is the action of reclaiming lands as disturbance occurs during mineral extraction.

The Surface Mining Control and Reclamation Act (SMCRA), and subsequent regulations and policy administered by the U.S. Department of the Interior's Office of Surface Mining Reclamation and Enforcement recognizes three discrete phases of reclamation for purposes of bond release (OSMRE Western Region 2009).

Phase I. Phase I bond release may occur upon the completion of backfilling, regrading (which may include the replacement of topsoil), and drainage control, in accordance with the approved reclamation plan. Major Phase I reclamation intent is

- Chemical and physical demonstration through sampling of permit compliance of topsoil and/or topsoil substitute;
- Demonstration of permanent drainage control, roads, and other permanent facilities by providing as-built drawings; and
- Justification of any variance of the as-built versus permitted design that such variance impacts are benign.

Phase II. Phase II bond release may occur after vegetation has been established in accordance with the approved reclamation plan, or after prime farmland productivity has been restored, if applicable, and when contributions of suspended solids to streamflow or runoff outside the permit area are not in excess of the requirements established in SMCRA, the regulations, and the approved permit. Major Phase II reclamation intent is

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- Vegetation successfully established and capable of stabilizing the soil surface from erosion;
- If applicable, documentation that soil productivity for prime farmlands has returned to the equivalent levels of yields;
- Demonstration that suspended solids from the reclaimed lands are not contributing to streamflow or runoff outside the permitted area in excess of the requirements set by regulations; and
- Documentation that the permittee or the landowner has provided for sound future maintenance of all approved permanent impoundments.

Phase III. Phase III bond release requires meeting the revegetation success standards and follows completion of the revegetation responsibility period, provided the site remains in compliance with all applicable reclamation requirements. Major Phase III reclamation intent is

- Information required for the Phase I and Phase II bond release applications if not previously submitted, and
- Demonstration that revegetation meets the success standards in the regulations and approved permit.

Reclamation demonstrates support of the approved post-mining land use(s):

- It must be demonstrated that all surface mining and reclamation activities have been conducted to minimize disturbance of the hydrologic balance within the permit and adjacent areas; prevent material damage to the hydrologic balance outside the permit area; assure quantity and quality are suitable to support approved post-mining land uses; assure the water rights of other users have been protected or replaced; and all is in accordance with terms and conditions of the approved permit.
- No bond shall be fully released until all the provisions of SMCRA, the performance standards of the regulatory program, the approved permit, and the approved reclamation plan demonstrate that all surface coal mining and reclamation operations have been successfully completed. Specific measures and methodologies may be required to achieve bond release. Criteria, such as agricultural production yields, stream turbidity, soil composition, or slope gradients, may be necessary along with the associated recordkeeping.

LIABILITIES

Liabilities are the aspects, elements, or components of an operation or facility that impose an obligation to do or refrain from doing something and deduct from the value of a property (Gifis 1975). Environmental liability relates to (1) permitting (i.e., Can the permits be obtained in a timely manner? Or, for an operating facility, are the permits in place and is the facility in compliance?) and (2) contingent liabilities associated with environmental claims (i.e., claims under an environmental statute, such as a hazardous waste cleanup statute) (Garver and Butler 1993).

Environmental Due Diligence

Future liability and financial risk can be minimized through environmental due diligence, which is the process of investigating and determining the existence of potential environmental liabilities or risk involved in the transfer of property ownership. The following is an abbreviated outline of an approach to environmental due diligence (Garver and Butler 1993).

- Identify constraints on the operation or expansion.
  - Identify all current permits imposed by regulating authorities and determine the expiration or renewal dates. For each permit:
    - Determine how long the renewal process is expected to take and when the application for renewal should be submitted.
    - Identify any substantive criteria that can affect renewal.
  - Determine the terms and conditions of the permits.
    - Determine what limits, if any, the permit places on operations or expansions. If there are limits, determine if the permit can be amended to accommodate any new operating plans.
    - Determine whether permits can be transferred.

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    - If the operation or facility is exempt from any current applicable permitting requirements, determine whether the exemption can remain applicable if the facility is transferred to a new owner.
    - Review applicable land use plans to confirm that contemplated uses are consistent.
  - Determine the *regulatory environment* in which key operations exist.
    - Identify regulatory agencies and determine the status of the relationship with the seller.
    - Determine the nature and extent of opposition to the project and assess the likelihood of appeal, citizen litigation, or other actions that could delay permitting or hinder operations.
  - Review the status of any performance bonds and liability policies. Determine how the transfer of authority to operate the facility may affect bonding.
  - Review company files and interview key employees.
  - Review regulatory agency records and interview key employees in regulatory agencies. (Caution: This must be done in cooperation with the target company.)
  - Review permit regulations, guidance, and correspondence.
- Identify significant contingent liabilities.
  - Identify potential significant contingent liabilities.
    - Define a *threshold of materiality* or a *risk tolerance* for the buyer.
    - Develop an understanding of the history of the company, facilities, or properties.
    - Define or identify potential contingent liabilities for the companies or properties to be acquired.
  - Identify potential sources of significant environmental liabilities.
    - Review laws defining liabilities and remediation for releases of hazardous substances.
    - Determine potential unfunded or underfunded reclamation expenses.
    - Review waste disposal practices for solid and hazardous wastes regulated under solid waste disposal statutes.
    - Determine presence of other hazardous or toxic substances that may trigger liability or cleanup obligations.
    - Determine presence and removal and cleanup obligations associated with any underground storage tanks.
    - Determine applicability of other environmental regulatory programs that might lead to financial liability for compliance or penalties.
    - If diligence discloses any historic or ongoing violations of permits or regulations, determine the potential for enforcement or citizen legal action.
    - Determine whether existing permits and approvals include terms that create remediation or closure obligations that extend beyond applicable law.
  - Review environmental indemnities in prior transactions.
  - Review applicable insurance policies.
  - Identify sources of information to review for environmental due diligence.
    - Target company files and interviews.
    - Perform site inspection; collect data on site where necessary.
    - Review regulatory agency records (if possible).
    - Review all information relating to threatened or pending litigation with environmental claims.
    - Review title information for evidence of possible environmental problems.

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CHAPTER

26

Key Resources for the Mining Industry

Steven A. Richards, P.E.

With the advent of powerful and various search engines on the Internet, such as Google, Bing, Yahoo, and Firefox, all you have to do is know what you are looking for and you can probably find it. For your convenience, this chapter offers a quick reference to engineering societies and boards, including societies listed by state and U.S. territory; and international mining industry resources and metals and minerals organizations. Mining industry resources are listed by country and also by mineral commodity. Mining history organizations are listed as well as U.S. state mining associations. And a selection of U.S. mining museums and mine tours wraps up the chapter.*

ENGINEERING SOCIETIES AND BOARDS

The National Council of Examiners for Engineering and Surveying (NCEES; **ncees.org**) administers the licensure process for the Professional Engineering (PE) certification.

The National Society of Professional Engineers maintains a list of all the engineering professional societies by state and U.S. territory. Following is the web address for the national organization and the URL for each state society.

National Society

National Society of Professional Engineers (NSPE): **nspe.org**

Societies Listed by State and U.S. Territory

Alabama Society of Professional Engineers: **myaspe.com**
Alaska Society of Professional Engineers: **alaskaspe.org**
Arizona Society of Professional Engineers: **arizonaspe.org**
Arkansas Society of Professional Engineers: **arkansasengineers.com**
California Society of Professional Engineers: **nspe-ca.org**
Colorado: NSPE-Colorado: **pec.org**
Connecticut Society of Professional Engineers: **ctspe.net**
DC Society of Professional Engineers: **nspe-dc.org**
[Delaware] NSPE-DE: **nspe-de.org**
Florida Engineering Society: **fleng.org**
Georgia Society of Professional Engineers: **gspe.org**
Guam Society of Professional Engineers: **nspe-gu.org**
Hawaii Society of Professional Engineers: **nspe-hi.org**
Hawaii Council of Engineering Societies: **hceshawaii.org**
Idaho Society of Professional Engineers: **idahospe.org**
Illinois Society of Professional Engineers: **illinoisengineer.com**
Indiana Society of Professional Engineers: **indspe.org**
Iowa Engineering Society: **iaengr.org**
Kansas Society of Professional Engineers: **kansasengineer.org**
Kentucky Society of Professional Engineers: **kyengcenter.org/page/A1**
Louisiana Engineering Society: **les-state.org**

* Disclaimer: SME makes no warranty of these aforementioned sites or information. Use of links is at user's own risk. Any site or information listed does not represent an endorsement by SME or its agents.

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Maine Society of Professional Engineers: **mespe.org**
Maryland Society of Professional Engineers: **mdspe.org**
Massachusetts Society of Professional Engineers: **mspe.com**
Michigan Society of Professional Engineers: **michiganspe.org**
Minnesota Society of Professional Engineers: **mnspe.org**
Mississippi Engineering Society: **msengsoc.org**
Missouri Society of Professional Engineers: **mspe.org**
Montana Society of Engineers: **mtengineers.org**
Nebraska Society of Professional Engineers: **nspenebraska.org**
Nevada: National Society of Professional Engineers Nevada: **nspenv.org**
New Hampshire Society of Professional Engineers: **nhspe.org**
New Jersey Society of Professional Engineers: **njspe.org**
New Mexico Society of Professional Engineers: **nmspe.org**
New York State Society of Professional Engineers: **nysspe.org**
North Carolina: Professional Engineers of North Carolina: **penc.org**
North Dakota Society of Professional Engineers: **ndspe.org**
Ohio Society of Professional Engineers: **ohioengineer.com**
Oklahoma Society of Professional Engineers: **ospe.org**
Oregon: Professional Engineers of Oregon: **oregonengineers.org**
Pennsylvania Society of Professional Engineers: **pspe.org**
Puerto Rico Society of Professional Engineers: **nspe-pr.org**
Rhode Island Society of Professional Engineers: **nspe-ri.org**
South Carolina Society of Professional Engineers: **scspe.org**
South Dakota Engineering Society: **sdes.org**
Tennessee Society of Professional Engineers: **tnspe.org**
Texas Society of Professional Engineers: **tspe.org**
Utah Society of Professional Engineers: **nspe-ut.org**
Vermont Society of Professional Engineers: **vtpe.org**
Virginia Society of Professional Engineers: **vspe.org**
Washington Society of Professional Engineers: **washingtonengineer.org**
West Virginia Society of Professional Engineers: **nspe-wv.org**
Wisconsin Society of Professional Engineers: **wspe.org**
Wyoming Society of Professional Engineers: **nspe-wy.org**

U.S. MINING ENGINEERING SCHOOLS

Colorado School of Mines, Mining Engineering program: **mines.edu**
Missouri University of Science and Technology, Mining Engineering program: **mining.mst.edu**
Montana Tech, School of Mines and Engineering: **mtech.edu/mines-engineering**
New Mexico Tech, New Mexico Institute of Mining and Technology, Department of Mineral Engineering: **nmt.edu/academics/mining**
PennState (Pennsylvania State University), College of Earth and Mineral Sciences, Mining Engineering program: **ems.psu.edu/academics/find-program-study/mining-engineering-bs**

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South Dakota School of Mines & Technology (SD Mines), Department of Mining Engineering and Management: **sdsmt.edu/MEM**

Southern Illinois University (SIU)-Carbondale, Mining and Mineral Resources Engineering Department: **engineering.siu.edu/mining**

University of Alaska Fairbanks, College of Engineering and Mines, Department of Mining and Geological Engineering: **cem.uaf.edu/mingeo**

University of Arizona, College of Engineering, Department of Mining and Geological Engineering: **mge.engineering.arizona.edu**

University of Kentucky, College of Engineering, Department of Mining Engineering: **engr.uky.edu/mng**

University of Nevada, Reno, Mackay School of Earth Sciences & Engineering, Mining and Metallurgical Engineering Department: **unr.edu/mining**

University of Utah, College of Mines and Earth Sciences, Department of Mining Engineering: **mining.utah.edu**

Virginia Tech, Department of Mining and Minerals Engineering: **mining.vt.edu**

West Virginia University, Mining Engineering Department: **mine.statler.wvu.edu**

INTERNATIONAL MINING INDUSTRY RESOURCES

Euromines [European Association of Mining Industries]: **euromines.org**

Federation of the International Geo-Engineering Societies (FedIGS): **geoengineeringfederation.org**

Global Mining Guidelines Group: **gmggroup.org**
Facilitates global mining collaboration on solutions to common industry problems, needs, and technology through standards, guidelines, and best practices

IMWA: International Mine Water Association: **imwa.info**

International Academy of Mine Safety & Health (IAMSH) of the Society for Mining, Metallurgy & Exploration (SME): **smecmsp.org/index.cfm/iamsh**

International Association for Engineering Geology and the Environment (IAEG): **iaeg.info**

International Association of Hydrogeologists: **iah.org**

International Commission on Large Dams/Commission Internationale des Grands Barrages (CIGB/ICOLD): **icold-cigb.org**

International Council on Mining & Metals (ICMM): **icmm.com**

International Geosynthetics Society: **geosyntheticssociety.org**

International Precious Metals Institute (IPMI): **ipmi.org**

International Society for Rock Mechanics and Rock Engineering (ISRM): **isrm.net**

International Society of Explosives Engineers (ISEE): **isee.org**

International Society of Soil Mechanics and Geotechnical Engineering (ISSMGE): **issmge.org**

International Tunnelling and Underground Space Association/Association Internationale des Tunnels et de l'Espace Souterrain (ITA/AITES): **ita-aites.org**
Based in Lausanne, Switzerland [in English and French]

International Women in Mining: **internationalwim.org**

IUGS [International Union of Geological Sciences]: **iugs.org**
121 national members; promotes development of the Earth sciences

Society for Mining, Metallurgy & Exploration (SME): **smenet.org**
International society with more than 13,000 members

Society of Mining Professors/Societät der Bergbaukunde: **miningprofs.org**

Women in Mining (WIM) Education Foundation: **womeninmining.org**
Nonprofit organization dedicated to promoting the employment, retention, and professional development of women in the mining and minerals sector with chapters in countries across the world

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METALS AND MINERALS ORGANIZATIONS

American Association of State Geologists (AASG): **stategeologists.org**

Copper Club: **copperclub.org**

Copper Development Association: **copper.org**

Florida Phosphate: **floridaphosphate.org**

International Copper Association: **copperalliance.org**

Minerals Council South Africa: **mineralscouncil.org.za**

National Stone, Sand & Gravel Association (NSSGA): **nssga.org**

North American Metals Council (NAMC): **namc.org**

Responsible Jewellery Council: **responsiblejewellery.com**

Rocky Mountain Mineral Law Foundation: **rmmlf.org**

Salt Institute: **saltinstitute.org**
Excellent archives from meetings and proceedings on salt, potash, and other minerals, as well as mining and processing

HISTORICAL ASSOCIATIONS

Australasian Mining History Association (AMHA): **mininghistory.asn.au**
Promotes and coordinates activities and programs relating to Australasian mining history

Gold Country Visitors Association, California: **visitgoldcountry.com**
Seeks to develop open communication throughout California's Gold Country and promote tourism

History of Mining in Idaho, Idaho Mining Association: **mineidaho.com/education/history**

Mine Explorer Society: **mineexplorer.org.uk**
Mine exploring, mining history, and historical mining archives with photographs and information on disused mines in the United Kingdom

Mining Heritage Trust of Ireland (MHTI): **industrialheritageireland.info/mhti**
Promotes awareness, appreciation, and conservation of Ireland's mining heritage

Mining History Association (MHA): **mininghistoryassociation.org**
Independent organization of individuals interested in the history of mining and metallurgy

National Association of Mining History Organisations (NAMHO): **namho.org**
National body for mining history in the United Kingdom and Ireland

National Mining Memorabilia Association: **mining-memorabilia.co.uk**
Background and history of mining memorabilia collected and researched in the United Kingdom

Peak District Mines Historical Society: **pdmhs.co.uk**
Seeks to preserve and promote interest in the mining history of the U.K.'s Derbyshire Peak district and surrounding areas

Welsh Mines Society: **welshmines.org**
Preservation of Welsh mining sites and history

MINING INDUSTRY RESOURCES BY COUNTRY

Australia

AMMA: Australian Resources and Energy Group: **amma.org.au**
Provides industry advocacy, workplace advice, and training solutions for companies in the resources and energy sector in Australia

AMPLA Limited: **ampla.org**
Not-for-profit organization for lawyers practicing in resources and energy law

Association of Mining and Exploration Companies (AMEC): **amec.org.au**
Industry body for junior and mid-size mineral exploration and mining companies within Australia

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AUSTMINE: **austmine.com.au**
Leading industry body for the Australian Mining, Equipment, Technology and Services (METS) sector; mining industry export association

Australasian Institute of Mining and Metallurgy (AusIMM): **ausimm.com**

Australasian Institute of Surface Finishing (AISF): **aisf.org.au**
Metal finishing suppliers, classifieds, resources, bookshop, magazine

Australasian Joint Ore Reserves Committee (JORC): **jorc.org**
Sets the professional code of practice for reporting of minerals exploration results, mineral resources, and ore reserves

Australian Tunnelling Society (ATS): **ats.org.au**

Australian Drilling Industry Association: **adia.com.au**

Australian Gas Association (AGA): **aga.asn.au**

Australian Geomechanics Society: **australiangeomechanics.org**

Australian Geoscience Council (AGC): **agc.org.au**
Council of earth science societies in Australia; represents eight major geoscientific societies

Australian Institute of Geoscientists: **aig.org.au**

Australian Institute of Petroleum: **aip.com.au**

Australian Mineral Economics (AME): **usa.amegroup.com**
Provides resource engineering and industry analysis

Australian Petroleum Production and Exploration Association (APPEA): **appea.com.au**

Australian Society of Exploration Geophysicists: **aseg.org.au**

Australian Water Association: **awa.asn.au**

Chamber of Minerals and Energy of Western Australia (CME): **cmewa.com**

Engineers Australia: **engineersaustralia.org.au**
The Institution of Engineers Australia (IEAust) or Engineers Australia; Australia's principal engineering association

Geological Society of Australia: **gsa.org.au**

MCA Victoria Minerals Tertiary Education Council: **minerals.org.au/mca-victoria**
Through MCA Victoria, assists the education community in Victoria to deliver useful information about Australia's minerals industry

Melbourne Mining Club: **melbourneminingclub.com**

Mine Managers Association of Australia (MMAA): **minemanagers.com.au**

Minerals Council of Australia (MCA): **minerals.org.au**
Representing Australia's minerals industry

National Safety Council of Australia (NSCA): **nsca.org.au**

New South Wales Minerals Council (NSWMin): **nswmin.com.au**

Petroleum Exploration Society of Australia (PESA): **pesa.com.au**

Queensland Resources Council (QRC): **qrc.org.au**

South Australian Chamber of Mines and Energy (SACOME): **sacome.org.au**

Surveying & Spatial Sciences Institute (SSSI): **sssi.org.au**
South Australian institute supporting professional disciplines of land, hydrographic, engineering, and mining surveying

Sydney Mineral Exploration Discussion Group (SMEDG): **smedg.org.au**
Meetings information, news, photo library, conference publications

Sydney Mining Club: **sydneyminingclub.org**

Tasmanian Minerals, Manufacturing & Energy Council (TMEC): **tasminerals.com.au**

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Brazil

Instituto de Brasileiro de Mineração (IBRAM): **ibram.org.br** [in Portuguese]
Brazilian Mining Association

International Copper Association (ICA), Latin America: **procobre.org/es** [in Portuguese and Spanish]

Canada

Alberta Centre for Reclamation and Restoration Ecology (ACRRE)/Oil Sands Research and Information Network (OSRIN): **acrre.ualberta.ca/Resources/OSRIN-2009-2014**

Association for Mineral Exploration (AME) British Columbia: **amebc.ca**
Issues, events, courses, and workshops related to the mineral exploration industry

Association minière du Québec (AMQ): **amq-inc.com** [in French]

Association of Consulting Engineering Companies of Canada (ACEC): **acec.ca**

Association of Professional Engineers & Geoscientists of Saskatchewan (APEGS): **apegs.ca**

Association of Professional Engineers and Geoscientists of Alberta (APEGA): **apega.ca**

Boundary Mining Association: **boundarymining.com**
Located in Grand Forks, British Columbia

Canada Mining Innovation Council (CMIC)/Conseil canadien de l'innovation minière (CCIM): **cmic-ccim.org**
Network of industry, academic, and government leaders whose purpose is to improve the competitiveness of a responsible mining industry in Canada

Canadian Dam Association (CDA)/Association canadienne des barrages (ACB): **cda.ca** [in French and English]

Canadian Institute of Mining, Metallurgy and Petroleum (CIM): **cim.org**

Canadian International Resources and Development Institute (CIRDI): **cirdi.ca**
Coalition of the University of British Columbia, Simon Fraser University, Polytechnique Montréal

Canadian Land Reclamation Association/Association canadienne de réhabilitation des sites dégradés: **clra.ca** [in French and English]

Canadian Water Resources Association/Association canadienne des resources hydriques: **cwra.org** [in French and English]

Centre of Training Excellence in Mining (CTEM): **bc-ctem.ca**
Information about training and career opportunities in British Columbia's mineral industries

Chamber of Mines of Eastern British Columbia: **cmebc.com**

Devonshire Initiative: **devonshireinitiative.org**
Forum for leading international development nongovernmental organizations (NGOs) and mining companies to come together in response to the emerging social agenda surrounding mining

Engineering Institute of Canada (EIC): **eic-ici.ca**

Engineers & Geoscientists New Brunswick/Ingénieurs Geoscientifiques (Association of Professional Engineers and Geoscientists of New Brunswick [APEGNB]): **apegnb.com**

Engineers and Geoscientists BC (Association of Professional Engineers and Geoscientists of the Province of British Columbia): **egbc.ca**

Engineers Canada/Ingénieurs Canada: **engineerscanada.ca** [in French and English]

Engineers Geoscientists Manitoba (Association of Professional Engineers and Geoscientists of the Province of Manitoba [APEGM]): **apegm.mb.ca**

Engineers PEI (Association of Professional Engineers of the Province of Prince Edward Island [APEPEI]): **engineerspei.com**

Engineers Yukon (Association of Professional Engineers of Yukon [APEY]): **apey.yk.ca/about-us.php**

Fraser Institute: **fraserinstitute.org**
Independent nonpartisan research and educational organization

Geoscientists Canada/Géoscientifiques Canada: **geoscientistscanada.ca** [in French and English]

Kamloops Exploration Group: **keg.bc.ca**

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Klondike Placer Miners' Association: **kpma.ca**

Mineralogical Association of Canada/Association mineralogique du Canada: **mineralogicalassociation.ca**

Minesite Drainage Assessment Group: **mdag.com**
Group of companies that investigate, interpret, and publish on the environmental and geochemical effects of mining

Mining Association of British Columbia (MABC): **mining.bc.ca**

Mining Association of Canada (MAC): **mining.ca**

Mining Association of Nova Scotia (MANS): **tmans.ca/home**

Mining Society of Nova Scotia: **miningsocietyns.ca**

MIRARCO [Mining Innovation Rehabilitation and Applied Research Corporation]: **mirarco.org**
Promotes mining innovation and provides a bridge between researchers and industry; formed through collaboration with Laurentian University

MSTA Canada (Mining Suppliers Trade Association): **mstacanada.ca**

NAPEG: **napeg.nt.ca**
Responsible for licensing professional engineers and geoscientists in the Northwest Territories and Nunavut

Not Your Grandfather's Mining Industry: **notyourgrandfathersmining.ca**
Mining Association of Nova Scotia–sponsored site about what has changed in the mining industry over the years.

Ontario Mining Association (OMA): **oma.on.ca/en/index.asp**

Ordre des ingénieurs du Québec (OIQ): **oiq.qc.ca/pages/accueil.aspx?lang=en** [in French and English]

Pembina Institute: **pembina.org**

Professional Engineers Ontario (PEO): **peo.on.ca**

Prospectors & Developers Association of Canada (PDAC): **pdac.ca**

Saskatchewan Mining Association (SMA): **saskmining.ca**

Smithers Exploration Group: **smithersexplorationgroup.com**

Sudbury Area Mining Supply and Service Association (SAMSSA): **samssa.ca**

Wetland Stewardship Partnership (WSP): **bcwetlands.ca**
Group of government and non-government organizations dedicated to the conservation of wetlands and other sensitive ecosystems in British Columbia

Women in Mining, Toronto Branch: **women-in-mining.com**

Chile

Instituto de Ingenieros de Minas de Chile (IIMCH) [Chilean Institute of Mining Engineers]: **iimch.cl** [in Spanish]

Sociedad Nacional de Minería (SONAMI) [National Mining Society]: **sonami.cl/site** [in Spanish]

Indonesia

Indonesian Association of Geologists (IAGI): **iagi.or.id**

JATAM: Mining Advocacy Network: **english.jatam.org**

Peru

Instituto de Ingenieros de Minas del Perú: **iimp.org.pe** [in Spanish]

Sociedad Geológica del Perú (SGP): **sgp.org.pe** [in Spanish]

Sociedad Nacional de Minería Petróleo y Energía (SNMPE): **snmpe.org.pe** [in Spanish]

Philippines

Philippine Society of Mining Engineers: **psem.ph**

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South Africa

Association of Mine Managers South Africa (AMMSA): **ammsa.org.za**

Congress of South African Trade Unions (COSATU): **cosatu.org.za**

Geological Society of South Africa (GSSA): **gssa.org.za**

Institute of Safety Management: **iosm.co.za**

Maputo Corridor Logistics Initiative (MCLI): **mcli.co.za**
Nonprofit organization of infrastructure investors, service providers, and stakeholders

Mine Medical Professionals Association: **mmpa.org.za**

Mine Ventilation Society of South Africa (MVSSA): **mvssa.co.za**

Minerals Council of South Africa: **mineralscouncil.org.za**
News, data, and policy information

Mining Industry Association of Southern Africa (MIASA): **miasa.org.za**
Membership includes the chambers of mines of Botswana, Namibia, South Africa, Tanzania, Zambia, and Zimbabwe

Mining Lekgotla: **mininglekgotla.co.za**
Collaboration between the Chamber of Mines of South Africa, Department of Mineral Resources, and the National Union of Mineworkers

Mining Qualifications Authority (MQA): **mqa.org.za**

National Union of Mineworkers (NUM): **num.org.za**
Largest recognized collective bargaining agent representing workers in the mining, construction, and electrical energy industries in South Africa

SA Colliery Managers' Association (SACMA): **sacollierymanagers.org.za**
Organization of colliery (coal) mining managers

Samda [South African Mining Development Association]: **samda.co.za**
An industry body that represents black investors

South African Association for Geotechnology (SAAG): **saag.co.za**

South African Council for Natural Scientific Professions (SACNASP): **sacnasp.org.za**

South African Geophysical Association (SAGA): **sagaonline.co.za**

South African Institute for Engineering & Environmental Geologists: **saieg.co.za**

Southern African Coal Processing Society: **sacoalprep.co.za**

Southern African Institute of Mining and Metallurgy: **saimm.co.za**

Sweden

Geological Society of Sweden: **geologiskaforeningen.se/index_eng.html** [in Swedish and English]

Nordic Galvanizers: **zincinfo.se** [in Swedish, Norwegian, Danish, and English]

Swedish Mineral Processing Research Association: **minfo.se** [in Swedish]

Swedish Mining Association (SveMin): **svemin.se** [in Swedish and English]

United Kingdom

Association of Mining Analysts: **ama.org.uk**

Black Country Geological Society: **bcgs.info**

Black Country Society: **blackcountrysociety.com**

British Geological Survey (BGS): **bgs.ac.uk**

British Hydrological Society: **hydrology.org.uk**

Carn Brea Mining Society: **carnbreaminingsociety.org.uk**

Geological Society of London: **geolsoc.org.uk**

Hydrogeological Group: **hydrogroup.org.uk**
Special interest group of the Geological Society of London

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Institute of Explosives Engineers: **iexpe.org**

Institute of Materials Finishing: **materialsfinishing.org**

Institute of Materials, Minerals and Mining (IOM3): **iom3.org**

Minor Metals Trade Association (MMTA): **mmta.co.uk**

NAFEMS: **nafems.org**
International association for the engineering modelling, analysis, and simulation community

North of England Institute of Mining and Mechanical Engineers: **mininginstitute.org.uk**

Pipe Jacking Association (PJA): **pipejacking.org**
Represents leading contractors, pipe suppliers, machinery manufacturers, consultants, and other suppliers in the United Kingdom

United States

American Chemical Society Geochemistry Division (ACS GEOC): **geochemistrydivision.sites.acs.org**

American Exploration & Mining Association: **miningamerica.org**

American Geophysical Union (AGU): **sites.agu.org**

American Geosciences Institute (AGI): **americangeosciences.org**

American Institute of Hydrology (AIH): **aihydrology.org**

American Institute of Mining, Metallurgical, and Petroleum Engineers (AIME): **aimehq.org**

American Institute of Professional Geologists (AIPG): **aipg.org**

American Rock Mechanics Association: **armarocks.org**

American Society of Civil Engineers: **asce.org**

American Society of Mining & Reclamation (ASMR): **asmr.us**

American Water Resources Association: **awra.org**

Association of State Wetland Managers (ASWM): **aswm.org**

Energy & Mineral Law Foundation (EMLF): **emlf.org**

Federation of International Trade Associations (FITA): **fita.org**

Geological Society of America (GSA): **geosociety.org**

Groundwater Resources Association of California (GRA): **grac.org**

International Institute of Minerals Appraisers: **mineralsappraisers.org**

International Marine Minerals Society (IMMS): **immsoc.org**

Joseph A. Holmes Safety Association: **holmessafety.org**
Promotes health and safety in the mining industry

Minerals Education Coalition (MEC): **mineralseducationcoalition.org**

Minerals, Metals & Materials Society (TMS): **tms.org**

Mining & Metallurgical Society of America (MMSA): **mmsa.net**

Mining Association of South Carolina (SC Mines): **scmines.com**

Mining Foundation of the Southwest: **miningfoundationsw.org**

National Association for Surface Refinishing: **nasf.org**

NGWA: The Groundwater Association: **ngwa.org**

NMA: National Mining Association: **nma.org**

NSSGA: National Stone, Sand & Gravel Association: **nssga.org**

Quincy Mine Hoist Association: **quincymine.com**

Rocky Mountain Coal Mining Institute (RMCMI): **rmcmi.org**

Society of Wetland Scientists: **sws.org**

Surface Technology Environmental Resource Center (STERC): **nmfrc.org**

United Mine Workers of America (UMWA): **umwa.org**

United States Mine Rescue Association: **usminedisasters.miningquiz.com/members.htm**

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Other Countries

A3M [Alliance of Minerals and Metals]: **a3m-asso.fr** [in French]
Aluminum, copper, lead, nickel, and zinc

Association Francaise de Micromineralogie (AFM) [French Micromineralogical Society]: **micromineral.org** [in French]

Associazione Nazionale Industrie Metalli non Ferrosi (ASSOMET): **assomet.it** [in Italian]
Aluminum, copper, lead, and zinc

Chamber of Mining Engineers of Turkey (Maden Mühendisleri Odasi): **maden.org.tr/index_en.php** [in Turkish and English]

Finnish Association of Mining and Metallurgical Engineers: **vuorimiesyhdistys.fi** [in Finnish]

Finnish Mining Association (FinnMin): **kaivosteollisuus.fi/en/finnmin-finnish-mining-association** [in English] Branch association of Technologies Industries of Finland

Korean Society of Environmental Engineers: **kosenv.or.kr/eng** [in Korean and English]

La société de l'industrie minérale (SIM): **lasim.org** [in French]

New Zealand Petroleum & Minerals: **nzpam.govt.nz**

Országos Magyar Bányászati és Kohászati Egyesület [Hungarian Mining and Metallurgical Society] (OMBKE): **ombkenet.hu/index.php/bkl-banyaszat** [in Hungarian]

MINING ASSOCIATIONS BY COMMODITY

Aluminum

Aluminium Association of Greece (AAG): **aluminium.org.gr** [in Greek and English]

Aluminium Association of India: **aluminium-india.org**

Aluminium Federation (ALFED): **alfed.org.uk**
Houses the largest library of aluminium-related material

Aluminium Federation of South Africa: **afsa.org.za**

Aluminium Packaging Recycling Organisation (Alupro): **alupro.org.uk**
Representing leading aluminium packaging producers and reprocessors in the United Kingdom

Aluminium Verband Schweiz/Association Suisse de l'Aluminium: **alu.ch/fr** [in German and French]

Aluminiumindustriens Miljøsekretariat (AMS): **ams-aluminium.net**
Association for the primary aluminium industry in the Nordic countries

Aluminum Association of Canada/l'Association de l'aluminium du Canada (AAC): **aluminium.ca/en/association**
Association of Canada's three primary aluminium producers: Alcoa, Aluminerie Alouette, and Rio Tinto.

Aluminum Association: **aluminum.org**

Asociación para el Reciclado de Productos de Aluminio (ARPAL): **aluminio.org** [in Spanish]

Associação Brasileira do Alumínio/Brazilian Aluminum Association: **abal.org.br/en** [in Portuguese, English, and Spanish]

Australian Aluminium Council Ltd: **aluminium.org.au**

European Aluminium: **european-aluminium.eu**

GDA: Gesamtverband der aluminiumindustrie: **aluinfo.de** [in German and English]

IGORA Genossenschaft: **igora.ch** [in German, French, and Italian]
Aluminium recycling cooperative

Institut pour l'histoire de l'aluminium: **histalu.org** [in French]

Japan Aluminium Association: **aluminum.or.jp/english** [in Japanese and English]

World Aluminium: International Aluminum Institute: **world-aluminium.org**

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Coal

Alabama Coal Association (ACA): **alcoal.com**

American Coal Council (ACC): **americancoalcouncil.org**

American Coalition for Clean Coal Electricity (ACCCE): **americaspower.org**

Assocarboni: **assocarboni.it/assocarboni** [in Italian]
Italian association of coal operators

Association of U.K. Coal Importers and Producers (CoalImP): **coalimp.org.uk**

Carbunión (Federación Nacional de Empresarios de Minas de Carbón): **carbunion.com** [in Spanish]

Coal Association of Canada: **coal.ca**

Coal Institute: **thecoalinstitute.org**
Formerly the North Carolina Coal Institute

Coal Preparation Society of America (CPSA): **www.coalprepsociety.org/default.aspx**

DEBRIV: Bundesverband Braunkohle: **braunkohle.de/1-1-DEBRIV.html** [in German and English]
Association of all lignite-producing companies and affiliated organizations

EURACOAL (European Association for Coal and Lignite): **euracoal.eu**

Friends of Coal: **friendsofcoal.org**

GVSt: Gesamtverband Steinkohle: **gvst.de [in German]**
Hard coal producers association

Illinois Coal Association: **illinoiscoal.org**

Indiana Coal Council: **indianacoal.com**

Kentucky Coal Association (KCA): **kentuckycoal.org**

Lignite Energy Council: **lignite.com**

Maryland Coal Association: **iqcweb.com/mca**

Montana Coal Council: **montanacoalcouncil.com**

Ohio Coal Association: **ohiocoal.com**

Pennsylvania Coal Alliance (PCA): **betterwithcoal.com**

South African Coal Processing Society: **sacoalprep.co.za**

Svenska Kolinstitutet [Swedish Coal Institute]: **kolinstitutet.se** [in Swedish]

Virginia Coal and Energy Alliance (VCEA): **virginiacoalenergy.com**

West Virginia Coal Association (WV Coal): **wvcoal.com**

World Coal Association: **worldcoal.org**

Copper

Comisión Chilena del Cobre [Chilean Copper Commission] (COCHILCO):
cochilco.cl/Paginas/English/Home.aspx [in Spanish and English]

Copper Alliance Websites: **procobre.org/es/copper-alliance-websites**
Links to and addresses of worldwide trade associations (Latin America, Asia, Europe, North America, Australia, and South Africa)

Federación de Trabajadores del Cobre Chile (FTC): **ftc.cl** [in Spanish]

International Copper Study Group: **icsg.org**

Organisation of European Copper Alloy Ingot Makers (OECAM): **oecam.eu**

Diamonds

Antwerp Diamond High Council: **hrdantwerp.com/en**
Belgium diamond industry

Rapaport Diamonds.net: **diamonds.net/**
Diamond trade network; subscription needed

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Gold

Arizona Association of Gold Prospectors: **arizonagoldprospectors.org**

Denver Gold Group: **denvergold.org**

European Precious Metals Federation: **epmf.be**

Gold Prospectors Association of America (GPAA): **goldprospectors.org**

Gold Prospectors Association of Tucson: **gpatucson.org**

LBMA: London Bullion Market Association: **lbma.org.uk**

Nova Scotia Prospectors Association: **prospectors.ns.ca**

Ontario Prospectors Association: **ontarioprospectors.com**

Portland Gold Prospectors: **portlandgoldprospectors.org**
Portland, Oregon

Prospectors' and Miners' Association of Victoria: **pmav.org.au**
Victoria, Australia

World Gold Council: **gold.org**

Iron

Association for Iron & Steel Technology: **aist.org**

American Iron and Steel Institute (AISI): **steel.org**

Brazilian Steel Institute: **acobrasil.org.br/site2015/eng**

Euroalliages: Association of European Ferro-Alloy Producers: **euroalliages.com**

Iron Mining Association of Minnesota: **taconite.org**

Lead

India Lead Zinc Development Association (ILZDA): **ilzda.com**

International Lead and Zinc Study Group (ILZSG): **ilzsg.org**
Formed by the United Nations to ensure transparency in the worldwide markets for lead and zinc

International Lead Association (ILA): **ila-lead.org**
International Lead Management Center tools [in English and Spanish]

International Lead Zinc Research Organization (ILZRO): **ilzro.org**
Manages environment, health, sustainable development, and product-related research programs on behalf of the International Lead Association (ILA) and the International Zinc Association (IZA)

Iran Lead & Zinc Industries & Mines Association (ILZIMA): **ilzima.ir/en**

Lead Sheet Training Academy: **leadsheet.co.uk**
U.K.-based association

Nickel

International Nickel Study Group (INSG): **insg.org**

Nickel Institute: **nickelinstitute.org**

Nickel Producers Environmental Research Association (NiPERA): **nickelinstitute.org/science**

Oil Sand

Alberta Chamber of Resources: **acr-alberta.com**

Canadian Association of Petroleum Producers: **capp.ca**

Canadian Heavy Oil Association: **choa.ab.ca**

Canadian Society of Petroleum Geologists (CSPG): **cspg.org**

CIM/ICM [Canadian Institute of Mining and Metallurgy] Oil Sands: **branches.cim.org/oil-sands**

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In Situ Oil Sands Alliance (IOSA): **iosa.ca**

Orphan Well Association: **orphanwell.ca**

Society of Petroleum Engineers (SPE): **spe.org/en**

Silver

European Precious Metals Federation: **epmf.be**

Precious Metals Association of North America: **silverusersassociation.org**
Formerly the Silver Users Association

Silver Institute: **silverinstitute.org**

Silver Research Consortium: **ilzro.org**
Program of the International Lead Zinc Research Organization

Titanium

Association Française du Titane [The French Titanium Association]: **titane.asso.fr** [in French and English]

International Titanium Association (ITA): **titanium.org**

Uranium

Canadian Nuclear Association (CNA): **cna.ca**

IAEA: International Atomic Energy Agency: **iaea.org**

Nuclear Industry Association (NIA): **niauk.org**

World Nuclear Association: **world-nuclear.org**

Zinc

American Galvanizers Association: **galvanizeit.org**

Asociación Latinoamericana de Zinc (Zinc Latin America): **latiza.zinc.org**

Associazione Italiana Zincatura: **aiz.it** [in Italian]

Galvanizers Association of Australia: **gaa.com.au**

Galvanizers Association: **galvanizing.org.uk**

Galvazinc Association: **old.galvazincparis.fr** [in French]

India Lead Zinc Development Association (ILZDA): **ilzda.com**

Initiative Zink: **initiative-zink.de** [in German]

International Lead and Zinc Study Group (ILZSG): **ilzsg.org**
Formed by the UN to ensure transparency in the markets for lead and zinc worldwide

International Lead Zinc Research Organization (ILZRO): **ilzro.org**
Manages environment, health, sustainable development, and product-related research programs on behalf of the International Lead Association (ILA) and the International Zinc Association (IZA)

International Zinc Association (IZA): **zincworld.org**

Iran Lead & Zinc Industries & Mines Association (ILZIMA): **ilzima.ir/en**

North American Die Casting Association (NADCA): **diecasting.org**

Vereinigung Schweizerischer Verzinkereien/Union Suisse Des Usines de Zingage: **verzinkereien.ch**
[in German and French]

Other

International Molybdenum Association (IMOA): **imoa.info**

International Platinum Group Metals Association (IPA): **ipa-news.com**

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U.S. STATE MINING ASSOCIATIONS

Alaska Miners Association (AMA): **alaskaminers.org**
Arizona Mining Association: **azmining.com**
CALCIMA: California Construction and Industrial Materials Association: **calcima.org**
California Cast Metals Association (CCMA): **foundryccma.weebly.com**
Colorado Mining Association (CMA): **coloradomining.org**
Eastern Oregon Mining Association (EOMA): **h2oaccess.com**
Georgia Mining Association (GMA): **georgiamining.org**
Idaho Mining Association: **mineidaho.com**
Kentucky Mining Institute: **miningusa.com/kmi**
Mining Minnesota: **miningminnesota.com**
Montana Mining Association: **montanamining.org**
Nevada Mining Association: **nevadamining.org**
New Mexico Mining Association: **nmmining.org**
Northern Utah Prospector's Association (NUPA): **nupagoldclub.blogspot.com**
Tennessee Mining Association: **tnmining.com**
Texas Aggregates & Concrete Association: **tx-taca.org**
Texas Mining & Reclamation Association (TMRA): **tmra.com**
Utah Mining Association: **utahmining.org**
Washington Prospectors Mining Association: **washingtonprospectors.org**
Wyoming Mining Association: **wyomingmining.org**
Wyoming Prospectors Association: **wpagold.com**

MINING MUSEUMS AND TOURS

One of the offerings of the National Mining Association is its list of museums and tours: "There are opportunities all over the country to visit mines and mining museums and learn more about U.S. mining" (**nma.org**). Here is a sampling of what is offered:

Alaska

Usibelli Coal Mine
usibelli.com
Located near Healy along the Nenana River, the Usibelli mine feeds six power plants in interior Alaska and ships 750,000 tons of coal to Seward via the Alaska Railroad.

Arizona

Asarco Mineral Discovery Center
asarco.com/about-us/our-locations/asarco-mineral-discovery-center
Located in Sahuarita, the discovery center offers open pit copper mine and mill tours; exhibits on geology, mining, and uses of minerals; and historic and present-day mining equipment.

Gem & Mineral Museum
uamineralmuseum.com
Located at the University of Arizona in Tucson, the University Mineral Museum "is dedicated to providing public education and the preservation of minerals and meteorites while also serving the research needs of professionals, students, and collectors. The collection is worldwide in scope, but with specific emphasis on minerals from Arizona and Mexico."

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Queen Mine Historic Mining Tours
queenminetour.com

Located in Bisbee, the tour takes visitors to one of the oldest copper mines in Arizona. Former miners show how turn-of-the-century mines were operated. The tour covers the leaching plant, mine shafts, dumps, and the pit perimeter.

Canada

Estevan Energy Tour
tourismsaskatchewan.com/listings/2707/estevan-energy-tour

Located in Saskatchewan, the tour covers the Estevan strip-mine site, Shand coal-fired power station, and Shand greenhouse, which uses excess heat from the power station.

Colorado

National Mining Hall of Fame and Museum
mininghalloffame.org

Located in Leadville, the "Smithsonian of the Rockies houses 25,000 square feet of interactive and informative exhibits." The National Mining Hall of Fame is chartered by Congress, with more than 244 inductees.

Newmont GoldCorp
newmont.com/operations-and-projects/north-america/cripple-creek-and-victor-us/tours/default.aspx

Located near the town of Victor, public tours of modern mining operations are offered in partnership with the Victor Lowell Thomas Museum (in Victor, Colorado). Visitors tour the Cripple Creek & Victor gold mine.

Old One Hundred Mine
minetour.com

Located in Silverton, the Old Hundred Gold Mine Tour is a guided tour into Galena Mountain. The tour offers an electric mine train ride with a view of 1930's era operating mining equipment. Children on tour can pan free for silver and gold in the Old Hundred sluice box.

Western Museum of Mining & Industry
wmmi.org

Located in Colorado Springs, the museum collects, restores, and displays mining artifacts. Steam engines, drills, pumps, and other mining equipment are shown operating. There are both indoor and outdoor exhibits.

Idaho

Crystal Gold Mine
goldmine-idaho.com

Located in Kellog, Crystal Gold Mine offers a tour of an authentic 1880's underground gold mine. Crystal and stalactite formations and mining artifacts are exhibited.

Sierra Silver Mine Underground Tour
silverminetour.org

Located in Wallace, Sierra Silver Mine tour takes visitors underground at the mine site. An experienced miner guides visitors through exhibits and describes the methods and techniques used in hard-rock silver mining.

Wallace District Mining Museum
visitnorthidaho.com/activity/wallace-district-mining-museum

Located in Wallace, which is the largest silver-producing mine district in the world, the museum displays artifacts and interactive displays.

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Illinois

Museum of Science and Industry
msichicago.org
msichicago.org/explore/whats-here/exhibits/coal-mine
Located in the city of Chicago, the museum offers a one-of-a-kind coal mine exhibit with a walk down the Old Ben No. 17 coal mine shaft and a "working coal mine" experience.

Kansas

Strataca: Kansas Underground Salt Museum
underkansas.org
Located in Hutchinson, the museum offers a "subterranean learning adventure 650 feet below the earth's surface in salt deposits."

Kentucky

Coal Miners' Museum
kentuckytourism.com/coal-miners-museum
Located in Van Lear, the museum is the former headquarters of the Miller's Creek Division of the Consolidation Coal Company. Exhibits include a collection of mining tools and a company doctor's office.

Kentucky Coal Mining Museum
kingdomcome.org/the-kentucky-coal-mining-museum
Located in central Benham, the four-story museum houses artifacts, photographs, and machinery, all exhibits on "the history of mining and life of a coal miner."

Kentucky Coal Museum
kycoalmuseum.southeast.kctcs.edu
Located in Benham, the museum houses a "comprehensive collection of mining memorabilia … [which] uniquely portrays life in a coal camp."

Portal 31
portal31.org
Located in Lynch (Harlan County), Portal 31 is Kentucky's first exhibition coal mine.

Michigan

A.E. Seaman Mineral Museum of Michigan Tech
museum.mtu.edu
Located in Houghton, the museum is famous for its world-class collection of Lake Superior copper and iron district minerals. The museum houses 60,000 specimens from around the world.

Michigan Iron Industry Museum
michigan.gov/IronIndustryMuseum
Located in Negaunee, the museum highlights the state's iron mining industry with interpretive exhibits of life in mining communities, a timeline of the evolution of mining, and examples of early mining tools.

Minnesota

Hill Annex Mine State Park
dnr.state.mn.us/state_parks/park.html?id=spk00176#homepage
Located in Calumet, the park offers an open pit tour of the iron ore mine and facilities, including a museum.

Lake Vermilion-Soudan Underground Mine State Park
dnr.state.mn.us/state_parks/park.html?id=spk00285#homepage
Located in Soudan, visitors can take an underground tour of Minnesota's first iron ore mine, 2,341 feet below the surface, and then about a mile to the stope area.

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Minnesota Museum of Mining
mnmuseumofmining.org

Located in West Chisolm, the exhibits include historic mining equipment, a simulated underground mine, and mining history displays.

Museum of the Iron Range
mndiscoverycenter.com

Located in West Chisolm, in the Minnesota Discovery Center, the museum is perched on open pit iron mine. Iron mining history is related through lifelike displays and audio, including living history tapes.

Missouri

Joplin History and Mineral Museum
joplin-museum.org

Located in Joplin, the Everett J. Ritchie Tri-State Mineral Museum is housed in the Joplin museum complex. The Ritchie Museum houses an extensive mineral collection, and has exhibits on mining tools and the evolution of mining, as well as a mineral-in-product exhibit.

Montana

Bannack State Park
bannack.org

Located in Dillon, Bannock State Park offers the best preserved ghost town west of the Mississippi. The park is the site of the first major gold rush in Montana. More than 60 historic structures still stand with free daily tours and gold panning.

World Museum of Mining
miningmuseum.org

Located in Butte, the World Museum of Mining is one of very few mining museums on the site of an underground metal mine. The museum houses an extensive collection of artifacts in the outdoor mine yard, including the original 100-foot-high headframe of Orphan Girl mine. There are period artifacts in Hellroaring Gulch, a recreation of an 1890's mining town with 50 buildings and 66 major exhibits of mining equipment ranging from the 1860s to the 1970s.

North Dakota

Beulah Energy & Mine Tours
visitbeulah.com/play/explore

Known as the "energy capital of North Dakota," Beulah has tours of local coals mines and power plants, including the Coteau Properties, Great Plains Synfuels, and Basin Electric Power Cooperative.

Center Mine
bnicoal.com/contact/tours

Located in Center and owned and operated by BNI Coal, the mine supplies lignite to the Milton R. Young Station.

Falkirk Mine
ndtourism.com/underwood/wind-farms-mines-power-plants/falkirk-mine

Located in Underwood and operated by the Falkirk Mining Company (a subsidiary of the North American Coal Corporation), the mine supplies lignite to the Coal Creek Station.

Freedom Mine
ndtourism.com/beulah/wind-farms-mines-power-plants/freedom-mine

Located in Beulah, the mine supplies lignite to the Great Plains Synfuels Plant, Antelope Valley Station, Leland Olds Station, and Stanton Station. It is the largest lignite mine in the United States, operating two 120-cubic-yard draglines and a fleet of 200-ton overburden trucks. Tours are by appointment only.

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Nevada

McCaw School of Mines
mccawmines.org

This park is located in Henderson, the "Jewel of the Desert" is a simulated mine tunnel with exhibits of historical mining, mining in Nevada, geology and minerals, and modern-day mining. It is specifically geared toward elementary school students.

Tonopah Historic Mining Park
tonopahminingpark.com

This park is located "on the site of the original mining claims that started the rush to Tonopah, making it queen of the silver camps." Covering more than 100 acres, the park has portions of four of the original major mining companies. Exhibits include preserved and restored equipment and buildings, with video presentations and a self-guided tour.

W.M. Keck Earth Science and Mineral Engineering Museum
unr.edu/keck

Located in the Mackay School of Earth Sciences & Engineering at the University of Nevada, Reno, the museum is Nevada's old geology museum (founded in 1908) with a collection of minerals, ores, fossil specimens, photographs, and mining-related relics.

New Jersey

Sterling Hill Mining Museum
sterlinghillminingmuseum.org

Located in Ogdensburg, an extensive collection of mining memorabilia is housed in two museums. The tour includes an underground portion of the mine, which was considered one of the world's richest zinc ore deposits; mining equipment, a drilling and blasting demonstration, and displays of fluorescent minerals are also included.

New Mexico

Mineral Museum
geoinfo.nmt.edu/museum

Located at the New Mexico Tech Campus in Socorro, the mineral museum is supported by the New Mexico Bureau of Geology & Mineral Resources.

New Mexico Mining Museum
grants.org/museums-galleries.aspx

Located in Grants, the museum is the only uranium mining museum in the world.

New York

Garnet Mine Tours
garnetminetours.com

Located in North River, the Barton Mines, established in 1878, holds one of the world's largest garnet deposits. The tour includes the history and geology of the mine, exploration of the No. 1 pit, with demonstrations of how to find garnet mineral specimens.

Pennsylvania

Earth and Mineral Sciences (EMS) Museum & Art Gallery
museum.ems.psu.edu

Located in State College (University Park), the exhibits include "collections of rocks, minerals, and fossils totaling more than 22,000 specimens … collections of glasses, ceramics, metals, plastics, synthetic materials, old mining and scientific equipment, and archaeological artifacts," as well as paintings and sculpture depicting mining and related industries.

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Eureka Coal Heritage Foundation
echf.org

Located near Windber, the non-profit Eureka Coal Heritage Foundation (ECHF) seeks to commemorate the role coal played in the U.S. industrial heritage by "preserving and [restoring] local historical facilities, commemorating the lifestyle and sacrifices of coal miners and their families, and educating the public of the role coal has played in both local and national economies."

Washington, D.C.

Janet Annenberg Hooker Hall of Geology, Gems, and Minerals
naturalhistory.si.edu/exhibits

Located in the Smithsonian National Museum of Natural History in Washington, D.C., the exhibits included in the Janet Annenberg Hooker Hall of Geology, Gems, and Minerals range from the Hope diamond to the walk-through mine.

West Virginia

Beckley Exhibition Coal Mine and Youth Museum
beckley.org/general-information-coal-mine

Located in Beckley, the exhibition includes an underground mine tour, showing a low-seam coal mine from its earliest manual stages to modern mechanized operation, and surface buildings, with a recreated coal camp.

Coal Heritage Museum
wvcoalmuseum.org

Located in Madison (Boone County), the Bituminous Coal Heritage Foundation Museum was created by the Boone County Development Corporation and chartered by the State of West Virginia. The downtown coal museum showcases the history of coal and its people.

National Coal Heritage Area and Coal Heritage Trail
coalheritage.org

The National Coal Heritage Area "encompasses 13 counties" in southern West Virginia. The 187-mile Coal Heritage Trail is located within the National Coal Heritage Area. From 1950–2000, unemployed miners and their families left West Virginia, and the remnants of the structures that housed and employed the miners, such as company stores, company offices, and railroad yards still exist with the Heritage Area.

Scotts Run Museum & Trail
scottsrunmuseumandtrail.org

Located in Osage, the musem "includes biographies of coal operators, profiles of ethnic groups, statistics on mines operating in Scotts Run, histories of schools, churches and communities, and information on mine disasters and glass workers living in the area."

West Virginia Mine Wars Museum
wvminewars.com

Located in Matewan, the museum "preserves and interprets artifacts and historical records of the local communities affected by the West Virginia Mine Wars, exploring historical events from multiple perspectives through the lives of ordinary people."

West Virginia University Royce J. and Caroline B. Watts Museum
wattsmuseum.wvu.edu

Located in Morgantown, the Watts museum is dedicated to preserving and promoting the social, cultural, and technological history of West Virginia's coal and petroleum industries through the collection, preservation, research, and exhibition of historical objects and archival materials. The museum is housed in the Benjamin M. Statler College of Engineering and Mineral Resources at West Virginia University.

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Wisconsin

Mining & Rollo Jamison Museums
mining.jamison.museum

Located in Platteville, museum exhibits trace the development of lead and zinc mining in the Upper Mississippi Valley through models, dioramas, artifacts, and photographs.

Wyoming

Black Thunder Mine
archcoal.com/news/video.aspx?video=0

Located in the Powder River Basin, "Black Thunder is one of the largest single coal mining complexes in the world" and is currently owned by Arch Coal. Virtual tours are available via the web link provided.

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